CHAPTER: 3

INVENTORY MODELS UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND PRICE CHANGE
3.0 INTRODUCTION:

The interaction between marketing policies and the EOQ has been identified and studied by Kotler (1971) and Ladaney and Sternlieb (1974) who have considered the effect of price variation on demand with the objective of profit maximization. There are several studies on these lines Viz., Sankara Subramanyam and Kumarswamy (1981), Agrawal and Ayalawadi (1988), Jani (1978), Brahmbhatt (1981), Shah and Jha (1991). All these earlier studied models deal with deterministic situation under the assumption that units in inventory do not deteriorate with passage of time and cost of a unit remains constant during the period under consideration. In this chapter, by relaxing these conditions, we develop and analyze following four models.

3.1 : AN INVENTORY MODEL UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND PRICE CHANGE ANTICIPATION.

3.2 : AN ANALYTICAL INVENTORY MODEL FOR EXPONENTIALLY DETERIORATING INVENTORY UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND PRICE CHANGE ANTICIPATION.

3.3 : AN INVENTORY MODEL UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND TEMPORARY PRICE DISCOUNT
3.4: AN ANALYTICAL INVENTORY MODEL FOR EXPONENTIALLY DETERIORATING INVENTORY UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND TEMPORARY PRICE DISCOUNT IN UNIT COST
3.1 AN INVENTORY MODEL UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND PRICE CHANGE ANTICIPATION

In this section, an inventory model is developed when vendor announces fixed price increase in the unit cost from some future date, which considers the variable markup concept where actual discounting for large purchases is available. The effect of price increase on markup, procurement units and net profit has been studied with a suitable numerical illustration.

3.1.1 : ASSUMPTIONS AND NOTATIONS :

The mathematical model is developed with the following assumptions:

1. The time $T$ between two successive orders is constant.
2. At the beginning of each period, a quantity $Q$ is replenished. $Q$ is a decision variable.
3. Shortages are not allowed. Lead time is zero.
4. The unit cost $C(Q)$ is a function of purchase quantity $Q$, which includes standard cost $h_1$ and discount cost $h_2$. $C(Q)$ is assumed to have the form

   
   \[ C(Q) = h_1 - h_2 Q \]  

   \[ (3.1.1) \]

where $h_2 < h_1$ such that $C(Q) > 0$, for all $Q$. And that satisfies

\[
\frac{\delta C(Q)}{\delta Q} = -h_2 < 0 \quad \text{for all } Q.
\]

The vendor announces unit cost to be $C(Q) + \lambda$ for some specified time, where $\lambda$ is known constant. i.e. the unit purchased during
specified time will cost \( C(Q) \) per unit whereas after specified time retailer will have to pay \( C(Q) + \lambda \) per unit.

(5) Any unit unsold at the end of the period has no economic value.

(6) The demand \( R(P) \) is a function of selling price \( P \) per unit which is given by

\[
R(P) = K P^{-\eta} \tag{3.1.2}
\]

where \( K > 0 \) is a constant, \( \eta > 1 \) is the elasticity of demand and, generally, selling price \( P \) is taken as a markup of unit cost, i.e.

\[
P = \alpha C(Q) \tag{3.1.3}
\]

with \( \alpha > 1 \) is the markup parameter, \( \alpha \) is also decision variable.

(7) The inventory holding charge \( i \) per rupee per unit time and replenishment cost \( A \) per order are known and they are constant during the period under consideration.

The notations for the mathematical model are:

- \( GR(\alpha, Q) \) = Gross Revenue
- \( TC(\alpha, Q) \) = Total cost of an inventory system during the period.
- \( NP_1(\alpha, Q) \) = Net profit when unit cost is \( C(Q) \).
- \( NP_2(\alpha, Q) \) = Net profit when unit cost is \( C(Q) + \lambda \).
- \( \alpha_{01} \) = Optimal markup parameter when unit cost is \( C(Q) \).
- \( \alpha_{02} \) = Optimal markup parameter when unit cost is \( C(Q) + \lambda \).
- \( Q_{01} \) = Optimal procurement quantity when unit cost is \( C(Q) \).
- \( Q_{02} \) = Optimal procurement quantity when unit cost is \( C(Q) + i \).

3.1.2 MATHEMATICAL MODEL:

Under the above assumptions and notations, in general we have,

\[
NP_1(\alpha, Q) = GR(\alpha, Q) - TC(\alpha, Q)
\]
For the system under consideration,

$$GR(\alpha, Q) = (P - C(Q))R(P)$$

and

$$TC(\alpha, Q) = \frac{C(Q)iQ}{2} + A\left(\frac{R(P)}{Q}\right)$$

Hence, net profit, when unit cost is $C(Q)$, is

$$NP_i(\alpha, Q) = (\alpha - 1)K\alpha^{-\eta}C(Q)^{-\eta - 1} - \frac{C(Q)iQ}{2} - \frac{AK\alpha^{-\eta}C(Q)^{-\eta}}{Q}$$

(3.1.4)

Optimum value of $\alpha$ and $Q$ can be obtained as follows:

$$\frac{\delta NP_i(\alpha, Q)}{\delta \alpha} = 0 \text{ gives us}$$

$$\alpha = \frac{\eta}{\eta + 1}\left\{1 + \frac{A}{Q \ C(Q)}\right\}$$

(2.1.5)

and

$$\frac{\delta NP_i(\alpha, Q)}{\delta Q} = 0 \text{ gives us}$$

$$h_1(\alpha - 1)K\alpha^{-\eta}(\eta - 1)C(Q)^{-\eta} - \left(\frac{h_1}{2} - h_2Q\right)i$$

$$+ \frac{AK\alpha^{-\eta}C(Q)^{-\eta - 1}}{Q^2}\{h_1 - (1 + \eta)h_2Q\} = 0$$

(3.1.6)

The optimum values of markup $\alpha = \alpha_0$ and procurement quantity $Q = Q_0$, when unit cost is $C(Q)$, can be obtained by solving (3.1.5) and (3.1.6) simultaneously.

We use Gauss-Seidel iterative method for solving these equations.
We know that Net Profit $NP_i(\alpha, Q)$ obtained by solving (3.1.5) and (3.1.6) at $\alpha = \alpha_0$ and $Q = Q_0$ is maximum iff Hessian matrix $H = \begin{bmatrix} L & N \\ N & M \end{bmatrix}$ is negative definite.

i.e. $LM - N^2 < 0, \ L < 0$ (3.1.7)

where

$$L = \frac{\delta^2 NP_i(\alpha, Q)}{\delta \alpha^2}$$

$$M = \frac{\delta^2 NP_i(\alpha, Q)}{\delta Q^2}$$

$$N = \frac{\delta^2 NP_i(\alpha, Q)}{\delta \alpha \delta Q}$$

Now assume that supplier announces fixed price increase (say Rs. $\lambda$) per unit from some future date, then net profit $NP_n(\alpha, Q)$ for the system is

$$NP_n(\alpha, Q) = (\alpha - \lambda)K\alpha^{-q}(C(Q) + \lambda)^{-q+1} \frac{(C(Q) + \lambda) Q}{2} - \frac{AK\alpha^{-q}(C(Q) + \lambda)^{-q}}{Q}$$

(3.1.8)

$$\delta NP_n(\alpha, Q) = 0 \quad \text{gives us}$$
\[
\alpha = \frac{\eta}{\eta + 1} \left( 1 + \frac{A}{Q (C(Q) + \lambda)} \right)
\]  

(3.1.9)

and

\[
\frac{\delta NP_n(\alpha, Q)}{\delta Q} = 0 \text{ gives us }
\]

\[
h_1(\alpha - 1)K\alpha^{-\eta}(\eta - 1)(C(Q) + \lambda)^{-\eta} - \left( \frac{(h_1 + \lambda)}{2} - h_2 Q \right) i
\]

\[
+ \frac{AK\alpha^{-\eta}(C(Q) + \lambda)^{-\eta - 1}}{Q^2} \{h_1 + \lambda - (1 + \eta)h_2 Q\} = 0
\]

(3.1.10)

The optimum values of markup \( \alpha = \alpha_0 \) and optimum procurement quantity \( Q = Q_0 \), when unit cost is \( (C(Q) + \lambda) \), can be obtained by solving (3.1.8) and (3.1.9) simultaneously. We use Gauss-Seidel iterative method for solving these equations.

We know that Net Profit \( NP_n(\alpha, Q) \) obtained by solving (3.1.9) and (3.1.10) at \( \alpha = \alpha_0 \) and \( Q = Q_0 \) is maximum iff Hessian matrix \( H = \begin{bmatrix} L & N \\ N & M \end{bmatrix} \) is negative definite.

i.e. \( LM - N^2 < 0, \ L < 0 \)

(3.1.11)

where

\[
L = \frac{\delta^2 NP_n(\alpha, Q)}{\delta \alpha^2}
\]

\[
M = \frac{\delta^2 NP_n(\alpha, Q)}{\delta Q^2}
\]

\[
N = \frac{\delta^2 NP_n(\alpha, Q)}{\delta \alpha \delta Q}
\]
3.1.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

The effect of price increase of a unit on optimum markup and procurement quantity is studied in the following numerical illustration.

Consider an inventory system with the following hypothetical parameters,

- \( K = 3375000 \)
- \( h_1 = 100 \)
- \( h_2 = 0.01 \)
- \( A = Rs. \ 250 \) per order
- \( i = Rs. \ 0.24 \) per rupee per unit time
- \( \lambda = Rs. \ 10.00, 20.00, 30.00 \)

The optimum values of \( \alpha \) and \( Q \) for different values of \( \eta (= 1.6, 1.65, 1.7) \) and \( \lambda (= 0, 10, 20, 30) \) are obtained and listed below in Tables 3.1.1 to 3.1.5. Also, corresponding graphs are plotted in fig. 3.1.1 to fig. 3.1.2.
TABLE 3.1.1
Effect of Elasticity of demand on Markup, Procurement units and Net Profit

<table>
<thead>
<tr>
<th>η</th>
<th>α₀₁</th>
<th>Q₀₁</th>
<th>NP₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>2.67</td>
<td>2228</td>
<td>65097.81</td>
</tr>
<tr>
<td>1.65</td>
<td>2.54</td>
<td>2539</td>
<td>44896.15</td>
</tr>
<tr>
<td>1.7</td>
<td>2.43</td>
<td>2822</td>
<td>29210.67</td>
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</tbody>
</table>

TABLE 3.1.2
Effect of Elasticity of demand and Price increase on Markup, Procurement units and Net profit

\[ \lambda = 10 \]

<table>
<thead>
<tr>
<th>η</th>
<th>α₀₂</th>
<th>Q₀₂</th>
<th>NP₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>2.669</td>
<td>2906</td>
<td>55604.30</td>
</tr>
<tr>
<td>1.65</td>
<td>2.54</td>
<td>3211</td>
<td>35764.40</td>
</tr>
<tr>
<td>1.7</td>
<td>2.431</td>
<td>3486</td>
<td>20409.15</td>
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</tbody>
</table>

TABLE 3.1.3
Effect of Elasticity of demand and Price increase on Markup, Procurement units and Net profit

\[ \lambda = 20 \]

<table>
<thead>
<tr>
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<th>α₀₂</th>
<th>Q₀₂</th>
<th>NP₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>2.668</td>
<td>3571</td>
<td>45705.92</td>
</tr>
<tr>
<td>1.65</td>
<td>2.54</td>
<td>3868</td>
<td>26223.08</td>
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<tr>
<td>1.7</td>
<td>2.430</td>
<td>4137</td>
<td>11191.54</td>
</tr>
</tbody>
</table>
### TABLE 3.1.4

Effect of Elasticity of demand and Price increase on Markup, Procurement units and Net profit

\( \lambda = 30 \)

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>( \alpha_{o2} )</th>
<th>( Q_{o2} )</th>
<th>( NP_{II} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>2.668</td>
<td>4224</td>
<td>35392.71</td>
</tr>
<tr>
<td>1.65</td>
<td>2.54</td>
<td>4514</td>
<td>16259.91</td>
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<tr>
<td>1.7</td>
<td>2.430</td>
<td>4774</td>
<td>1544.02</td>
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</table>

### TABLE 3.1.5

Comparative study of Tables 3.1.2, 3.1.3 & 3.1.4

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \eta )</th>
<th>% ↑ in ( \eta )</th>
<th>( \alpha_{o2} ) % ↑ in ( \alpha_{o2} )</th>
<th>( Q_{o2} ) % ↑ in ( Q_{o2} )</th>
<th>( NP_{II} ) % ↑ in ( NP_{II} )</th>
<th>% ↑ in ( NP_{II} ) % ↑ in ( \eta )</th>
</tr>
</thead>
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<td>2.669</td>
<td>–</td>
<td>2906</td>
<td>–</td>
</tr>
<tr>
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<td>1.65</td>
<td>3.125</td>
<td>2.54</td>
<td>-4.48</td>
<td>3211</td>
<td>10.50</td>
</tr>
<tr>
<td></td>
<td>1.7</td>
<td>6.25</td>
<td>2.43</td>
<td>-8.92</td>
<td>3486</td>
<td>19.96</td>
</tr>
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<td>20</td>
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<td>–</td>
<td>2.668</td>
<td>–</td>
<td>3571</td>
<td>–</td>
</tr>
<tr>
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<td>1.65</td>
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<td>-4.79</td>
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<td>8.329</td>
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<td>1.7</td>
<td>6.25</td>
<td>2.43</td>
<td>-8.92</td>
<td>4137</td>
<td>15.85</td>
</tr>
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<td>30</td>
<td>1.6</td>
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<td>2.669</td>
<td>–</td>
<td>4224</td>
<td>–</td>
</tr>
<tr>
<td></td>
<td>1.65</td>
<td>3.125</td>
<td>2.54</td>
<td>-4.79</td>
<td>4514</td>
<td>6.86</td>
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<tr>
<td></td>
<td>1.7</td>
<td>6.25</td>
<td>2.43</td>
<td>-8.92</td>
<td>4774</td>
<td>13.02</td>
</tr>
</tbody>
</table>
Research Finding:

From above comparative study table it is evident that for increase in $\eta$ (the elasticity of demand with respect to initial value)

1. There is increase in the $\%$ of quantity procured.
2. There is decrease in the $\%$ of markup value $\alpha$.
3. There is decrease in the $\%$ of net profit.

Also, for increase in unit cost $\lambda$ there is a decrease in the net profit.
Fig: 3.1.1: EFFECT OF ELASTICITY OF DEMAND ON PROCUREMENT QUANTITY

Fig: 3.1.2: EFFECT OF ELASTICITY OF DEMAND ON NET PROFIT

Fig: 3.1.1: EFFECT OF ELASTICITY OF DEMAND ON PROCUREMENT QUANTITY

Fig: 3.1.2: EFFECT OF ELASTICITY OF DEMAND ON NET PROFIT
3.2 AN ANALYTICAL INVENTORY MODEL FOR EXPONENTIALLY DETERIORATING INVENTORY UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND PRICE CHANGE ANTICIPATION

In this section, an inventory model is developed when units in storage are subject to deteriorate and vendor announces fixed price increase in unit cost from some future date that considers the variable markup concept where actual discounting for large purchase is available. The effect of deterioration, price increase and elasticity of demand on markup, procurement units and net profit has been studied with a suitable numerical example.

3.2.1 : ASSUMPTIONS AND NOTATIONS :

The mathematical model is developed with the following assumptions :

(1) The time T between two successive orders is constant.

(2) At the beginning of each period, a quantity Q is replenished. Q is a decision variable.

(3) Shortages are not allowed. Lead time is zero.

(4) The unit cost C(Q) is a function of purchase quantity Q, which includes standard cost \( h_1 \) and discount cost \( h_2 \). C(Q) is assumed to have the form

\[
C(Q) = h_1 - h_2 Q
\]

where \( h_2 < h_1 \) such that \( C(Q) > 0 \), for all \( Q \). And that satisfies

\[
\frac{\delta C(Q)}{\delta Q} = -h_2 < 0 \quad \text{for all } Q.
\]
The vendor announces unit cost to be $C(Q) + \lambda$ for some specified time, where $\lambda$ is a known constant. i.e. the unit purchased just will cost $C(Q)$ per unit whereas after specified time retailer will have to pay $C(Q) + \lambda$ per unit.

(5) Any unit unsold at the end of the period has no economic value.

(6) The demand $R(P)$ is a function of selling price $P$ per unit which is given by

$$R(P) = kP^n$$

where $K > 0$ is a constant, $\eta > 1$ is the elasticity of demand and generally, selling price $P$ is taken as a markup of unit cost, i.e.

$$P = \alpha C(Q)$$

with $\alpha > 1$ is the markup parameter. $\alpha$ is also decision variable.

(7) The inventory holding charge $i$ per annum and replenishment cost $A$ per order are known and constant during the period under consideration.

(8) A constant fraction $\theta$ of on hand inventory gets deteriorated per time unit.

(9) There is no repair or replacement of the deteriorated inventory during the period under consideration.

The notations for the mathematical model are:

- $GR(\alpha, Q) =$ Gross Revenue
- $TC(\alpha, Q) =$ Total cost of an inventory system during the period.
- $NP(\alpha, Q) =$ Net profit when unit cost is $C(Q)$.
- $NP_{h}(\alpha, Q) =$ Net profit when unit cost is $C(Q) + \lambda$.
- $\alpha_{o1} =$ Optimal markup parameter when unit cost is $C(Q)$.
- $\alpha_{o2} =$ Optimal markup parameter when unit cost is $C(Q) + \lambda$. 


Q_{01} = \text{Optimal procurement quantity when unit cost is } C(Q).

Q_{02} = \text{Optimal procurement quantity when unit cost is } C(Q) = \lambda.

3.2.2 MATHEMATICAL MODEL:

Under the above assumptions and notations, in general

\[ NP_1(\alpha, Q) = GR(\alpha, Q) - TC(\alpha, Q) \]

For the system under consideration,

\[ GR(\alpha, Q) = (P - C(Q)) R(P) \]

and assuming \( \theta \) to be very small and neglecting the higher powers of \( \theta \), we get

\[
TC(\alpha, Q) = \frac{C(Q) \theta Q}{2} + \frac{C(Q) i R(P)}{2} \left( \frac{Q}{R(P)} - \frac{5 \theta Q^2}{6 R(P)^2} \right) \\
+ A \left[ \frac{R(P)}{Q} + \frac{\theta}{2} \right]
\]

Hence, net profit when unit cost is \( C(Q) \) is

\[
NP_1(\alpha, Q) = (\alpha - 1) K \alpha^{-\eta} C(Q)^{-\eta + 1}
\]

\[
- \frac{C(Q) \theta Q}{2} - \frac{C(Q) i R(P)}{2} \left( \frac{Q}{R(P)} - \frac{5 \theta Q^2}{6 R(P)^2} \right) \\
- A \left[ \frac{R(P)}{Q} + \frac{\theta}{2} \right]
\]

(3.2.1)
Optimum values of $\alpha$ and $Q$ can be obtained as follows:

$$\frac{\delta NP_t(\alpha, Q)}{\delta \alpha} = 0 \text{ gives us}$$

$$\frac{\delta NP_t(\alpha, Q)}{\delta \alpha} = (1 - \eta)C(Q) + \eta \alpha^{-1}\left\{C(Q) + \frac{A}{Q}\right\}$$

$$+ \frac{5\eta C(Q)^{2\pi+1}i \theta Q^2 \alpha^{2\pi-1}}{12K^2} = 0$$

(3.2.2)

and

$$\frac{\delta NP_t(\alpha, Q)}{\delta Q} = 0 \text{ gives us}$$

$$\frac{\delta NP_t(\alpha, Q)}{\delta Q} = (\eta - 1)(\alpha - 1)K \alpha^{-\eta}C(Q)^{-\eta}h_2 - \frac{(\theta + i)(C(Q) - Qh_s)}{2}$$

$$+ \frac{5C(Q)^{4i\theta} Q^\alpha}{6K} \left\{C(Q) - \frac{(\eta + 1)Qh_s}{2}\right\}$$

$$+ \frac{AK\alpha^{-\eta}C(Q)^{-\eta-1}}{Q^2} \left\{C(Q) - \eta h_s Q\right\} = 0$$

(3.2.3)

The optimum values of markup $\alpha = \alpha_{o1}$ and procurement quantity $Q = Q_{o1}$, when unit cost is $C(Q)$, can be obtained by solving (3.2.2) and (3.2.3) simultaneously.

We use Gauss-Seidel iterative method for solving these equations.

The net profit $NP_t(\alpha, Q)$ obtained by solving (3.2.2) and (3.2.3) at $\alpha = \alpha_{o1}$ and $Q = Q_{o1}$ is maximum iff equation (3.1.7) holds.

Now assume that the supplier announces fixed price increase (say Rs. $X$) per unit from some future date, then the net profit $NP_{n}(\alpha, Q)$ for the system is
\[ NP_n(\alpha, Q) = (\alpha - 1)K\alpha^{-\eta}(C(Q) + \lambda)^{-\eta_1} \]

\[ -\frac{(C(Q) + \lambda)\theta Q}{2} \left( C(Q) + \lambda \right) \text{Re}(P) \left\{ \frac{Q}{R(P)} - \frac{5\theta Q^2}{6R(P)^2} \right\} \]

\[ -A\left\{ \frac{R(P)}{\theta} + \frac{\theta}{2} \right\} \]

(3.2.4)

\[ \frac{\delta NP_n(\alpha, Q)}{\delta \alpha} = 0 \] gives us

\[ \frac{\delta NP_n(\alpha, Q)}{\delta \alpha} = (1 - \eta)(C(Q) + \lambda) + \eta\alpha^{-1}\left\{ C(Q) + \lambda + \frac{A}{Q} \right\} \]

\[ + \frac{5\eta(C(Q) + \lambda)^{2\eta_1}i\theta Q^2\alpha^{2\eta_1}}{12K^2} = 0 \]

(3.2.5)

and

\[ \frac{\delta NP_n(\alpha, Q)}{\delta Q} = 0 \] gives us

\[ \frac{\delta NP_n(\alpha, Q)}{\delta Q} = (\eta - 1)(\alpha - 1)K\alpha^{-\eta}(C(Q) + \lambda)^{-\eta}\eta_2 - \frac{(\theta + i)(C(Q) + \lambda - \eta h_2)}{2} \]

\[ + \frac{5(C(Q) + \lambda)^{\eta}i\theta Q\alpha^\eta}{6K} \left\{ (C(Q) + \lambda - \frac{(\eta + 1)Qh_2}{2} \right\} \]

\[ + \frac{AK\alpha^\eta(C(Q) + \lambda)^{-\eta_1}}{Q^2} \left\{ C(Q) + \lambda - \eta h_2Q \right\} = 0 \]

(3.2.6)
The optimum values of markup $a = a_{02}$ and procurement quantity $Q = Q_{02}$, when unit cost is $(C(Q) + \lambda)$, can be obtained by solving (3.2.5) and (3.2.6) simultaneously. We use Gauss-Seidel iterative method for solving these equations.

The net profit $NP_n(a, Q)$ obtained by solving (3.2.5) and (3.2.6) at $a = a_{02}$ and $Q = Q_{02}$ is maximum iff equation (3.1.11) holds.

### 3.2.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

The effect of deterioration and price increase in a unit cost on markup and procurement quantity is studied in the following numerical illustration.

Consider an inventory system with following hypothetical parameters,

- $K = 3375000$
- $h_1 = 100$
- $h_2 = 0.01$
- $A = Rs. 250$ per order
- $i = Rs. 0.24$ per rupee per unit time
- $\lambda = Rs. 10.00, 20.00, 30.00$

The optimum values of $a$ and $Q$ for different values of $\eta (= 1.4, 1.6, 1.8)$, $\lambda (= 0, 10, 20, 30)$ and $6 (= 0.01, 0.02, 0.03)$ are obtained and listed below in Tables 3.2.1 to 3.2.6. Also corresponding graphs are plotted in fig. 3.2.1 to fig. 3.2.2.
TABLE 3.2.1

Effect of Elasticity of demand and Deterioration on Markup, Order quantity and Net profit

<table>
<thead>
<tr>
<th>η</th>
<th>α₀₁</th>
<th>Q₀₁</th>
<th>NP₁</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
</tr>
</thead>
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<td>0.01</td>
<td>3.525</td>
<td>356</td>
<td>229902</td>
<td>2.724</td>
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<td>2.725</td>
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</table>
### TABLE 3.2.2

Effect of Elasticity of demand and Deterioration on Markup, Order quantity and Net profit

\( \lambda = \text{Rs. 10.00} \)

<table>
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<tr>
<th>( \eta )</th>
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<th>( Q_{02} )</th>
<th>( NP_{II} )</th>
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**TABLE 3.2.3**

Effect of Elasticity of demand and Deterioration on Markup, Order quantity and Net profit

\[ \lambda = \text{Rs. } 20.00 \]

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<td>2.737</td>
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### TABLE 3.2.4

Effect of Elasticity of demand and Deterioration on Markup.

Order quantity and Net profit

\( \lambda = \text{Rs. 30.00} \)

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<td>18624</td>
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<td>61129</td>
<td>18597</td>
</tr>
<tr>
<td>0.02 ( \theta )</td>
<td>3.554</td>
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<td>2.351</td>
</tr>
<tr>
<td>( \alpha_{02} )</td>
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<td>18597</td>
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<td>( NP_{II} )</td>
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### Table 3.2.5

Comparative study of Tables 3.2.2, 3.2.3 & 3.2.4

(for elasticity of demand \( \eta \))

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<th>( \alpha_{Q2} )</th>
<th>% ↑ in ( \alpha_{Q2} )</th>
<th>( Q_{Q2} )</th>
<th>% ↑ in ( Q_{Q2} )</th>
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<th>% ↑ in ( \text{NP}_n )</th>
<th>% ↑ in ( \text{NP}_n )</th>
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<td>-50.52</td>
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<td>-69.81</td>
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</table>

**Research Finding:**

From above comparative study table it is evident that for increase in \( \eta \) (the elasticity of demand with respect to initial value)

1. There is decrease in the % ↑ of quantity procured.
2. There is decrease in the % ↑ of markup value \( \alpha \).
3. There is decrease in the % ↑ of net profit.

Also, for increase in unit cost \( \lambda \) there is slight decrease in the % ↑ of net profit.
**TABLE 3.2.6**

Comparative study of Tables 3.2.2, 3.2.3 & 3.2.4

(for deterioration $\theta$)

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<th>$\alpha_{02}$</th>
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<td>-5.47</td>
<td>61084</td>
<td>-0.148</td>
<td>-0.074</td>
</tr>
</tbody>
</table>

Research Finding:

From above comparative study table it is evident that for increase in $\theta$ (the rate of deterioration with respect to initial value)

1. There is decrease in the $\% \uparrow$ of quantity procured.
2. There is increase in the $\% \uparrow$ of markup value $\alpha$.
3. There is decrease in the $\% \uparrow$ of net profit.

Also, for increase in unit cost $\lambda$ there is decrease in the $\% \uparrow$ of net profit.
3.3 AN INVENTORY MODEL UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND TEMPORARY PRICE DISCOUNT

In this section, an inventory model is developed under different marketing policies using the concept of variable markup when vendor announces temporary discount in unit cost for some specified time, which considers the variable markup concept where actual discounting for large purchase is available. The effect of price discount on markup, procurement quantity and net profit has been studied with a suitable numerical example.

3.3.1: ASSUMPTIONS AND NOTATIONS:

The mathematical model is developed with the following assumptions:

1. The time $T$ between two successive orders is constant.
2. At the beginning of each period, a quantity $Q$ is replenished. $Q$ is a decision variable.
3. Shortages are not allowed. Lead time is zero.
4. The unit cost is $C$ per unit. The vendor announces unit cost to be $C - \lambda$, where $\lambda$ is known constant for some specified time. i.e. the unit purchased during this specified time will cost $C - \lambda$ per unit whereas unit purchased before or after the specified discounted period will cost $C$ per unit.
5. Any unit unsold at the end of the period has no economic value.
6. The demand $R(P)$ is a function of selling price $P$ per unit which is given by $R(P) = KP^n$
where \( K > 0 \) is a constant, \( \eta > 1 \) is the elasticity of demand and, generally, selling price \( P \) is taken as a markup of unit cost, i.e.

\[
P = \alpha C
\]

with \( \alpha > 1 \) is the markup parameter. \( \alpha \) is also decision variable.

(7) The inventory holding charge \( i \) per annum and replenishment cost \( A \) per order are known and constant during the period under consideration.

The notations for the mathematical model are:

- \( \text{GR}(\alpha, Q) \) = Gross Revenue
- \( \text{TC}(\alpha, Q) \) = Total cost of an inventory system during the period.
- \( \text{NP}_i(\alpha, Q) \) = Net profit when unit cost is \( C \).
- \( \text{NP}_n(\alpha, Q) \) = Net profit when unit cost is \( C - \lambda \).
- \( \alpha_{01} \) = Optimal markup parameter when unit cost is \( C \).
- \( \alpha_{02} \) = Optimal markup parameter when unit cost is \( C - \lambda \).
- \( Q_{01} \) = Optimal procurement quantity when unit cost is \( C \).
- \( Q_{02} \) = Optimal procurement quantity when unit cost is \( C - \lambda \).

3.3.2 MATHEMATICAL MODEL:

Under the above assumptions and notations, in general

\[
\text{NP}_i(\alpha, Q) = \text{GR}(\alpha, Q) - \text{TC}(\alpha, Q)
\]

For the system under consideration,

\[
\text{GR}(\alpha, Q) = (P - C) R(P)
\]

and
\[ TC(\alpha, Q) = \frac{C_i Q}{2} + \frac{AR(P)}{Q} \]

Hence, net profit, when unit cost is \( C \), is

\[ NP_i(\alpha, Q) = (\alpha - i)K\alpha^{-\eta}C^{-\eta+1} - \frac{Ci}{2} \frac{AK\alpha^{-\eta}C^{-\eta}}{Q} \]  

(3.3.1)

\[ \frac{\delta NP_i(\alpha, Q)}{\delta \alpha} = 0 \] gives us

\[ \alpha = \frac{\eta}{\eta+1} \left( 1 + \frac{A}{QC} \right) \]  

(3.3.2)

and

\[ \frac{\delta NP_i(\alpha, Q)}{\delta Q} = 0 \] gives us

\[ Q = \sqrt[2]{\frac{2Ak}{(\alpha C)^\eta C i}} \]  

(3.3.3)

The optimum values of markup \( \alpha = \alpha_{01} \) and procurement quantity \( Q = Q_{01} \), when unit cost is \( C \), can be obtained by solving (3.3.2) and (3.3.3) simultaneously. We use Gauss-Seidel iterative method for solving these equations.

The net profit \( NP_i(\alpha, Q) \) obtained by solving (3.3.2) and (3.3.3) at \( \alpha = \alpha_{01} \) and \( Q = Q_{01} \) is maximum iff equation (3.1.7) holds.

Now, assume that the supplier announces temporary price discount (say Rs. \( \lambda \)) per unit for short duration date, then the net profit \( NP_d(\alpha, Q) \) for the system is
Now, when there is a temporary price discount the system pass it on the whole benefit of purchase cost to the customers. Hence in this case we use the same value of markup parameter as in the usual purchase cost.

and

$$\delta \frac{\delta N_{P_i}(\alpha, Q)}{\delta Q} = 0$$

gives us

$$Q = \sqrt{\frac{2AK}{(r(C - \lambda))^{r+1}(1 - \lambda)}}$$

(3.3.5)

Hence the optimum values of markup $\alpha = \alpha_{02}$ and optimum procurement quantity $Q = Q_{02}$ can be obtained by solving (3.3.2) and (3.3.5) simultaneously. We use the Gauss-Seidel iterative method for solving these equations. The net profit $NP_{ai}(\alpha, Q)$ at $\alpha = \alpha_{02}$ and $Q = Q_{02}$ is maximum iff (3.1.11) holds.
3.3.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

The effect of price discount in a unit cost for some specified time on optimum markup and procurement unit is studied in following numerical illustration.

Consider an inventory system with following hypothetical parameters,

\[ K = 3375000 \]
\[ C = 100 \]
\[ A = \text{Rs. 250 per order} \]
\[ i = \text{Rs. 0.24 per rupee per unit time} \]
\[ \lambda = \text{Rs. 10.00, 20.00, 30.00} \]

The optimum values of \( \alpha \) and \( Q \) for different values of \( \eta \) (= 1.6, 1.7, 1.8) and \( \lambda \) (= 10, 20, 30) are obtained and listed below in Tables 3.3.1 to 3.1.4. Also corresponding graphs are plotted in fig. 3.3.1 to fig. 3.3.2.

<table>
<thead>
<tr>
<th>Table 3.3.1</th>
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<tr>
<td>Effect of Elasticity of demand and Price discount on Markup, Procurement units and Net profit</td>
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<tr>
<td>( \lambda = 10 )</td>
</tr>
<tr>
<td>( \eta )</td>
</tr>
<tr>
<td>---</td>
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<tr>
<td>1.8</td>
</tr>
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</table>
### TABLE 3.3.2
Effect of Elasticity of demand and Price discount on Markup, Procurement units and Net profit

\( \lambda = 20 \)

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<th>( NP_{n} )</th>
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### TABLE 3.3.3
Effect of Elasticity of demand and Price discount on Markup, Procurement units and Net profit

\( \lambda = 30 \)

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### TABLE 3.3.4
Comparative study of Tables 3.3.1, 3.3.2 & 3.3.3

<table>
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<th>% ↑ in α₀₂</th>
<th>Q₀₂</th>
<th>% ↑ in Q₀₂</th>
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<th>% ↑ in NPᵣ</th>
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**Research Finding:**

From above comparative study table it is evident that for increase in η (the elasticity of demand w.r.t. initial value)

1. There is decrease in the % ↑ of quantity procured.
2. There is decrease in the % ↑ of net profit.

Also, for increase in discount λ there is an increase in the net profit.

Thus, it is always beneficial for the supplier to transfer the price discount offered to the customers.
3.4 AN ANALYTICAL INVENTORY MODEL FOR EXPONENTIALLY DETERIORATING INVENTORY UNDER THE INFLUENCE OF MARKETING POLICIES WITH VARIABLE MARKUP AND TEMPORARY PRICE DISCOUNT IN UNIT COST

In this section, an inventory model is developed when units in storage are subject to deteriorate and vendor announces fixed price discount in unit cost for short time period, which considers the variable markup concept. The effect of deterioration, price discount in unit cost of a unit and elasticity of demand on markup, procurement units and net profit has been studied with a suitable numerical example.

3.4.1 : ASSUMPTIONS AND NOTATIONS :

The mathematical model is developed with the following assumptions :

(1) The time T between two successive orders is constant.

(2) At the beginning of each period, a quantity Q is replenished. Q is a decision variable.

(3) Shortages are not allowed. Lead time is zero.

(4) The unit cost is $C$ per unit. The vendor announces unit cost to be $C - \lambda$, where $\lambda$ is known constant for some specified time. i.e. the unit purchased during this specified time will cost $C - \lambda$ per unit whereas unit purchased before or after the specified discounted period will cost $C$ per unit.

(5) Any unit unsold at the end of the period has no economic value.
The demand $R(P)$ is a function of selling price $P$ per unit which is given by

$$R(P) = KP^n$$

where $K > 0$ is a constant, $\eta > 1$ is the elasticity of demand and, generally, selling price $P$ is taken as a markup of unit cost, i.e.

$$P = \alpha C(Q)$$

with $\alpha > 1$ is the markup parameter. $\alpha$ is also decision variable.

The inventory holding charge $i$ per annum and replenishment cost $A$ per order are known and constant during the period under consideration.

A constant fraction $\theta$ of on hand inventory gets deteriorated per time unit.

There is no repair or replacement of the deteriorated inventory during the period under consideration.

The notations for the mathematical model are

- $\text{GR}(\alpha, Q) = \text{Gross Revenue}$
- $\text{TC}(\alpha, Q) = \text{Total cost of an inventory system during the period.}$
- $\text{NP}_i(\alpha, Q) = \text{Net profit when unit cost is } C.$
- $\text{NP}_n(\alpha, Q) = \text{Net profit when unit cost is } C - \lambda.$

- $\alpha_{01} = \text{Optimal markup parameter when unit cost is } C.$
- $\alpha_{02} = \text{Optimal markup parameter when unit cost is } C - \lambda.$
- $Q_{01} = \text{Optimal procurement quantity when unit cost is } C.$
- $Q_{02} = \text{Optimal procurement quantity when unit cost is } C - \lambda.$
3.4.2 MATHEMATICAL MODEL:

Under the above assumptions and notations, in general

\[ NP_i(\alpha, Q) = GR(\alpha, Q) - TC(\alpha, Q) \]

For the system under consideration,

\[ GR(\alpha, Q) = (P - C) R(P) \]

and assuming \( \theta \) to be very small and neglecting the higher powers of \( \theta \), we get

\[ TC(\alpha, Q) = \frac{C\theta Q}{2} + \frac{C i R(P)}{2} \left( \frac{Q}{R(P)} - \frac{5\theta Q^2}{6R(P)^2} \right) \]

\[ + A \left[ \frac{R(P)}{Q} + \frac{\theta}{2} \right] \]

Hence, net profit when unit cost is \( C \) is

\[ NP_i(\alpha, Q) = (\alpha - 1)K\alpha^{-n}C^{-q_1} \]

\[ - \frac{C\theta Q}{2} - \frac{C i R(P)}{2} \left( \frac{Q}{R(P)} - \frac{5\theta Q^2}{6R(P)^2} \right) \]

\[ - A \left[ \frac{R(P)}{Q} + \frac{\theta}{2} \right] \]

\[ (3.4.1) \]

\[ \frac{\delta NP_i(\alpha, Q)}{\delta \alpha} = 0 \] gives us
\[
\frac{\delta NP_t(\alpha, Q)}{\delta \alpha} = (1 - \eta)C + \eta \alpha^{-1} \left\{ C + \frac{A}{Q} \right\} \\
+ \frac{5\eta C^{2\eta+1} \theta Q^2 \alpha^{-2\eta-1}}{12K^2} = 0 
\]

(3.4.2)

and

\[
\frac{\delta NP_t(\alpha, Q)}{\delta Q} = 0 \text{ gives us} \\
\frac{\delta NP_t(\alpha, Q)}{\delta Q} = -\frac{(\theta + i)C}{2} + \frac{5C^{\eta+1}i\theta Q\alpha^\eta}{6K} + \frac{AK\alpha^{-\eta}C^{-\eta}}{Q^2} = 0 
\]

(3.4.3)

The optimum values of markup \( \alpha = \alpha_{01} \) and procurement quantity \( Q = Q_{01} \), when unit cost is \( C \), can be obtained by solving (3.4.2) and (3.4.3) simultaneously. We use Gauss-Seidel iterative for solving these equations method.

The net profit \( NP_t(\alpha, Q) \) obtained by solving (3.4.2) and (3.4.3) at \( \alpha = \alpha_{01} \) and \( Q = Q_{01} \) is maximum iff equation (3.1.7) holds.

Now assume that the supplier announces fixed price discount (say Rs. \( \lambda \)) per unit for short time period, then the net profit \( NP_{II}(\alpha, Q) \) for the system is

\[
NP_{II}(\alpha, Q) = (\alpha - 1)K\alpha^{-\eta}(C - \lambda)^{-\eta+1} \\
- \frac{(C - \lambda)\theta Q}{2} - \frac{(C - \lambda)}{2} \frac{R(P)}{R(P)} \left[ \frac{Q}{R(P)} - \frac{5\theta Q^2}{6R(P)^2} \right] \\
- A\left\{ \frac{R(P)}{Q} + \frac{\theta}{2} \right\} 
\]

(3.4.4)
\[ \frac{\delta NP_n(\alpha, Q)}{\delta \alpha} = 0 \] gives us
\[ \frac{\delta NP_n(\alpha, Q)}{\delta \alpha} = (1 - \eta)(C - \lambda) + \eta \alpha^{-1} \left\{ C - \lambda + \frac{A}{Q} \right\} \]
\[ + \frac{5\eta(C - \lambda)^{2 \eta+1} i \theta Q^2 \alpha^{-2 \eta}}{12K^2} = 0 \]

(3.4.5)

and
\[ \frac{\delta NP_n(\alpha, Q)}{\delta Q} = 0 \] gives us
\[ \frac{\delta NP_n(\alpha, Q)}{\delta Q} = \frac{(\theta + i)(C - \lambda)}{2} + \frac{5(C - \lambda)^{\eta+1} i \theta Q \alpha^\eta}{6K} + \frac{AK \alpha^{-\eta}(C - \lambda)^{-\eta}}{Q^2} = 0 \]

(3.4.6)

The optimum values of markup \( \alpha = a_{02} \) and procurement quantity \( Q = Q_{02} \), when unit cost is \( C - \lambda \), can be obtained by solving (3.4.5) and (3.4.6) simultaneously. We use Gauss-Seidel iterative method for solving these equations.

The net profit \( NP_n(\alpha, Q) \) obtained by solving (3.4.5) and (3.4.6) at \( \alpha = a_{02} \) and \( Q = Q_{02} \) is maximum iff equation (3.1.11) holds.

3.4.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

The effect of deterioration and price discount in a unit of a unit cost on markup and procurement quantity is studied in the following numerical illustration.
Consider an inventory system with following hypothetical parameters,

\[ K = 3375000 \]

\[ C = 100 \]

\[ A = \text{Rs. 250 per order} \]

\[ i = \text{Rs. 0.24 per rupee per unit time} \]

\[ \lambda = \text{Rs. 10.00, 20.00, 30.00} \]

The optimum values of \( \alpha \) and \( Q \) for different values of \( \eta (=1.4, 1.6, 1.8) \), \( \lambda (=0, 10, 20, 30) \) and \( \delta (=0.01, 0.02, 0.03) \) are obtained and listed below in Tables 3.4.2.1 to 3.4.6. Also corresponding graphs are plotted in fig. 3.4.1 to fig. 3.4.2.
**TABLE 3.4.1**

Effect of Elasticity of demand and Deterioration on Markup, Order quantity and Net profit

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TABLE 3.4.2
Effect of Elasticity of demand and Deterioration on Markup.
Order quantity and Net profit

\( \lambda = \text{Rs. 10.00} \)

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TABLE 3.4.3
Effect of Elasticity of demand and Deterioration on Markup.
Order quantity and Net profit

$\lambda = \text{Rs. 20.00}$

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TABLE 3.4.4
Effect of Elasticity of demand and Deterioration on Markup, Order quantity and Net profit

\( \lambda = \text{Rs. } 30.00 \)

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TABLE 3.4.5
Comparative study of Tables 3.4.2, 3.4.3 & 3.4.4
(for elasticity of demand $\eta$)

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Research Finding:

From above comparative study table it is evident that for increase in $\eta$ (the elasticity of demand with respect to initial value)

(1) There is decrease in the $\% \uparrow$ of quantity procured.

(2) There is decrease in the $\% \uparrow$ of markup value $\alpha$.

(3) There is decrease in the $\% \uparrow$ of net profit.

Also, with increase in $\lambda$ (discount in unit cost) there is increase in the $\% \uparrow$ of net profit.
TABLE 3.4.6
Comparative study of Tables 3.4.2, 3.4.3 & 3.4.4 
(for deterioration $\theta$ )

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Research Finding:

From above comparative study table it is evident that for increase in $\theta$ (the rate of deterioration with respect to initial value)

(1) There is decrease in the $\% \uparrow$ of quantity procured.

(2) There is increase in the $\% \uparrow$ of markup value $\alpha$.

(3) There is decrease in the $\% \uparrow$ of net profit.

Also, with increase in $\lambda$ (discount in the unit cost) there is increase in the $\% \uparrow$ of net profit.
Fig 3.4.1: EFFECT OF DETERIORATION ON PROCUREMENT QUANTITY

Fig 3.4.2: EFFECT OF DETERIORATION ON NET PROFIT

PROCUREMENT QUANTITY

DETERIORATION

NP

0.0 0.01 0.02 0.03 0.04

DETERIORATION

NP

Q_{\lambda}=10

Q_{\lambda}=20

Q_{\lambda}=30

PROCUREMENT QUANTITY

NP

0.0 0.01 0.02 0.03

DETERIORATION

NP

NP_{\lambda}=10

NP_{\lambda}=20

NP_{\lambda}=30

Fig : 3.4.2 : EFFECT OF DETERIORATION ON NET PROFIT

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