CHAPTER 4

INVENTORY MODELS WITH RANDOM CHANCE OF DISCOUNTS
4.0 INTRODUCTION:

An important aspect of an inventory control considered here is to develop a model for evaluating alternative discount proposals and finding the replenishment quantity under different situations like:

(i) units in inventory are subject to deteriorate,
(ii) units in inventory are subject to not deteriorate and
(iii) shortages are allowed.

These three cases are discussed in this chapter and effect of deterioration, discount in unit cost and availability of discount in unit cost are studied.

In section 4.1, we developed a deterministic inventory model to find the reorder procurement quantities at discounted and non-discounted price, which minimize the total cost of an inventory system.

In section 4.2, we developed a deterministic inventory model to find the reorder points when discount in unit cost is available and is not available, which minimize the total cost of an inventory system. We also find the effect of deterioration on reorder points and total cost of an inventory system.

In the last section 4.3, we extended the model 4.1 allowing shortages.

In all the three sections mathematical models are supported by numerical example and sensitivity analysis.

4.1 AN EOQ SYSTEM WITH A RANDOM CHANCE OF DISCOUNTS
4.2 AN EOQ MODEL FOR DETERIORATING INVENTORY AND RANDOM CHANCE OF DISCOUNTS

4.3 ORDER LEVEL LOT SIZE SYSTEM WITH RANDOM CHANCE OF DISCOUNTS.
4.1 AN EOQ SYSTEM WITH A RANDOM CHANCE OF DISCOUNTS

In this section, a model is developed to decide optimum purchase quantity when discount may or may not be available. The model is also supported by numerical illustration.

4.1.1 ASSUMPTIONS AND NOTATIONS:

The mathematical model is developed under following assumptions:

1. The demand rate of R units per time unit is known and constant.

2. P denotes the probability that a discount is available and \( (1-P) \) denotes the probability that a discount is not available at the time of ordering.

3. \( Q \) denotes order quantity to be procured when discount is not available and \( Q_1 \) denotes an order quantity to be purchased when discount is available. \( Q \) and \( Q_1 \) are decision variables.

4. The inventory holding charge fraction \( i \) per annum and set up cost \( A \) per order are known and are constant during the period under consideration.

5. The unit cost is \( C \) per unit. If \( d \) is discount as a percentage of unit cost then \( Q_1 \) units are available at unit cost \( C(1-d) \) per unit and \( Q \) units are available at unit cost \( C \) per unit.

6. Lead time is zero.

7. Shortages are not allowed.
4.1.2 MATHEMATICAL MODEL:

Under the above assumptions the partial costs of the inventory under availability and non-availability of discount are:

The cost $TC_1(Q_i)$ when discount is available, is

$$TC_1(Q_i) = A + C(1-d)Q_i + \frac{C(1-d)Q_i^2}{2R}$$

and the cost $TC_2(Q)$ when discount is not available, is

$$TC_2(Q) = A + CQ + \frac{CQ^2}{2R}$$

The total duration of cycle time is $\left(\frac{Q_i}{R}\right) + \left(\frac{Q}{R}\right)$

Since there is random chance of discount, the total cost $TC(Q,Q_i)$ per unit time of the inventory system is taken up as a weighted mean of the above two costs.

$$TC(Q,Q_i) = \frac{P(TC_1(Q_i)) + (1-P)(TC_2(Q))}{P\left(\frac{Q_i}{R}\right) + (1-P)\left(\frac{Q}{R}\right)}$$

$$= \left\{ 2AR + C(1-d)PRQ_i + C(1-d)P\left(\frac{Q_i^2}{2}\right) + C(1-P)RQ \right\}$$

$$P\left(\frac{Q_i}{R}\right) + (1-P)\left(\frac{Q}{R}\right)$$

For obtaining minimum value of $TC(Q,Q_i)$, we require to solve

$$\frac{\delta TC(Q,Q_i)}{\delta Q} = 0 \quad \text{and} \quad \frac{\delta TC(Q,Q_i)}{\delta Q_i} = 0$$

Differentiating equation (4.1.3) with respect to $Q_i$ and $Q$ respectively, we get
\[ C(1-d)iPQi_1^2 - C(1-P)iQ_1^2 + 2C(1-d)(1-P)iQQ_1 - 2Cd(1-P)RQ - 2AR = 0 \quad \rightarrow \times \] (4.1.4)

and

\[ C(1-d)iPQi_2^2 - C(1-P)iQ_2^2 - 2CPiQQ_2 - 2CdPRQ_1 + 2AR = 0 \quad \rightarrow \times \] (4.1.5)

Subtracting equation (4.1.5) from equation (4.1.4), we get

\[ Q_1 = \frac{2AR + C(1-P)dRQ}{C(PdR + i(1-(1-P)d)Q)} \quad \rightarrow \times \] (4.1.6)

Substituting value of \( Q_1 \) in equation (4.1.4), we get

\[ X_4Q^4 + X_3Q^3 + X_2Q^2 + X_1Q + X_0 = 0 \] (4.1.7)

where

\[ X_4 = C^2(1-P)i^3\alpha^2 \]

\[ X_3 = 2C^2P_1^2dR\alpha \]

\[ X_2 = CiR\{3CP^2d^2R - CP^3d^2R - C(1-d)Pd^3R + 4AP_1\alpha + 2CPd^2R_2 - 2A_1\alpha^2\} \]

\[ X_1 = 2CPdR^2\{2\alpha(1+d) + CPd^2R\} \]

\[ X_0 = 2\alpha P^2\{CPd^2R - 2i(1-d)A\} \]

\[ \alpha = 1 - (1-P)d \]
Using Newton-Raphson method with initial iterate \[ Q = \sqrt[2]{\frac{2C_i R}{Ci}} \], we solve (4.1.7) and obtain optimal value of \( Q \) as \( Q_0 \).

Once \( Q = Q_0 \) is established \( Q_1 \) can be found using equation (4.1.6).

**4.1.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:**

Consider an inventory system with following hypothetical data:

- Unit cost \( C = \text{Rs. } 10.00 \) per unit
- Inventory holding charge \( i = \text{Rs. } 0.24 \) per rupee per unit time
- Set up cost \( A = \text{Rs. } 250.00 \) per order
- Demand rate \( R = 1000 \) units per annum

The effect of discount in unit cost and the availability of discount on optimum purchase quantity to be procure at discounted /non-discounted price are studied in the following table. Also corresponding graphs are plotted for the sake of comparisons in fig.4.1.1 to fig.4.1.4.
### TABLE
Effect of changes in discount \(d\) and probability \(P\) on Purchase quantity \(Q, Q_1\) and Total cost \(TC_1, TC_2,\) and \(TC\)

<table>
<thead>
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<th>(d)</th>
<th>0.10</th>
<th>0.15</th>
<th>0.20</th>
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</thead>
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<tr>
<td>(d)</td>
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<td>(Q)</td>
<td>389</td>
<td>357</td>
<td>326</td>
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<tr>
<td>(Q_1)</td>
<td>895</td>
<td>859</td>
<td>826</td>
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<tr>
<td>(TC_1)</td>
<td>8306.18</td>
<td>7992.26</td>
<td>7685.34</td>
</tr>
<tr>
<td>(TC_2)</td>
<td>4322.68</td>
<td>3972.94</td>
<td>3647.02</td>
</tr>
<tr>
<td>(TC)</td>
<td>10736.86</td>
<td>10580.27</td>
<td>10439.03</td>
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</table>

<table>
<thead>
<tr>
<th>(0.20)</th>
<th>(Q)</th>
<th>(Q_1)</th>
<th>(TC_1)</th>
<th>(TC_2)</th>
<th>(TC)</th>
</tr>
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<tbody>
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<td>(Q)</td>
<td>257</td>
<td>162</td>
<td>70</td>
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<tr>
<td>(Q_1)</td>
<td>1363</td>
<td>1244</td>
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<tr>
<td>(TC_1)</td>
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<tr>
<td>(TC_2)</td>
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<td>(TC)</td>
<td>10132.80</td>
<td>9702.86</td>
<td>9302.10</td>
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**Observations:**

(i) Probability of availability of the discount increases, the ratio \(Q_1/Q\) increases and as a consequence, the total cost of the inventory system per time unit decreases.

(ii) With increase in \(d\), there is decrease in purchase quantity \(Q\) when discount is not available but increase in purchase quantity \(Q_1\) when discount is available. Also with increase in \(d\) there is decrease in the total cost of an inventory system.
**Figure 4.1.1: Effect of Probability of Discount on Procurement Quantity**

- Graph showing the relationship between procurement quantity and probability of discount.
- Two curves represent different conditions or scenarios.

**Figure 4.1.2: Effect of Probability of Discount on Total Cost**

- Graph showing the relationship between total cost and probability of discount.
- Multiple curves indicate different cost models or scenarios.

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Fig. 4.1.3 EFFECT OF DISCOUNT LEVEL ON PROCUREMENT QUANTITY

Fig. 4.1.4 EFFECT OF DISCOUNT LEVEL ON TOTAL COST
4.2 AN EOQ MODEL FOR DETERIORATING INVENTORY AND RANDOM CHANCE OF DISCOUNTS

In this section, we develop a mathematical model when items in inventory are subject to deteriorate and random chance of discounts are offered. A model is illustrated with numerical illustration.

4.2.1 ASSUMPTIONS AND NOTATION:

The mathematical model is developed under following assumptions:

1. The demand rate of \( R \) units per time unit is known and constant.
2. \( P \) denotes the probability that a discount is available and \( (1-P) \) denotes the probability that a discount is not available at the time of ordering.
3. \( T \) denotes cycle time when discount is not available and \( T_1 \) denotes cycle time when discount is available. \( T \) and \( T_1 \) are decision variables.
4. The inventory holding charge fraction \( i \) per annum and set up cost \( A \) per order are known and constant during the period under consideration.
5. The unit cost is \( C \) per unit. If \( d \) is discount as a percentage of unit cost then \( Q_1 \) units are available at unit cost \( C(1-d) \) per unit and \( Q \) units are available at unit cost \( C \) per unit.
6. Lead time is zero.
7. Shortages are not allowed.
8. A constant fraction \( \theta \) of on hand inventory gets deteriorated per time unit.
9. There is no repair or replacement of the deteriorated inventory during the period under consideration.
4.2.2 MATHEMATICAL MODEL:

Shah and Shah (1992) have considered an EOQ model under exponentially deteriorating units. They denoted by $Q(t)$ the on hand inventory of the system at time $t$ of a cycle and obtained the differential equation (that describe the instantaneous state of $Q(t)$) as

$$\frac{dQ(t)}{dt} + \theta Q(t) = -R \quad 0 \leq t \leq T$$

They have also obtained the solution of the above equation as

$$Q(t) = \frac{R}{\theta} \left( e^{\theta(T-t)} - 1 \right) \quad 0 \leq t \leq T$$

Ours being same assumptions regarding deterioration of units, we use their solution under the assumption of $t = 0$ and obtain the solution for cycle time $T$ as

$$Q = \frac{R}{\theta} \left( e^{\theta T} - 1 \right)$$

and for cycle time $T_1$ as

$$Q_1 = \frac{R}{\theta} \left( e^{\theta T_1} - 1 \right)$$

From these we obtain the expressions for $T$ and $T_1$ as follows:

$$T = \frac{1}{\theta} \log \left( 1 + \frac{\theta Q}{R} \right)$$

and

$$T_1 = \frac{1}{\theta} \log \left( 1 + \frac{\theta Q_1}{R} \right)$$
The partial costs of the inventory under availability and non-availability of the discount are:

The cost $TC_1(T_i)$ of the inventory when discount is available, is

$$TC_1(T_i) = A + \frac{C(1-d)R}{\theta} \left\{ e^{\theta T_i} - 1 \right\} + \frac{C(1-d)(i + \theta)R}{\theta^2} \left\{ e^{\theta T_i} - \theta T_i - 1 \right\}$$

and the cost $TC_2(T)$ of inventory when discount is not available, is

$$TC_2(T) = A + \frac{CR}{\theta} \left\{ e^{\theta T} - 1 \right\} + \frac{C(i + \theta)R}{\theta^2} \left\{ e^{\theta T} - \theta T - 1 \right\}$$

The total duration of cycle time is

$$(T_i) + (T)$$

Since there is a random chance of discount, the total cost $TC(T,T_i)$ per unit time of the inventory system is taken up as a weighted mean of the above two costs.

$$TC(T,T_i) = \frac{P(TC_1(T_i)) + (1-P)(TC_2(T))}{P(T_i) + (1-P)(T)}$$
\[
\begin{align*}
&\left[P\left(A + \frac{C(1-d)}{\theta} e^{\theta T_1} - 1\right) + \frac{C(1-d)(i+\theta)}{\theta^2} e^{\theta T_1} - \theta T_1 - 1\right] \\
&+ (1-P)\left[A + CR \left(e^{\theta T - 1}\right) + C(i+\theta)R \left(e^{\theta T} - \theta T - 1\right)\right] \\
&= P(T_1) + (1-P)(T)
\end{align*}
\]

(4.2.4)

For obtaining minimum value of TC(T, T_1), we require to solve

\[
\frac{\delta TC(T, T_1)}{\delta T} = 0 \quad \text{and} \quad \frac{\delta TC(T, T_1)}{\delta T_1} = 0
\]

Differentiating equation (4.2.4) with respect to T_1 and T respectively, we get

\[
\begin{align*}
&(P(T_1) + (1-P)(T))\left[CR \theta^2 e^{\theta T} + C(i+\theta)R \theta e^{\theta T - 1}\right] \\
&- (1-P)\left[A \theta^2 + CR \theta e^{\theta T - 1} + C(i+\theta)R e^{\theta T} - \theta T - 1\right] \\
&- P\left[A \theta^2 + C(1-d)R \theta e^{\theta T_1} - 1\right] + C(1-d)(i+\theta)R \left(e^{\theta T_1} - \theta T_1 - 1\right) = 0
\end{align*}
\]

(4.2.5)

and
\[
(P(T_i) + (1 - P(T)) \left[C(1 - d)R\theta^{T_i} + C(1 - d)(i + \theta)R\theta \left(e^{\theta T_i} - 1\right)\right]
\]
\[-(1 - P) \left[A\theta^2 + CR\theta \left(e^{\theta T_i} - 1\right) + C(i + \theta)R \left(e^{\theta T_i} - \theta T_i - 1\right)\right]
\]
\[-P \left[A\theta^2 + C(1 - d)R\theta \left(e^{\theta T_i} - 1\right) + C(1 - d)(i + \theta)R \left(e^{\theta T_i} - \theta T_i - 1\right)\right] = 0
\]
\[(4.2.6)\]

Optimum values of \( T = T_0 \) and \( T_i = T_{10} \) can be obtained by solving (4.2.5) and (4.2.6) simultaneously using Gauss-Seidel iterative method.

The total cost is minimum at \( T = T_0 \) and \( T_i = T_{10} \) iff \( LM - N^2 > 0 \) and \( L > 0 \)

where,

\[
L = \frac{\delta^2 TC(T, T_i)}{\delta T^2}, \quad M = \frac{\delta^2 TC(T, T_i)}{\delta T_i^2}, \quad N = \frac{\delta^2 TC(T, T_i)}{\delta T \delta T_i}
\]

The effect of deterioration of units in inventory and discount in unit cost on optimum purchase quantity to be procured at discounted/ not discounted cost are studied in following numerical illustration. Also corresponding graphs are plotted for the sake of comparisons in fig. 4.2.1 and fig. 4.2.4.
4.2.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

Consider an inventory system with following data:

Unit cost $C = Rs. 50.00\ per\ unit$

Inventory holding charge $i = Rs. 0.24\ per\ rupee\ per\ unit\ time$

Set up cost $A = Rs. 250.00\ per\ order$

Demand rate $R = 1000\ units\ per\ annum$
### TABLE
Effect of deterioration of units and discount in unit cost on Purchase quantity and Total cost

<table>
<thead>
<tr>
<th>D</th>
<th>0.06</th>
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<tr>
<td>θ</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>T</td>
<td>0.177</td>
<td>0.168</td>
<td>0.151</td>
</tr>
<tr>
<td>T₁</td>
<td>0.432</td>
<td>0.516</td>
<td>0.596</td>
</tr>
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<td>Q</td>
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<td>151</td>
</tr>
<tr>
<td>Q₁</td>
<td>432</td>
<td>517</td>
<td>597</td>
</tr>
<tr>
<td>TC</td>
<td>52284.60</td>
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<td>0.02</td>
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</tr>
<tr>
<td>T</td>
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<td>0.151</td>
</tr>
<tr>
<td>T₁</td>
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<td>Q</td>
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<td>Q₁</td>
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<td>TC</td>
<td>52394.58</td>
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</table>

**Observations:**

1. With increase in deterioration rate there is decrease in optimum cycle time T when Q units are to be procured at original cost and optimum cycle time T₁ when Q₁ units are to be procured at discounted cost. The increase in deterioration rate increases the total cost of an inventory system.

2. With increase in discount there is decrease in optimum cycle time T when Q units are to be procured at original cost but increase in optimum cycle time T₁ when Q₁ units are to be procured at discounted cost. The increase in discount decreases the total cost of an inventory system.
Fig: 4.2.1: EFFECT OF DISCOUNT ON CYCLE TIME

Fig: 4.2.2: EFFECT OF DISCOUNT ON TOTAL COST
Fig. 4.2.4 EFFECT OF DETERIORATION ON TOTAL COST

Fig. 4.2.3 EFFECT OF DETERIORATION ON CYCLE TIME

Fig. 4.2.4 EFFECT OF DETERIORATION ON TOTAL COST

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4.3 ORDER LEVEL LOT SIZE SYSTEM WITH RANDOM CHANCE OF DISCOUNTS

In this section, a model is developed to decide optimum purchase quantity and order level when discount may or may not be available at the beginning of each cycle. The model is also supported by numerical illustration.

4.3.1 ASSUMPTIONS AND NOTATIONS:

The mathematical model is developed under following assumptions and notations:

(1) The demand rate of $R$ units per time unit is known and constant.

(2) $P$ denotes the probability that a discount is available and $(1-P)$ denotes the probability that a discount is not available at the time of ordering.

(3) $Q$ denotes order quantity to be procured when discount is not available and $Q_i$ denotes an order quantity to be purchased when discount is available. $Q$ and $Q_i$ are decision variables.

(4) The inventory holding charge fraction $i$ per annum and set up cost $A$ per order are known and constant during the period under consideration.

(5) The unit cost is $C$ per unit. If $d$ is discount as a percentage of unit cost then $Q_i$ units are available at unit cost $C(1-d)$ per unit and $Q$ units are available at unit cost $C$ per unit.

(6) Lead time is zero.

(7) Shortages are allowed and made up as soon as fresh stock arrives.
4.3.2 MATHEMATICAL MODEL:

In each cycle, we allow shortages of \( Q-S \) units. Where, \( Q \) is lot size and \( S \) denotes the order level at the beginning of each cycle.

Under the above assumptions the partial costs of the inventory under availability and non-availability of the discount are:

The Cost \( TC_1(Q, Q_1, S) \) when discount is available, is

\[
TC_1(Q, Q_1, S) = A + C(1-d)Q_1 + \frac{C(1-d) i (Q_1 - Q + S)^2}{2R} + \frac{C_2(Q - S)^2}{2R}
\]

(4.3.1)

and the Cost \( TC_2(Q, S) \) when discount is not available, is

\[
TC_2(Q, S) = A + CQ + \frac{C_iS^2}{2R} + \frac{C_2(Q - S)^2}{2R}
\]

(4.3.2)

The total duration of cycle time is

\[
\left( \frac{Q_1}{R} \right) + \left( \frac{Q}{R} \right)
\]

Since there is a random chance of discount, the total cost \( TC(Q, Q_1, S) \) of inventory system per unit time is taken up as a weighted mean of the above two costs.

\[
TC(Q, Q_1, S) = \frac{P(TC_1(Q, Q_1, S)) + (1-P)(TC_2(Q, S))}{P\left(\frac{Q_1}{R}\right) + (1-P)\left(\frac{Q}{R}\right)}
\]

(4.3.3)

Differentiating equation (4.3.3) with respect to \( S, Q \) and \( Q_1 \) respectively and equating them with zero,
\[ \delta \frac{TC(Q, Q_1, S)}{\delta S} = 0 \text{ gives; } S = \frac{C_2 Q - C(1-d) i P(Q_1 - Q)}{C i (1-Pd) + C_2} \]

\[ \delta \frac{TC(Q, Q_1, S)}{\delta Q} = 0 \text{ gives; } \]
\[ 2 \left\{ P Q_1 + (1-P)Q \right\} \left\{ C_2 (Q - S) - C(1-d)iP(Q_1 - Q + S) + CR(1-P) \right\} \]
\[ - (1-P) \left\{ 2AR + C_2 (Q - S)^2 + C(1-d)iP(Q_1 - Q + S)^2 + 2C(1-d)PRQ + 2C(1-P)RQ + Ci(1-P)S^2 \right\} = 0 \]

(4.3.5)

and

\[ \delta \frac{TC(Q, Q_1, S)}{\delta Q_1} = 0 \text{ gives; } \]
\[ 2 \left\{ P Q_1 + (1-P)Q \right\} \left\{ C(1-d)iP(Q_1 - Q + S) + C(1-d)PR \right\} \]
\[ - P \left\{ 2AR + C_2 (Q - S)^2 + C(1-d)iP(Q_1 - Q + S)^2 + 2C(1-d)PRQ + 2C(1-P)RQ + Ci(1-P)S^2 \right\} = 0 \]

(4.3.6)

For obtaining minimum value of \( TC(Q, Q_1, S) \), we require to solve
\[ \frac{\delta TC(Q, Q_1, S)}{\delta S} = 0, \quad \frac{\delta TC(Q, Q_1, S)}{\delta Q} = 0, \quad \frac{\delta TC(Q, Q_1, S)}{\delta Q_1} = 0 \]
simultaneously using Gauss-seidel iterative method.
Also, from equation (4.3.4)

\[ \frac{\delta S}{\delta Q} = \frac{C_2 + C(1-d)iP}{Ci(1-Pd) + C_2} \geq 0, \text{ for all } Q. \]

So, increase in $S$ results in an increase in optimum purchase quantity $Q$ which are to be purchased at usual cost.

Again,

\[ \frac{\delta S}{\delta Q_1} = -\frac{C(1-d)iP}{Ci(1-Pd) + C_2} < 0, \text{ for all } Q_1 \]

Thus, order level $S$ and optimum purchase quantity $Q_1$, which are available at discounted price, are inversely related.

The above facts hold practically also.

The following numerical illustration also interpret the same fact. We study variations in purchase quantities and order level with change in discount price $d$. Also corresponding graphs are plotted for the sake of comparisons in fig.4.3.1 to 4.3.4.

4.3.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

Consider an inventory system with following hypothetical data:

- Unit cost $C = Rs. 50.00$ per unit.
- Inventory holding charge $i = 0.24$ per rupee per unit time
- Shortage cost $C_2 = Rs. 15.00$ per unit.
- Set up cost $A = Rs. 250.00$ per order.
- Demand rate $R = 1000$ units per year.
TABLE

Effect of changes in discount \( d \) and probability \( P \) on \( S \), \( Q \), \( Q_1 \) and \( TC \).

<table>
<thead>
<tr>
<th>( d )</th>
<th>0.10</th>
<th>0.15</th>
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<tbody>
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<td>0.06</td>
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<td>116.17</td>
<td>98.41</td>
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<td></td>
<td>Q</td>
<td>229.00</td>
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<td>( Q_1 )</td>
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</tr>
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<td></td>
<td>Q</td>
<td>200.00</td>
<td>165.00</td>
</tr>
<tr>
<td></td>
<td>( Q_1 )</td>
<td>571.00</td>
<td>533.00</td>
</tr>
<tr>
<td></td>
<td>TC</td>
<td>51150.87</td>
<td>50825.26</td>
</tr>
</tbody>
</table>

Observations:

(i) As probability \( P \) that a discount is available at the time of ordering increases, order level \( S \), purchase quantities \( Q \), \( Q_1 \) and total cost of an inventory system \( TC \) decreases.

(ii) As \( d \) (the discount in unit cost) increases there is decrease in the order level \( S \) and purchase quantity \( Q \) (without discount) but increase in the purchase quantity \( Q_1 \) (with discount). Also, with increase in \( d \) there is a decrease in the total cost of an inventory system.
Fig: 4.3.1: EFFECT OF PROBABILITY OF DISCOUNT ON ORDER LEVEL AND PROCUREMENT QUANTITY

Fig: 4.3.2: EFFECT OF PROBABILITY OF DISCOUNT ON TOTAL COST
Fig. 4.3.3  EFFECT OF DISCOUNT LEVEL ON ORDER LEVEL AND PROCUREMENT QUANTITY

Fig. 4.3.4  EFFECT OF DISCOUNT LEVEL ON TOTAL COST

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