CHAPTER 3

FUZZY LOGIC MODELLING

3.1 INTRODUCTION

Fuzzy logic provides a method to formalize reasoning when dealing with vague terms. Traditional computing requires finite precision, which is not always possible in real world scenarios. Not every decision is either true or false, or as with Boolean logic is either 0 or 1. Fuzzy logic allows for membership functions, or degrees of truthfulness and falsehoods, or as with Boolean logic, not only 0 and 1 but all the numbers that fall in between them.\(^8^1\)

The derivation of mathematical models can effectively describe real-world problems, but it is most of the time an impossible task due to the complexity and inherent ambiguity of characteristics the problem may possess.\(^8^2\) As the founder of the theory of fuzzy sets, Zadeh\(^8^3\), puts it, “… as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics”.

Fuzzy reasoning is based on the theory of fuzzy sets, introduced in 1965, and it encompasses artificial intelligence, information processing and theories from logic to pure and applied mathematics, including graph theory, topology and optimization. In his introductory paper, Zadeh noted that “…the
notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability, particularly in the fields of pattern classification and information processing.

Soon, the theory of fuzzy sets was more decisively established as a new approach to complex systems theory and decision processes. Since 1990, fuzzy logic has found applications in decision-making problems in diverse areas such as production, finance, marketing, microcontroller-based systems in home appliances, and large-scale process control systems (Sugeno and Yasukawa, Karr and Gentry, Lee). For systems involving nonlinearities and lack of a reliable analytical model, fuzzy logic control has emerged as the most promising of approaches.

The main merit in fuzzy logic techniques over conventional approaches lies in their capability of incorporating a prior qualitative knowledge and expertise about system behavior and dynamics in solving complex, nonlinear problems. This renders fuzzy logic systems almost indispensible for systems whose representation with mathematical models is poor or inadequate.

In addition, Wang stated that any continuous nonlinear function can be approximated as exactly as needed with a finite set of fuzzy variables, values and rules. Therefore, by applying appropriate design procedures, it is always possible to design a fuzzy controller for any nonlinear system.

Basically, the fuzzy inference system consists of three components, namely rule base, data base and reasoning mechanism. A rule base contains a selection of fuzzy rules; a data base defines the membership functions; and a
reasoning mechanism performs the fuzzy reasoning based on the rules and facts to derive a reasonable output or conclusion.  

A practical application of Fuzzy logic system is to calculate the height of persons. A person’s height membership function graph is shown with linguistic values of the degree of membership as very tall, tall, average, short and very short being replaced by 0.85, 0.65, 0.50, 0.45 and 0.15.

![Height Membership Functions](image)

In traditional logic, statements can be either true or false, and sets can either contain an element or not. These logic values and set memberships are typically represented with number 1 and 0. Fuzzy logic generalizes traditional logic by allowing statements to be somewhat true, partially true, etc. Likewise, sets can have full members, partial members, and so on. For example, a person whose height is 5’ 9” might be assigned a membership of 0.6 in the fuzzy set “tall people”. The statement “Joe is tall” is 60% true if Joe is 5’9”. Fuzzy logic is a set of “if-then” statements based on combining fuzzy sets.
3.2 FUZZY LOGIC SYSTEM

A Fuzzy Logic System (FLS) performs a nonlinear mapping of an input dataset to a scalar output data. It consists of four main parts: fuzzifier, rules, inference engine and defuzzifier. The architecture of an FLS is shown in Figure 3.1.

![Figure 3.1 Architecture of FLS](image)

The steps involved in developing an FLS are as follows:

- **Fuzzifier**: Converts the crisp input to a linguistic variable using the membership functions stored in the fuzzy knowledge base. Fuzzy linguistic variables are used to represent qualities spanning a particular spectrum; e.g., temperature spans are Freezing, Cool, Warm, or Hot.

- **In-Ference Engine**: Using If-Then type fuzzy rules converts the fuzzy input to the fuzzy output.
• **Defuzzifier:** Converts the fuzzy output of the inference engine to crisp using membership functions analogous to the ones used by the fuzzifier.

• **Fuzzy Knowledge Base:** Information storage for Linguistic variables definitions and Fuzzy rules.

(a) **Linguistic Variables**

Linguistic variables are input or output variables of the system whose values are expressed in a natural language (words or sentences) instead of numerical values. A linguistic variable is generally decomposed into a set of linguistic terms.

(b) **Membership Functions**

A membership function is used to quantify the linguistic term. Membership functions are used in the fuzzification and defuzzification steps to map the non-fuzzy input variables to fuzzy linguistic terms and vice versa. There are different forms of membership functions, such as triangular, trapezoidal, piecewise linear, Gaussian or singleton. The choice of membership functions is context dependent and based on user experience. In Figures 3.2 to 3.5 the X axis represents crisp values and Y axis represents membership functions.
Figure 3.2 Gaussian membership function

Figure 3.3 Triangular membership function
3.3 FUZZY RULES

The input to a fuzzy system is called crisp input since it contains precise information about the parameters. The fuzzifier converts this precise quantity to an imprecise quantity like ‘large’, ‘medium’ or ‘high’ with a
degree of belongingness. Typically the value ranges from 0 to 1. A fuzzy rule is a simple IF-THEN rule with a condition and a conclusion. The knowledge base is the main part of a fuzzy system where both rule base and data base are jointly referred. The data base defines the membership functions of the fuzzy sets used in the fuzzy rules, while the rule base contains a number of fuzzy IF-THEN rules.

3.4 FUZZY SET OPERATIONS

The evaluation of fuzzy rules is performed using fuzzy set operations. The commonly used operations for OR and AND operators are max and min, respectively. After evaluation, the results of different rules are combined to obtain a final result. This step is called inference. There are several methods to combine the individual rules. The inference system handles the way in which the rules are combined.

3.5 DEFUZZIFICATION

After the inference step, the result we get is a fuzzy value. This value should be defuzzified to obtain the final crisp output. Defuzzification is performed according to the membership function of the output variable. Several algorithms are available for defuzzification. The most commonly used algorithms are centre of gravity, centre of gravity for singletons, left most maximum and right most maximum.

Two fuzzy inference systems are well known: Mamdani fuzzy model and Sugeno fuzzy model. The Mamdani fuzzy model is based on the collection of IF-THEN rules with both fuzzy antecedent and consequent predicts. Because of ease of use, the Mamdani model is most commonly used for solving many real-world problems. The Mamdani fuzzy inference is also commonly utilized for modeling the EDM process.