CHAPTER : 5

OPTIMAL ORDERING POLICIES UNDER VARIOUS MARKETING CRITERIA WITH PRICE CHANGE ANTICIPATION
OPTIMAL ORDERING POLICIES UNDER VARIOUS MARKETING CRITERIA WITH PRICE CHANGE ANTICIPATION

5.0 INTRODUCTION:

In this chapter, an inventory model is developed when vendor announces fixed price increase in the unit cost from some future date, where actual discounting for large purchases is available. The effect of price increase in unit cost on optimum purchase quantity, selling price and net profit has been studied with a suitable numerical illustration.

5.1 : ASSUMPTIONS AND NOTATIONS :

The mathematical model is developed with the following assumptions:

(1) The time $T$ between two successive orders is constant.

(2) At the beginning of each period, a quantity $Q$ is replenished. $Q$ is a decision variable.

(3) Shortages are not allowed. Lead time is zero.

(4) The unit cost $C(Q)$ is a function of purchase quantity $Q$, which includes standard cost $h_1$ and discount cost $h_2$. $C(Q)$ is assumed to have the form

$$C(Q) = h_1 - h_2 Q$$

where $h_2 < h_1$ such that $C(Q) > 0$, for all $Q$. And that satisfies

$$\frac{\delta C(Q)}{\delta Q} = -h_2 < 0 \quad \text{for all } Q.$$
The vendor announces unit cost to be $C(Q) + \lambda$, where $\lambda$ is known constant for some specified time. i.e. the unit purchased just will cost $C(Q)$ per unit whereas after specified time retailer will have to pay $C(Q) + \lambda$ per unit.

(5) Any unit unsold at the end of the period has no economic value.

(6) The demand $R(P)$ is a function of selling price $P$ per unit which is given by

$$R(P) = a - bP$$

where $a$ and $b$ are positive constants and $a \gg b$. $P$ is the decision variable.

(6) The inventory holding charge $i$ per rupee unit time and replenishment cost $A$ per order are known and constant during the period under consideration.

The notations for the mathematical model are:

- $GR(P, Q)$ = Gross Revenue
- $TC(P, Q)$ = Total cost of an inventory system during the period.
- $NP_i(P, Q)$ = Net profit when unit cost is $C(Q)$.
- $NP_{n}(P, Q)$ = Net profit when unit cost is $C(Q) + \lambda$.
- $P_{01}$ = Optimal selling price when unit cost is $C(Q)$.
- $P_{02}$ = Optimal selling price when unit cost is $C(Q) + \lambda$.
- $Q_{01}$ = Optimal procurement quantity when unit cost is $C(Q)$.
- $Q_{02}$ = Optimal procurement quantity when unit cost is $C(Q) + \lambda$.

5.2 MATHEMATICAL MODEL:

Under the above assumptions and notations, in general

$$NP_{n}(P, Q) = GR(P, Q) - TC(P, Q)$$
For the system under consideration,

\[ GR(P, Q) = (P - C(Q)) R(P) \]

and

\[ TC(P, Q) = \frac{C(Q) i Q}{2} + \frac{AR(P)}{Q} \]

**CRITERION-I: NET PROFIT**

Net profit \( NP_1(P, Q) \), when unit cost is \( C(Q) \), is

\[
NP_1(P, Q) = \left( P - (h_1 - h_2 Q) - \frac{A}{Q}(a - bP) - \frac{(h_1 - h_2 Q) i Q}{2} \right)
\]

(5.1)

The optimum values of selling price \( P = P_{o1} \) when unit cost is \( C(Q) \), can be obtained by solving

\[
\frac{\delta NP_1(P, Q)}{\delta P} = 0
\]

i.e.

\[
P = \left\{ \frac{a}{2} + h_1 - h_2 Q + \frac{A}{Q} \right\}
\]

(5.2)

and hence, net profit \( NP_1(P, Q) \) reduces to

\[
NP_1(Q) = \frac{1}{4b} \left( a - b(h_1 - h_2 Q) - \frac{Ab}{Q} \right)^2 - \frac{(h_1 - h_2 Q) i Q}{2}
\]

Similarly, optimum value of \( Q = Q_{o1} \) can be obtained by solving

\[
\frac{\delta NP_1(Q)}{\delta Q} = 0
\]
i.e. \[ h_2(bh_2 + 2i)Q^4 - (h_1h_2b - ah_2 + ih_1)Q^3 + A(a - bh_1)Q - A^2b = 0 \]  
(5.3)

We know that Net Profit \( NP_1(P, Q) \) obtained by solving (5.2) and (5.3) at \( P = P_0 \) and \( Q = Q_{0i} \) is maximum iff Hessian matrix \( H = \begin{bmatrix} L & N \\ N & M \end{bmatrix} \) is negative definite.

i.e. \( LM - N^2 < 0, \ L < 0 \)  
(5.4)

where

\[ L = \frac{\delta^2 NP_1(P, Q)}{\delta P^2} \]

\[ M = \frac{\delta^3 NP_1(P, Q)}{\delta Q^2} \]

\[ N = \frac{\delta^2 NP_1(P, Q)}{\delta P \delta Q} \]

Now assume that supplier announces fixed price increase (say Rs. \( \lambda \)) per unit from some future date, then net profit \( NP_n(P, Q) \) for the system is

\[
NP_n(P, Q) = \left( P - (h_1 - h_2Q + \lambda) - \frac{A}{Q} (a - bP) - \frac{(h_1 + \lambda - h_2Q)i}{2} \right) Q
\]

(5.5)
The optimum values of selling price \( P = P_0 \) when unit cost is \( C(Q) + \lambda \), can be obtained by solving \( \frac{\delta \text{NP}_E(P,Q)}{\delta P} = 0 \)

i.e. \( P = \frac{1}{2} \left[ \frac{a}{b} + h_1 + \lambda - h_2 Q + \frac{A}{Q} \right] \)

(5.6)

and hence, net profit \( \text{NP}_E(P,Q) \) reduces to

\[
\text{NP}_E(Q) = \frac{1}{4b} \left( a - b(h_1 + \lambda - h_2 Q) - \frac{A}{Q} \right)^2 - \frac{(h_1 + \lambda - h_2 Q)Q}{2}
\]

Similarly, optimum value of \( Q = Q_0 \) can be obtained by solving \( \frac{\delta \text{NP}_E(Q)}{\delta Q} = 0 \)

i.e. \( h_2(bh_2 + 2\lambda)Q^2 - (h_1 h_2 b + bh_2 \lambda - ah_2 + i(h_1 + \lambda))Q + A(a - bh_1 - b \lambda) - A^2 b = 0 \)

(5.7)

The values obtained by solving (5.6) and (5.7) maximize \( \text{NP}_E(P,Q) \) provided equation (5.4) holds for \( \text{NP}_E \).

**CRITERION-II : RESIDUAL INCOME**

Residual Income \( RI(P,Q) \), when unit cost is \( C(Q) \), is

\[
RI_I(P,Q) = \text{NP}_I(P,Q) - \frac{1}{2} QC(Q)I
\]

\[
= \left( P - (h_1 - h_2 Q) - \frac{A}{Q}(a - bP) - \frac{(h_1 - h_2 Q)Q}{2}(i + I) \right)
\]

(5.8)
Evidently selling price \( P = P_{0i} \), when unit cost is \( C(Q) \), is same as (5.2) therefore, substituting the value of \( P \) in \( RI_i(P, Q) \) and then to obtain the optimum purchase quantity

\[
Q = Q_{0i}, \quad \text{solving } \frac{\delta RI_i(P, Q)}{\delta Q} = 0
\]

i.e.

\[
h_2(bh_2 + 2(i + l_e))Q^4 - (h_1h_2b - ah_2 + (i + l_e)h_1)Q^3 + A(a - bh_1)Q - A^2b = 0
\]

(5.9)

We know that Residual Income \( RI_i(P, Q) \) obtained by solving (5.2) and (5.9) at \( P = P_{0i} \) and \( Q = Q_{0i} \) is maximum provided equation (5.4) holds for \( RI_i(P, Q) \).

Now assume that supplier announces fixed price increase (say Rs. \( \lambda \)) per unit from some future date, then Residual Income \( RI_{ii}(P, Q) \) for the system is

\[
RI_{ii}(P, Q) = NP_{ii}(P, Q) - \frac{1}{2} Q(C(Q) + \lambda)I_e
\]

(5.10)

The optimum values of selling price \( P = P_{02} \) when unit cost is \( C(Q) + \lambda \), can be obtained by solving

\[
\frac{\delta RI_{ii}(P, Q)}{\delta P} = 0
\]

i.e.

\[
P = \frac{1}{2} \left[ \frac{a}{b} + h_1 + \lambda - h_1Q + \frac{A}{Q} \right]
\]

(5.11)

and

optimum value of \( Q = Q_{02} \) can be obtained by solving

\[
\frac{\delta RI_{ii}(P, Q)}{\delta Q} = 0
\]
The values obtained by solving (5.11) and (5.12) maximize \( R_{\text{In}}(P,Q) \) provided equation (5.4) holds for \( R_{\text{In}} \).

**CRITERION-III: RETURN ON INVESTMENT**

Return On Investment \( \text{ROI}_i(P,Q) \), when unit cost is \( C(Q) \), is

\[
\text{ROI}_i(P,Q) = \frac{NP_i(P,Q)}{2QC(Q)}
\]

Evidently selling price \( P = P_0i \), when unit cost is \( C(Q) \), is same as (5.2) therefore, substituting the value of \( P \) in \( \text{ROI}_i(P,Q) \) and then to obtain the optimum purchase quantity

\[
Q = Q_{0i}, \text{ solving } \frac{\delta \text{ROI}_i(P,Q)}{\delta Q} = 0
\]

i.e.

\[
bh_i^2(2a - bh_i)Q^4 + 2h_i(3a^2 + b^2h_i - 2abh_i - 2b^2 Ah_i)Q^3
\]
\[
+ (8b^2 Ah_i h_i - 6abAh_i - a^2h_i - b^3h_i^3 + 2abh_i^2)Q^2
\]
\[
+ 4Ab(ah_i + Abh_i - bh_i^3)Q - 3A^2b^2h_i = 0
\]

(5.13)
We know that Return On Investment $ROI_i(P, Q)$ obtained by solving (5.2) and (5.13) at $P = P_0$ and $Q = Q_0$ is maximum provided equation (5.4) holds for $ROI_i(P, Q)$.

Now assume that supplier announces fixed price increase (say Rs. $\lambda$) per unit from some future date, then Return On Investment $ROI_n(P, Q)$ for the system is

$$ROI_n(P, Q) = \frac{NP_n(P, Q)}{Q(C(Q) + \lambda)}$$

The optimum values of selling price $P = P_{02}$ when unit cost is $C(Q) + \lambda$, can be obtained by solving

$$\frac{\delta ROI_n(P, Q)}{\delta P} = 0$$

i.e.

$$P = \frac{1}{2} \left\{ \frac{a}{b} + h_1 + \lambda + h_2 Q + \frac{A}{Q} \right\}$$

and optimum value of $Q = Q_{02}$ can be obtained by solving

$$\frac{\delta ROI_n(P, Q)}{\delta Q} = 0$$

i.e.

$$bh_2^2(2a-b(h_1+\lambda)) Q^4 + 2h_2(a^2+b^2 h_1(h_1 + 2 \lambda) + b^2 \lambda^2 - 2ab(h_1 + \lambda) - 2b^2 A h_2) Q^3$$

$$+ \left( (8b^3 A(h_1 + \lambda)h_2 - 6abA h_2 - (a^2 + b^2 h_1^2)(h_1 + \lambda) ight) Q^2$$

$$- 2b^3 h_1 \lambda (h_1 + \lambda) - b^3 \lambda^2 (h_1 + \lambda) + 2ab(h_1 + \lambda) ) \right\} Q^2$$

$$+ \left( 4 Aba(h_1 + \lambda) - 2 A b^2 (h_1 + \lambda)(2 h_1 + \lambda) + 4 A^2 b^2 h_2 - 2 A b^2 \lambda \right) Q$$

$$- 3 A^2 b^2 (h_1 + \lambda) = 0$$

(5.15)
The values obtained by solving (5.14) and (5.15) maximize $\text{ROI}_n(P, Q)$ provided equation (5.4) holds for $\text{ROI}_n$.

5.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

The effect of price increase of a unit on selling price and procurement quantity is studied for three marketing policies in following numerical illustration.

Consider an inventory system with following hypothetical parameters,

- $A = \text{Rs. 250 per order}$
- $b = 10$
- $h_1 = 50$
- $h_2 = 0.1$
- $i = \text{Rs. 0.2 per rupee per unit time}$
- $I_s = 0.12 \text{ per rupee per unit time}$
- $\lambda = \text{Rs. 10.00, 20.00, 30.00}$
- $a = 1000, 1100, 1200 \text{ units}$

Now the results of various increase in $\lambda$-value and $a$-value for the marketing policies NP, RI, and ROI with respect to above data are exhibited below in tables 5.1, 5.2 and 5.3 respectively.
**TABLE 5.1**

NP Policy

<table>
<thead>
<tr>
<th>λ \ a</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>P</td>
<td>66.38</td>
<td>72.58</td>
<td>78.31</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>18</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>NP</td>
<td>10.00</td>
<td>589.81</td>
<td>886.81</td>
</tr>
<tr>
<td>1100</td>
<td>P</td>
<td>64.64</td>
<td>71.50</td>
<td>76.42</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>21</td>
<td>17</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td>NP</td>
<td>278.48</td>
<td>1264.00</td>
<td>1690.37</td>
</tr>
<tr>
<td>1200</td>
<td>P</td>
<td>62.25</td>
<td>70.62</td>
<td>77.20</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>25</td>
<td>19</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>NP</td>
<td>314.37</td>
<td>2066.00</td>
<td>2993.46</td>
</tr>
</tbody>
</table>

Observations:

(i) As λ increases there is increase in P and NP but decrease in Q.

(ii) As a increases there is increase in Q and NP but decrease in F.
### TABLE 5.2
RI Policy

<table>
<thead>
<tr>
<th>λ</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>P</td>
<td>82.25</td>
<td>84.00</td>
<td>86.22</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>20</td>
<td>17</td>
<td>14</td>
</tr>
<tr>
<td></td>
<td>RI</td>
<td>2984</td>
<td>2414</td>
<td>1774</td>
</tr>
<tr>
<td>1100</td>
<td>P</td>
<td>85.51</td>
<td>87.40</td>
<td>89.50</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>26</td>
<td>21</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>RI</td>
<td>5784</td>
<td>4928</td>
<td>4053</td>
</tr>
<tr>
<td>1200</td>
<td>P</td>
<td>89.48</td>
<td>91.00</td>
<td>92.58</td>
</tr>
<tr>
<td></td>
<td>Q</td>
<td>31</td>
<td>26</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>RI</td>
<td>9060</td>
<td>8187</td>
<td>7328</td>
</tr>
</tbody>
</table>

Observations:

(i) As λ increases there is increase in P but decrease in Q and NP.

(ii) As a increases there is increase in P, Q and RI.
TABLE 5.3
ROI Policy

<table>
<thead>
<tr>
<th>λ</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>P 82.63</td>
<td>82.75</td>
<td>82.90</td>
<td>83.08</td>
</tr>
<tr>
<td></td>
<td>Q 19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>ROI 5.90</td>
<td>5.41</td>
<td>4.93</td>
<td>4.55</td>
</tr>
<tr>
<td>1100</td>
<td>P 89.58</td>
<td>90.08</td>
<td>90.30</td>
<td>90.51</td>
</tr>
<tr>
<td></td>
<td>Q 15</td>
<td>15</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>ROI 10.39</td>
<td>9.69</td>
<td>8.98</td>
<td>8.37</td>
</tr>
<tr>
<td>1200</td>
<td>P 95.97</td>
<td>96.47</td>
<td>96.96</td>
<td>97.46</td>
</tr>
<tr>
<td></td>
<td>Q 13</td>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>ROI 16.16</td>
<td>15.67</td>
<td>14.72</td>
<td>13.83</td>
</tr>
</tbody>
</table>

Observations:

(i) As λ increases there is increase in P and Q but decrease in ROI.

(ii) As a increases there is increase in P and ROI but decrease in Q.

Note: Since a >> b in R(P) = a-bP, increase in R(P) is effective with respect to increase in a. Hence only increase in a is considered.

Summary:

For increase in λ there is increase in NP but decrease in RI. Hence increase observed in NP is illusory. Thus the model throws light on fixed λ and a so as to go for non-illusory NP.

Increase in λ-value can be justified with respect to sale of second hand items. Hence the model opens a new area of application.
Fig : 5.1 : EFFECT OF DEMAND \( a \) ON PROCUREMENT QUANTITY

Fig : 5.2 : EFFECT OF PRICE INCREASE \( \lambda \) ON PROCUREMENT QUANTITY

137