CHAPTER : 7

EOQ MODELS WITH PERMISSIBLE CREDIT PERIOD
REQUIRING PARTIAL PAYMENT
7.0 INTRODUCTION:

In the classical, Economic Order Quantity (EOQ) model it is assumed that the payments for the goods received should be made immediately after the receipt of the order. It's a general practice that supplier offers some credit period to the retailer to settle the account. This, indirectly, gives a kind of incentive in the form of supplying goods as an interest free loan to the retailer. The retailer, in turn, can make use of this incentive by keeping the amount in an interest bearing account and hence earning some interest. Goyal (1985) has studied an EOQ system with deterministic demand and delay in payments is permissible which was reinvestigated by Chand and Ward (1987). Mandal and Phaujdar (1989) extended Goyal(1985)'s model to the case of shortages. Shah (1991) studied the same problem with uncertainty in the quantity received, resulting in a random duration between successive orders. Using the first two moments of the distribution of the random quantity received, Shah (1991) arrived at modified results of Goyal(1985) by using probabilistic demand. The aspect of admissible delay in payments has been extended to the case of two level of storage by Shah and Shah (1992), which combines the ideas of Goyal(1985) and that of Hartely (1970). Shah (1993) developed model for exponentially deteriorating items when delay in payments is permissible by assuming deterministic demand. Then after, under the same situation, Shah (1993) developed model for probabilistic demand.

In this chapter, we have introduced the aspect of part payment taking (i) demand as known constant and (ii) demand as a function of selling price. A part of the purchased cost is to be paid during the permissible delay period. What quantity of the
part is to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods. This led us to develop and analyze following two models:

7.1 DETERMINATION OF THE EOQ MODEL FOR DETERIORATING ITEMS WITH PERMISSIBLE CREDIT PERIOD REQUIRING PARTIAL PAYMENT.

7.2 AN INVENTORY MODEL UNDER THE EFFECT OF PART PAYMENT ON SELLING PRICE AND CYCLE TIME.

Sensitivity analysis of the model with respect to some of the vital parameters is carried out with the help of numerical example. The chapter concludes with conclusion section.
7.1 DETERMINATION OF THE EOQ MODEL FOR DETERIORATING ITEMS WITH PERMISSIBLE CREDIT PERIOD REQUIRING PARTIAL PAYMENT.

In this section, the aspect of part payment required by the supplier prior to the complete settlement of the account, when units in inventory is subject to deteriorate, is studied.

7.1.1 ASSUMPTIONS AND NOTATIONS:

1. The rate of replenishment is infinite.
2. Lead time is zero.
3. Shortages are not allowed.
4. The deteriorated units can neither be replaced nor repaired during the cycle time.

Notations:

\[
\begin{align*}
C &= \text{Unit cost of an item.} \\
i &= \text{Inventory holding charge fraction per rupee per year.} \\
A &= \text{Ordering cost.} \\
R &= \text{Deterministic demand rate per year.} \\
I_e &= \text{Rate of interest earned.} \\
I_c &= \text{Rate of interest charged and } I_c > I_e \\
T^* &= \text{Permissible credit period expressed as a known fraction of a year.} \\
T_1 &= \text{Time at which a part payment is to be made.} \\
\alpha &= \text{Part of the purchase cost to be paid at } T_1 \\
T &= \text{Length of a cycle time(years).}
\end{align*}
\]
0 = The constant rate of deterioration of units in an inventory system during cycle time.

7.1.2 DETERMINATION OF COST FUNCTION:

The total cost function of an inventory system consists of an ordering cost, inventory holding cost, the interest charged and earned. Since $T^*$ is constant and $T$ is decision variable which depends on the lot size ordered. Two cases may arise Viz $T^* \leq T$ and $T^* > T$. In both the cases, the interest charges for the items in stock are different, and will be computed as given below:

CASE-1: $T^* < T$ (i.e. when permissible delay period is less than or equal to the cycle time)

In this case the account has to be settled before the cycle ends and part payment $\alpha CRT_1$ is to be made at $T_1$, where $\alpha$ and $T_1$ are specified at the time of the deal. The cycle situation is shown below in the fig. 1.

(fig. 1)

The components of interest earned:

- The interest earned during $(0, T_1) = \frac{CI_1 RT_1^2}{2}$

- The cash on hand at $T_1 = (1 - \alpha) CRT_1 + \frac{CI_1 RT_1^2}{2} - \frac{CR}{\theta} [e^{-\theta T} - e^{-\theta (T - T_1)} - \theta T_1]$
The interest earned during \((T_i, T^*)\)

\[
\left\{ (1 - \alpha)CRT_i + \frac{Cl_iRT^2_i}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - e^{\theta (T - T_i)} - \theta T_i \right] \right\} I_e(T^* - T_i)
\]

\[+ \frac{Cl_eR(T^* - T_i)^2}{2} \]

The total interest earned

\[
I_{en} = \left\{ (1 - \alpha)CRT_i + \frac{Cl_iRT^2_i}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - e^{\theta (T - T_i)} - \theta T_i \right] \right\} I_e(T^* - T_i)
\]

\[+ \frac{Cl_eR(T^* - T_i)^2}{2} + \frac{Cl_eRT^2_i}{2} \]

Now,

The interest charged during \((T^*, T)\)

\[
I_{ch} = \frac{Cl_eR}{\theta^2} \left\{ e^{\theta (T - T^*)} - e^{\theta (T - T^*)} - 1 \right\}
\]

The inventory holding cost, IHC, is

\[
IHC = \frac{Cr}{\theta^2} \left\{ e^{\theta T} - e^{\theta T} - 1 \right\}
\]

Ordering cost, OC, is \(OC = A\)

Cost due to deterioration, CD, is \(CD = \frac{Cr}{\theta} \left\{ e^{\theta T} - e^{\theta T} - 1 \right\}\)
The total cost function of a cycle is

\[ TC_1(T) = \frac{(OC + CD + IHC + Ich - Ie^\theta)}{T} \]

\[ = \frac{A}{T} + \frac{CR}{\theta T} \left\{ e^\theta T - \theta T - 1 \right\} + \frac{CIeR}{\theta^2 T} \left\{ e^\theta T - \theta T - 1 \right\} \]

\[ + \frac{CI_2R}{\theta^2 T} \left\{ e^\theta (T - T^*) - \theta (T - T^*) - 1 \right\} \]

\[ = \left\{ (1 - \alpha)CR\tau + \frac{CI_2RT_1^2}{2} - \frac{CR}{\theta} \left[ e^\theta T - e^\theta (T - T_i) - \theta T_i \right] \right\} I_1(T^* - T_1) \]

\[ - \frac{CI_2R(T^* - T_1)^2}{2} - \frac{CI_2RT_1^2}{2} \quad (7.1.1) \]

The optimum value of \( T = T_1^0 \) can be obtained by solving \( \frac{dTC_1(T)}{dT} = 0 \) i.e. by solving

\[ \left\{ \frac{A\theta^2}{CR} + (i + \theta ) (e^\theta T - \theta T - 1) + I_c \left\{ e^\theta (T - T^*) - \theta (T - T^*) - 1 \right\} \right\} \]

\[ + \frac{\theta^2 I_1}{2} (T^* - T_1)^2 - \frac{\theta^2 I_2T_1^2}{2} + \left[ (1 - \alpha)T_i \theta^2 + \frac{\theta^2 I_2T_1^2}{2} \right] \]

\[ - \theta \left\{ e^\theta T - e^\theta (T - T_i) - \theta T_i \right\} \right] - T \left(i\theta \right) (e^\theta T - 1) \]

\[ + \theta \left\{ e^\theta T - 1 \right\} + I_c \theta \left(e^\theta (T - T^*) - 1 \right) \]

\[ - \theta \left\{ e^\theta T - e^\theta (T - T_i) \right\} I_1(T^* - T_1) = 0 \quad (7.1.2) \]
The optimum $T^0_{(0)}$ of $T$ minimizes the cost function $TC_i(T)$ whenever $\frac{d^2TC_i(T)}{dT^2} > 0$.

This $T = T^0_{(0)}$ gives optimum procurement quantity $Q_i = \frac{R}{\theta} \left( e^{\theta T^0_{(0)}} - 1 \right)$ and the optimum cost $TC_i(T^0_{(0)})$.

The model due to Shah (1993) turns out to be a particular case of our model when we take $T_i = T^*$, i.e. when part payment is not allowed and the entire payment has to be made at the end of the permissible delay period, i.e. $\alpha = 1$. Further the Goyal's (1985) model happens to be a particular case of our model if we take $T_i = T^*$ and $\theta = 0$.

**CASE : 2**  $T^* > T$  (i.e. when the permissible delay period is more than the cycle time)

In this case the retailer will prefer to settle the account of the current at time $T^*$, after placing another order at $T$. There are again two possibilities, Viz, $T_1 < T$ (fig. 2) or $T < T_1 < T^*$. These two sub cases are discussed separately.

**Case : 2.1**  $T^* > T$  and  $T_1 < T$. 

![Diagram](fig. 2)
Here part payment $\alpha CRT_1$ will be made at $T_1$ and at $T$ another order is placed.

Thus, the retailer can earn interest during $(0, T)$, but interest charged is zero.

**The components of interest earned:**

- The interest earned during $(0, T_1) = \frac{C I_e RT_1^2}{2}$

- The cash on hand at $T_1 = (1-\alpha)CRT_1 + \frac{C I_e RT_1^2}{2} - CR [e^\theta T - e^\theta (T - T_1) - \theta T_1 ]$

- The interest earned during $(T_1, T) = \left( (1-\alpha)CRT_1 + \frac{C I_e RT_1^2}{2} - CR \left[e^\theta T - e^\theta (T - T_1) - \theta T_1 \right] \right) I_e (T - T_1)$

\[ + \frac{C I_e R(T - T_1)^2}{2} \]

- The cash on hand at $T$ is

\[ \left\{ (1-\alpha)CRT_1 + \frac{C I_e RT_1^2}{2} - CR \left[e^\theta T - e^\theta (T - T_1) - \theta T_1 \right] \right\} [1 + I_e (T - T_1)] \]

\[ + \frac{C I_e R(T - T_1)^2}{2} - CR \left[e^\theta (T - T_1) - \theta (T - T_1) - 1 \right] \]

- The interest earned during $(T, T^*)$ is

\[ \left\{ (1-\alpha)CRT_1 + \frac{C I_e RT_1^2}{2} - CR \left[e^\theta T - e^\theta (T - T_1) - \theta T_1 \right] \right\} [1 + I_e (T - T_1)] \]

\[ + \frac{C I_e R(T - T_1)^2}{2} - CR \left[e^\theta (T - T_1) - \theta (T - T_1) - 1 \right] I_e (T^* - T) \]
The total interest earned

\[ I_{em} = \left\{ (1 - \alpha \text{CRT})_1 + \frac{CI\text{e}RT_i^2}{2} - \frac{CR}{\theta} \left[ e^\theta (T - e^\theta (T - T_i) - \theta T_i) \right] \right\}. \]

\[ \left[ I_s(T^* - T_i) + I_s^2(T^* - T)(T - T_i) \right] + \frac{CI\text{e}RT_i^2}{2} \]

\[ + \left( 1 + I_s(T^* - T) \right) \frac{CI\text{e}R(T - T_i)^2}{2} \]

\[ - \frac{I_s(T^* - T)CR}{\theta} \left( e^\theta (T - T_i) - \theta (T - T_i) - 1 \right) \]

The total cost function per time unit is given by

\[ TC_2(T) = (OC + CD + IHC + I_{ch} - I_{ea}) / T \]

\[ = A + \frac{CR}{\theta T} \left\{ e^\theta T - \theta T - 1 \right\} + \frac{CI\text{e}R}{\theta^2 T} \left\{ e^\theta T - \theta T - 1 \right\} \]

\[ - \left\{ (1 - \alpha \text{CRT})_1 + \frac{CI\text{e}RT_i^2}{2} - \frac{CR}{\theta} \left[ e^\theta (T - e^\theta (T - T_i) - \theta T_i) \right] \right\} \]

\[ \left[ I_s(T^* - T_i) + I_s^2(T^* - T)(T - T_i) \right] + \frac{CI\text{e}RT_i^2}{2} \]

\[ + \left( 1 + I_s(T^* - T) \right) \frac{CI\text{e}R(T - T_i)^2}{2} \]

\[ - \frac{I_s(T^* - T)CR}{\theta} \left( e^\theta (T - T_i) - \theta (T - T_i) - 1 \right) \]

(7.1.3)
The optimum value of $T = T^{\circ}_{(2)}$ can be obtained by solving \( \frac{dT C_2(T)}{dT} = 0 \). i.e.

by solving

\[
\left\{ -\frac{A \theta^2}{CP} (i + \theta)(e^{\theta T} - \theta - 1) + \frac{I_s T^2 \theta^2}{2} \\
+ \frac{\theta^2 I_s}{2} (T - T_1)^2 [1 + I_s(T^* - T)] + \left[ (1 - \alpha) T_i \theta^2 + \frac{\theta^2 I_s T_i^2}{2} \right] \\
- \theta \left[ e^{\theta T} - e^{\theta (T^* - T)} - \theta (T^* - T) - 1 \right] I_s (T^* - T_i) + I_s^2 (T^* - T)(T - T_i) \\
- \theta \left[ I_s (T^* - T_i) \left[ e^{\theta (T - T_i)} - \theta (T - T_i) - 1 \right] \right] \\
- \theta \left[ (i + \theta) \left( e^{\theta T} - 1 \right) \theta + [1 + I_s(T^* - T)] I_s (T - T_i) \theta^2 \right] \\
- \frac{I_s^2 (T - T_i)^2 \theta^2}{2} + \left[ (1 - \alpha) T_i \theta^2 + \frac{I_s T^2 \theta^2}{2} \right] \\
+ \theta \left[ e^{\theta (T - T_i)} - e^{\theta T} + \theta T_i \right] \left[ I_s^2 (T^* - 2T + T_i) \right]
\]

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The optimum value of $T = T^0$ minimizes the cost function $TC_2(T)$ whenever \( \frac{d^2TC_2(T)}{dT^2} > 0 \). This $T = T^0_{(2)}$ gives optimum procurement quantity $Q^0_2 = \frac{R}{\theta} \left( e^{\theta T^0_{(2)}} - 1 \right)$ and the optimum cost $TC_2(T^0_{(2)})$.

The model due to Shah (1993) turns out to be a particular case of our model when we take $T_1 = T^*$, i.e. when part payment is not allowed and the entire payment has to be made at the end of the permissible delay period, i.e. $\alpha = 1$. Further the Goyal’s (1985) model happens to be a particular case of our model if we take $T_1 = T^*$ and $\theta = 0$.

Case: 2.2: $T^* > T$ and $T < T_1 < T^*$

\[
- \left[ \theta^2 \left( e^{\theta T} - e^{\theta (T - T_1)} \right) \right] I_1(T^* - T) + I_2^2(T^* - T)(T - T_1) \]

\[ - I_4(T^* - T)CR_2 \left( e^{\theta (T - T_1)} - 1 \right) \]

\[ + I_5 \theta \left( e^{\theta (T - T_1)} - \theta (T - T_1) - 1 \right) = 0 \quad (7.1.4) \]
In this case, the account is to be settled at $T^*$ and the part payment is made at $T_1$.

The components of interest earned:

- The interest earned during $(0, T) = \frac{CIR^2}{2}$

- The cash on hand at $T = CRT + \frac{CIR^2}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - \theta T - 1 \right]

- The interest earned during $(T, T_1)$

$$= \left( CRT + \frac{CIR^2}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - \theta T - 1 \right] \right) I_e(T_1 - T)$$

- The cash on hand at $T_1$ is

$$\left( CRT + \frac{CIR^2}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - \theta T - 1 \right] \right) \left[ 1 + I_e(T_1 - T) \right]$$

- The interest earned during $(T_1, T^*)$ is

$$\left( CRT + \frac{CIR^2}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - \theta T - 1 \right] \right) \left[ I_e(T^* - T_1) + I_e^2(T_1 - T)(T^* - T_1) \right]$$

- The total interest earned, $I_{Te}$, is

$$\left( CRT + \frac{CIR^2}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - \theta T - 1 \right] \right) \left[ I_e(T^* - T_1) + I_e(T_1 - T)(T^* - T_1) + (T_1 - T) \right] I_e$$

$$+ \frac{CIR^2}{2}$$

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• The total cost function of a cycle per time unit is

\[ TC_3(T) = \frac{(OC + CD + IHC + Icu - I_e)}{T} \]

\[ = \frac{A}{T} + \frac{CR}{T} \left\{ \theta T - \theta T - 1 \right\} + \frac{CR}{T} \left\{ \theta T - \theta T - 1 \right\} - \]

\[ \left\{ CRT + \frac{CR^2 T^3}{2} - \frac{CR}{\theta} \left[ e^{\theta T} - \theta T - 1 \right] \right\} \left\{ (T^* - T_i) + I_e (T_i - T)(T^* - T_i) + (T_i - T) \right\} \frac{I_e}{T} \]

\[ - \frac{CR^2 T^3}{2} \]

(7.1.5)

The optimum value of \( T = T_{(3)}^0 \) can be obtained by solving \( \frac{dTC_3(T)}{dT} = 0 \) i.e. by solving

\[ \left\{ \frac{-A\theta^2}{CR} - i(\theta T - \theta T - 1) \cdot \frac{I_e T^3 \theta^2}{2} + (i + \theta) \theta (e^{\theta T} - 1) \right\} \]

\[ - \left[ I_e (T^* - T_i) + I_e^2 (T^* - T_i)(T_i - T) + I_e (T_i - T) \right] . \]

\[ \left[ \theta \left( e^{\theta T} - \theta T - 1 \right) + \frac{I_e T^2 \theta^2}{2} - \theta^2 T \left( e^{\theta T} - 1 \right) \right] \]

\[ + \left[ \frac{I_e T^3 \theta^2}{2} - \theta T \left( e^{\theta T} - \theta T - 1 \right) + \theta^2 T^2 \right] \left[ I_e + I_e^2 (T^* - T_i) \right] = 0 \]

(7.1.6)
The optimum value of $T = T^{\circ}_3$ minimizes the cost function $TC_3(T)$ whenever
\[
\frac{d^2 TC_3(T)}{dT^2} > 0.
\]
This $T = T^{\circ}_3$ gives optimum procurement quantity
\[
Q_3^* = \frac{R}{\theta} \left( e^{\theta T^{\circ}_3} - 1 \right)
\]
and the optimum cost $TC_3(T^{\circ}_3)$.

**Algorithm to find optimum solution:**

Since there are three cases depending on the values of $T^*$ and $T$, the feasibility of each case should be checked before evaluating cost function. When $T = T^*$, the interest charge $I_{Ch}$ will be zero and hence in this case the cost function $TC(T^*)$ can be obtained by substituting $T = T^*$ in equation (7.1.1).

\[
TC(T^*) = \frac{A}{T^*} + \frac{CR}{\theta T^*} \left( e^{\theta T^*} - \theta T^* - 1 \right) \left( 1 + \frac{I}{\theta} \right) - \frac{CLR}{2T^*} \left( 2T^* T^* - T^{*2} \right)
\]

\[
+ \left\{ \left( 1 - \alpha \right) CRT_1 + \frac{CLR T^3}{2} - \frac{1}{\theta} e^{\theta T} \left( e^{\theta (T - T_1)} - \theta T \right) \right\} \frac{I_e(T^* - T_1)}{T^*}
\]

(7.1.7)
To find optimum solution, we shall use following steps:

**Step : 1 :**
Given the input parameters, find $T_{(0)}^0$, $T_{(2)}^0$, $T_{(3)}^0$. If $T_{(0)}^0 > T^*$, Evaluate TC$_1(T_{(0)}^0)$ using (7.1.7). Otherwise evaluate TC(T*) using (7.1.7).

Define $J_1 = \min \{ \text{TC}_1(T_{(0)}^0), \text{TC}(T^*) \}$ and designate replenishment interval for $J_1$ by $X_1$.

**Step : 2 :**
If $T_{(1)} < T_{(2)}^0 < T^*$, evaluate TC$_2(T_{(2)}^0)$ using (7.1.3). If $T_{(2)}^0 < T^*$ and $T_{(2)}^0$ does not lie between $T_1$ and $T^*$, evaluate TC$_2(T_{(1)})$ using (7.1.3) and TC(T*) using (7.1.7).

Define $J_2 = \min \{ \text{TC}_2(T_{(2)}^0), \text{TC}_2(T_{(1)}), \text{TC}(T^*) \}$ and designate replenishment interval for $J_2$ by $X_2$.

**Step : 3 :**
If $T_{(3)}^0 < T^*$ and $T_1$ lies between $T_{(3)}^0$ and $T^*$, evaluate TC$_3(T_{(3)}^0)$ using (7.1.5) and if $T_{(3)}^0 < T^*$ and $T_1$ does not lie between $T_{(3)}^0$ and $T^*$, evaluate TC$_3(T_{(1)})$ using (7.1.5) and TC(T*) using (7.1.7).

Define $J_3 = \min \{ \text{TC}_3(T_{(3)}^0), \text{TC}_3(T_{(1)}), \text{TC}(T^*) \}$ and designate replenishment interval for $J_2$ by $X_3$.

**Step : 4 :**
Define $J = \min \{ J_1, J_2, J_3 \}$ and let $I = X_1$ or $X_2$ or $X_3$ denotes the replenishment interval that yields $J$. Thus the optimal value of the decision variable and the corresponding minimum cost is obtained. Hence terminate the algorithm.
7.1.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

Consider the following hypothetical data,

\[ a = 10000 \text{ units per year} \]

\[ C = \text{Rs.10 per unit} \]

\[ C_i = \text{Rs.2 per unit per year} \]

\[ i = 0.1 \text{ per rupee per unit time} \]

\[ I_e = 0.10 \text{ per rupee per unit time} \]

\[ I_c = 0.15 \text{ per rupee per unit time} \]

\[ \alpha = 0.2 \circ \]

\[ T^* = 0.10 \text{ years} \]

\[ T_1 = \beta T^* \ (\beta = 0.5) \]

\[ A = 150 \text{ per order} \]

For the above set of parameters case-1 is feasible. It is interesting to study the effect of \( T_1 \), time at which part payment is to be made, on decision variable and on total cost. For convenience, consider \( T_1 = \beta T^* \), \( 0 < \beta < 1 \) so that the time at which part payment is to be made is expressed as a fraction of the credit period.

The effect of variations in values of \( \alpha \), \( \beta \) and \( \theta \) on decision variables and total cost of an inventory system are given in following tables. Also corresponding graphs are plotted for the sake of comparisons in fig.7.1.1 to fig.7.1.4.
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**Observations:**

1. As θ increases there is decrease in cycle time T and procurement quantity Q but increase in the total cost TC.

2. As α increases there is decrease in cycle time T, procurement quantity Q and the total cost TC.
**TABLE 7.1.2**

Variations in $\beta$ and $\theta$

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<th>$\beta$</th>
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<th>0.05</th>
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**Observations:**

1. As $\theta$ increases there is decrease in cycle time $T$ and procurement quantity $Q$ but increase in the total cost $TC$.

2. As $\beta$ increases there is decrease in cycle time $T$, procurement quantity $Q$ and the total cost $TC$. 

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### TABLE 7.1.3
Variations in $\alpha$ and $\beta$

<table>
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<th>$T_C$</th>
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</table>

**Observations:**

1. As $\alpha$ increases there is decrease in cycle time $T$, procurement quantity $Q$ and the total cost $T_C$.
2. As $\beta$ increases there is decrease in cycle time $T$, procurement quantity $Q$ and the total cost $T_C$. 
Fig: 7.1.3: EFFECT OF $\beta$ ON PROCUREMENT QUANTITY

Fig: 7.1.4: EFFECT OF $\beta$ ON TOTAL COST
7.2 AN INVENTORY MODEL UNDER THE EFFECT OF PART PAYMENT ON SELLING PRICE AND CYCLE TIME.

In this section, we have introduced the aspect of part payment and taking demand as a function of selling price. A part of the purchased cost is to be paid during the permissible delay period. The part of the purchased cost to be paid and the time at which it has to be paid can be fixed up at the time of the deal of purchasing the goods. Sensitivity analysis of the model with respect to some of the vital parameters is carried out with the help of numerical example.

7.2.1 ASSUMPTIONS AND NOTATIONS:

1. The rate of replenishment is infinite.
2. Lead time is zero.
3. Shortages are not allowed.
4. The demand rate, \( R(P) \), is a function of selling price \( P \), which is defined by
   \[ R(P) = a - bP, \] where \( a >> b \) and \( a, b \) are constants.

Notations:

- \( C \) = Unit cost of an item.
- \( P \) = Selling price of an item.
- \( i \) = Inventory holding charge fraction per rupee per unit time.
- \( A \) = Ordering cost.
- \( I_e \) = Interest rate earned.
- \( I_c \) = Interest rate charged and \( I_c > I_e \)
- \( R \) = Deterministic annual demand.
- \( T^* \) = Permissible credit period expressed as a known fraction of a year.
- \( T_1 \) = Time (period) at which a part payment is to be made.
\[ \alpha = \text{Part of the purchase cost to be paid at } T_1 \]

\[ T = \text{Length of a cycle time.} \]

7.2.2 DETERMINATION OF COST FUNCTION:

The total cost function of an inventory system consists of an ordering cost, inventory holding cost, the interest charged and earned. Since \( T^* \) is constant and \( T \) is decision variable which depends on the lot size ordered. Mainly two cases may arise \( \text{Viz } T^* \leq T \) and \( T^* > T \). From Gujarathi C.C. et. al. (2001) the case of \( T^* > T \) is not feasible. So in this section we considered the case of \( T^* \leq T \) only, i.e. when permissible delay period is less than or equal to cycle time, the account has to be settled before the cycle ends and part payment \( \alpha CRT_1 \) is to be made at \( T_1 \). Where \( \alpha \) and \( T_1 \) are specified at the time of the deal. The cycle situation is shown below in the figure.

The components of interest earned:

- The interest earned during \((0,T_1)\) = \( \frac{PRT_1^2}{2} \)
- The cash on hand at \( T_1 = (PRT_1 - \alpha CRT_1) + \frac{PRT_1^2}{2} \)
• The interest earned during \((T_i, T^*)\)

\[
= \left[ (P - \alpha C)RT_i + \frac{PI_eRT_i^2}{2} \right] I_e(T^* - T_i) + \frac{PI_eR(T^* - T_i)^2}{2}
\]

• Cash on hand at \(T^*\)

\[
= \left[ (P - \alpha C)RT_i + \frac{PI_eRT_i^2}{2} \right] \left[ 1 + I_e(T^* - T_i) \right] + PI_eR(T^* - T_i)^2 \\
+ PR(T^* - T_i) - \left[ CRT^* - \alpha CRT_i \right]
\]

• The interest earned during \((T^*, T)\)

\[
= \left[ (P - \alpha C)RT_i + \frac{PI_eRT_i^2}{2} \right] \left[ 1 + I_e(T^* - T_i) \right] + PI_eR(T^* - T_i)^2 \\
+ PR(T^* - T_i) - \left[ CRT^* - \alpha CRT_i \right] \left[ I_e(T - T^*) \right]
\]

• The total interest earned

\[
I_{10n} = \frac{PI_eRT_i^2}{2} + \left[ (P - \alpha C)RT_i + \frac{PI_eRT_i^2}{2} \right] I_e(T^* - T_i) + \frac{PI_eR(T^* - T_i)^2}{2} \\
+ \left[ (P - \alpha C)RT_i + \frac{PI_eRT_i^2}{2} \right] \left[ 1 + I_e(T^* - T_i) \right] + PI_eR(T^* - T_i)^2 \\
+ PR(T^* - T_i) - \left[ CRT^* - \alpha CRT_i \right] \left[ I_e(T - T^*) \right]
\]

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• The interest charged during \((T^*, T)\)

\[
I_{\text{ch}} = \frac{CI_c R (T - T^*)^2}{2}
\]

• The total cost function per time unit is given by

\[
TC = \frac{A}{T} + \frac{C_i R T^2}{2T} - \frac{I_{E_0}}{T} + \frac{I_{\text{ch}}}{T}
\]

\[
= \frac{A}{T} + \frac{C_i RT}{2} - \frac{P_i R T_i^2}{2T}
\]

\[
- \left[ (P - \alpha C) R T_i + \frac{P_i R T_i^2}{2} \right] \frac{I_e (T - T_i)}{T} - \frac{P_i R (T^* - T_i)^2}{2T}
\]

\[
- \left\{ \left[ (P - \alpha C) R T_i + \frac{P_i R T_i^2}{2} \right] \left[ 1 + I_e (T^* - T_i) \right] + \frac{P_i R (T^* - T_i)^2}{T} \right\}
\]

\[
+ \frac{P R (T^* - T_i)}{T} - \left[ \frac{C R T^* - \alpha C R T_i}{T} \right] \left( I_e (T - T^*) \right)
\]

\[
+ \frac{C I_c R (T - T^*)^2}{2T}
\]

Net Profit \(NP = (P - C) R - TC\)
\[ NP(P,T) = (P - C)(a-bP) - \frac{A}{T} - \frac{C_l(a-bP)T}{2} + \frac{P_{I_e}(a-bP)T^2}{2T} \]

\[ + \left[ (P - C)(a-bP)T_i + \frac{P_{I_e}(a-bP)T^2_i}{2} \right] \frac{I_e(T^* - T_i)}{T} \]

\[ + \frac{P_{I_e}(a-bP)(T^* - T_i)^2}{2T} \]

\[ + \left[ (P - C)(a-bP)T_i + \frac{P_{I_e}(a-bP)T^2_i}{2} \right] \frac{1+I_e(T^* - T_i)}{T} \]

\[ + \frac{P_{I_e}(a-bP)(T^* - T_i)^2}{2T} + \frac{P(a-bP)(T^* - T_i)}{T} \]

\[ - \left[ T^* - \alpha T_i \right] \frac{C(a-bP)}{T} \left[ I_e(T^* - T^*) \right] - \frac{C_l(a-bP)(T - T^*)}{2T} \]

\[ \frac{\delta NP(P,T)}{\delta P} = (a + bC - 2bP) + \frac{C_l b}{2} + \frac{I_e T^2(a - 2bP)}{2T} \]

\[ + \left[ (a + \alpha bC - 2bP)T_i + \frac{I_e T^2(a - 2bP)}{2} \right] \]

\[ \left[ I_e(T^* - T_i) + I_e(T - T^*) \left[ 1 + I_e(T^* - T_i) \right] \right] \frac{T}{T} \]

\[ + \frac{I_e(T^* - T_i)^2(a - 2bP)}{2T} \left[ 1 + I_e(T - T^*) \right] \]

\[ + \frac{I_e(T - T^*)}{T} \left[ (a - 2bP)(T^* - T_i) + (T^* - \alpha T_i)b \right] \]
\[
\frac{\delta N(P,T)}{\delta P} = 0 \text{ gives; }
\]
\[
\begin{align*}
\left\{-5I_2^* T_1 a - bT(2P - C) - 5aT^2 (1 + I_2 T^*) + aT + 5CibT^2 + aTT^* I_2^* & \\
-bl_2 T^*(2PT - PT^* - \alpha CT.) + I_2^* T^3 (5aT^* - PbT^* + T_1 Pb) & \\
-I_1 T_1 Cb\alpha (T_1 - I_2 TT^* + I_1 I_2 T + I_2 T^2 - I_2 T_1 T^*) + 5I_2^* T_1^3 (aTT^* - aT^2) & \\
-aTT_1 - aT^2 + aT_1 T^* - 2PbTT^* + 2PbTT_1 + 2PbT^2 - 2PbT_1 T^* & \\
+ T^2 I_2^*(5aT - bPT + bPT^*) + I_2 TT^* Cb(T - T^*) & \\
+ 5C_i b(T^2 - 2TT^* + T^2) & \\
\right\}
\end{align*}
\]
\[
\int_T = 0 \quad (7.2.1)
\]
and hence from (7.2.1)
\[
P = -\left( -\frac{I_1 T_1 bC\alpha}{T} (T_1 - T^* - I_2 T^2 I_2 T^* + I_2 T^2 - I_1 T_1 T^*) - \frac{5aT^2 I_1^3 T}{T} (1 + I_2) - aI_2 T^3 \left( \frac{5T^2 I_1 + 5I_1^3 T^2 T^* - 5I_1 T^2 - 5I_2 T^3}{T} \right) \right.
\]
\[
-1 - \frac{T^* I_2 - I_2^2 I_1^2}{2} + Cb \left( 1 + \frac{5I_2 T^2 - I_2^2 T^2}{T} - I_1 T^* + T^* I_2 \right).
\]
Now, substituting the value of $P$ in $NP(P,T)$, we get

$$NP(T) = \left\{ a + I_s^2 T_b C \alpha T^* + C b - \frac{a l_e T^*}{2T} - \frac{I_s T_b a T_e}{2T} - \frac{T^* a l_e}{2T} + \frac{C i T b}{2} \right\}$$

$$- \frac{I_s^2 T_b a T^*}{2T} + \frac{I_s^2 T_b a T_e}{2T} + \frac{b \alpha C T^*}{I_s T_e} + \frac{T^* a l_e}{2T} + \frac{I_s T_b a T_e}{2T} + a l_e T^*$$

$$- I_s^2 T_b C \alpha - \frac{I_s T_b C b}{T} + \frac{T^* a l_e}{2} - \frac{I_s T_s a}{2} - \frac{I_s^2 T_b C \alpha T^*}{2}$$

$$+ \frac{I_s T_s b C \alpha T^*}{2} + \frac{I_s T_b a T^*}{2} - \frac{I_s T_s b C \alpha T^*}{2}$$

$$+ \frac{I_s T_b C \alpha T^*}{2} + \frac{T^* a l_e}{2} - \frac{I_s T_s a}{2} - \frac{I_s^2 T_b C \alpha T^*}{2}$$

$$+ \frac{b l_e T^*}{2T} - \frac{b l_e T^*}{T} - T^* b l_e T^* - 2b l_e T^*$$

$$+ \frac{b l_e T^*}{2T}$$
\[ \begin{align*}
- \frac{I^2 T^2 a T^{*2}}{2T} &+ \frac{I^2 T^3 a T^{*}}{2T} + \frac{b C a T^{*}}{T} + \frac{I^2 T^2 a T^{*}}{2T} + a l_e T^{*} \\
- I^2 T^2 b C a &- \frac{I^2 T^3 b C a}{T} + T^{*2} a l_e^2 + \frac{I^2 T^3 a}{2} - \frac{I^2 T^3 b C a T^{*2}}{T} \\
+ \frac{I^2 T^3 b C a T^{*}}{T} &+ \frac{I^2 T^3 a T^{*}}{2} - \frac{I^2 C b T^{*2}}{T} - C l_c b T^{*} + I_c C b T^{*} \\
+ \frac{C l_c b T^{*2}}{2} &+ \frac{T C l_c b}{2} \\
\begin{bmatrix}
-2b & + \frac{I^2 T^3 b}{T} - \frac{I^2 T^3 b T^{*}}{T} + I^2 T^3 b \\
+ \frac{I^2 T^3 b T^{*2}}{T} - I^2 T^3 b T^{*} & + \frac{T^{*2} b l_e^2}{T} - T^{*2} b l_e^2 \\
-2b l_e T^{*} & + \frac{b l_e T^{*2}}{T} - \alpha C \left[ a + \frac{b S_2}{S_1} \right] T_1 \\
S_2 I_e T_1^2 \left[ a + \frac{b S_2}{S_1} \right] & - \frac{2 S_1}{1 + I_e (T^{*} - T_1)} \left[ I_e (T^{*} - T_1) + I_e (T^{*} - T) \right].
\end{bmatrix}
\end{align*} \]
\[
S_1 \left[ a + \frac{b S_2}{S_1} \right] (T^* - T_1) - (T^* - \alpha T_1) C \left[ a + \frac{b S_2}{S_1} \right]
\]

\[
- C_l c \left( a + \frac{b S_2}{S_1} \right) \frac{(T - T^*)^2}{2T}
\]

where:

\[
S_1 = -2b + \frac{I^2 T_1^2 a}{T} \left( T_1 - T^* + I_x T^* \right) - I_x^2 T_1^2 b (T^* - T_1)
+ \frac{b I_x T^*}{T} \left( I_x T^* - I_x^2 T_1^* - I_x^2 T^* T - 2T + T^* \right)
\]

\[
S_2 = \frac{I^2 T_1^3 a}{2T} - \frac{I_x T_1^2 C b \alpha}{T} + C b - \frac{a l_x T^*}{2T} - \frac{T^3 a l_x^2}{2T} + a + \frac{C l T b}{2}
\]

\[
- \frac{I_x^2 T_1^3 a}{2} - \frac{I_x^2 T_1^2 a T^*}{2T} + \frac{b \alpha C T^* I_x T_1}{T} + \frac{I_x^2 T_1^2 a T^*}{2T} + a l_x T^*
\]

\[
+ I_x T_1 b \alpha C T \left( 1 - \frac{T^*}{T} \right) + \frac{a l_x^2 T^*}{2} \left( T^* + I_x T_1^2 \right) - I_x^2 T_1^2 b \alpha C
\]

\[
+ C l_c b \left( \frac{T^3}{2T} - T^* + \frac{T}{2} \right) + \frac{I_x C b T^*}{T} \left( I_x T_1^3 \alpha + T - T^* \right)
\]

And hence:

\[
\frac{d N P(T)}{d T} = \begin{bmatrix}
- \left( \frac{a l_x T^*}{2T^2} + \frac{I_x^2 T_1^2 a}{2T^3} \left( T_1 - I_x T_1 T^* + I_x T^* \right) \right) + \frac{C b}{2}
\end{bmatrix}
\]

\[
- \frac{I_x T_1 b C \alpha T^*}{T^2} \left( 1 - \frac{T}{T^*} - I_x^2 T^* + I_x \right) + \frac{I_x C b T^*}{T^2}
\]

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\[ + \frac{I_2^2 a T^*}{2 T^2} \left( T^* - T_i^2 \right) + \frac{C_l c b}{2 T^2} \left( T^2 - T_i^2 \right) \]  
\( (S_1) \)

\[ S_7 \left[ \frac{I_2^2 T^3 b}{T^4} \left( \frac{T^*}{T_i} - \frac{T^*}{T_i^2} + I_2 T_i T^* - \frac{T^2}{T_i^2} \right) \right] \]  
\( (S_8) \)

\[ \left( \frac{S_7}{S_1} - C \right) S_5 + \frac{A}{T^2} - \frac{C_l}{2} (S_8 + T S_8) - \frac{I_2 T_i^3}{2 S T_i} (S_8 S_i + S_2 S_i) \]

\[ \frac{S_2 I_2 T_i^2 S_8}{2 S T_i} \left( \frac{S_8}{S_1} + \frac{1}{T_i} \right) + \left[ - \left( \frac{I_2 T_i^3 a T^*}{2 T^2} \left( T^* - 1 \right) + \frac{C_l b}{2} \right) \right] \]

\[ \frac{a I_2 T^2}{2 T^2} \left( 1 + T^* + I_2 T^* \right) + \frac{I_2 T_i^3 a}{2 T^2} + \frac{C_l c b}{2 T} \left( T^2 - T_i^2 \right) \]

\[ \frac{b \alpha C l T_i}{T^2} \left( T_i + I_2 T^2 - I_2 T_i T^* - T^* \right) - \frac{I_2 T_i^2 a T^*}{2 T^2} \]

\[ + \frac{I_2 C l b T_i^2}{T^2} \left( T^* - T_i - I_2 T^2 - \frac{T^3}{T_i} + I_2 T_i T^* - \frac{T^2}{I_2 T_i} \right) \]

\[ \left( \frac{S_7}{S_1} + \alpha C \right) S_5 T_i \left( S_6 + \alpha C \right) S_1 \]

\[ - \left( \frac{S_7}{S_1} + \alpha C \right) S_5 T \left( S_5 + \alpha C \right) \left( S_8 S_i + S_2 S_i \right) \]

\[ - \left( \frac{S_7}{S_1} + \alpha C \right) S_5 T_i \]

\[ - \left( \frac{S_7}{S_1} + \alpha C \right) S_8 T_i \]

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\[
\begin{align*}
&-\frac{I_eT^2S_4S_8}{S_1} S_6 + \frac{I_e(T' - T_0)^2 S_8}{2S_1T} \left( \frac{S_2}{T} \right) \\
&- \frac{I_eS_8}{S_1T} \left( S_2(T' - T_0) + S_1(T' - \alpha T_0)C \right) \\
&+ I_e(T - T') \left( \frac{T - T_0}{S_4T} \right) \left( S_3S_4S_6 - S_1S_4S_6 - S_2S_5 \right) \\
&- \left( \frac{T' - \alpha T_0}{S_3T + S_4} + \frac{S_2S_6(T' - T_0)}{T^2} \right) \\
&+ \frac{Cl_e(T - T')^2 S_6}{T} \left( \frac{1}{2T} - \frac{S_3}{2S_4} - \frac{1}{(T - T')} \right)
\end{align*}
\]

where;

\[
S_1 = -2b + \frac{I_e^2T^2b}{T} \left[ \frac{T^3}{T^3} + T_0 - T' + I_eT^2 - I_eT'T + I_eT_0(T - T') \right]
\]

\[
S_2 = \frac{I_eT_0CbaT^*}{T} \left[ 1 - \frac{T}{T} + I_eT - I_eT^* - \frac{I_eTT_0}{T^*} + I_eT_0 \right]
\]

\[
+ \frac{al_eT^*}{2T} \left( 2T - T' - I_eT^2 - I_eT_0^2T^* + I_eT_0^2 + I_eTT^* + I_eTT_0^2 \right)
\]

\[
+ I_eT_0 \left[ \frac{I_eT^3a}{2T} [1 + \Pi T_e] + Cb \left[ \frac{iT}{2} + I_eT^* - \frac{I_eT^2}{T} \right] \right]
\]

\[
+ \frac{Cl_eT}{2T} \left( T^2 + T_0^2 \right)
\]

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The optimum value of $T = T_0$ can be obtained by solving $\frac{dNP(T)}{dT} = 0$. This optimum value $T = T_0$ maximizes the Net profit whenever $\frac{d^2NP(T)}{dT^2} < 0$. 

$$S_3 = \frac{I_e^2T^3b}{T} \left[ T^* - T_i - I_eT^* + I_eT_iT^* - \frac{T^3}{T_i^2} - \frac{T^2}{I_eT_i} \right]$$

$$S_4 = \frac{aI_e^2T^*}{2T^2} \left[ T^* + I_eT^3 + I_e^2(T^* - 1 - T_i) \right] + \frac{I_e^2T_i^3a}{2T^2}$$

$$\frac{bC\alpha T_i}{T} \left[ T_i - T^* + I_eT^2 - I_eT_iT^* \right] + \frac{Cb}{2} \left( i + I_e \right)$$

$$+ \frac{CbT^*}{2T^2} \left( 2I_e - I_c \right)$$

$$S_5 = \frac{bS_a}{S_1} - \frac{bS_aS_5}{S_2}$$

$$S_6 = \left[ I_e(T^* - T_i) + I_e(T - T^*)(1 + I_e(T^* - T_i)) \right]$$

$$S_7 = \frac{b\alpha C T^* I_e T_i}{T} \left[ 1 + I_eT^* - \frac{I_eT_iT}{T^*} - I_eT^* + I_eT_i \right] + a$$

$$+ \frac{I_e^2T_i^3aT^*}{2T} \left[ 1 - I_eT^* + I_eT_i + I_eT \right] + I_eCbT^* \left( 1 - \frac{T^*}{T} \right)$$

$$+ Cb \left( 1 + \frac{iT}{2} - I_cT^* + \frac{I_cT^2}{2T} + \frac{I_cT}{2} \right) - \frac{I_e^2T_i^3a}{2T} \left( 1 + I_eT \right)$$

$$+ aI_eT^* \left( 1 - \frac{T^*}{2T} - \frac{I_eT^2}{2T} + \frac{I_eT^*}{2} \right)$$

$$S_8 = \left( a + \frac{bS_a}{S_1} \right)$$
7.2.3 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS:

Consider,

- \( a = 1000 \) units per year
- \( b = 5 \)
- \( C = \text{Rs. 10 per unit} \)
- \( i = 0.1 \) per rupee per unit time
- \( I_c = 0.11 \) per rupee per unit time
- \( I_c = 0.18 \) per rupee per unit time
- \( \alpha = 0.2 \)
- \( T^* = 0.25 \) years
- \( T_1 = \beta \cdot T^* = 0.125 \) \((\beta = 0.5)\)
- \( A = 250 \) per order

The effect of variations in values of \( \alpha, \beta, I_c, I_o, a, b, C \) and \( T^* \) on decision variable and Total cost of an inventory system are given in the following tables. Also corresponding graphs are plotted for the sake of comparisons in fig.7.2.1 to fig.7.2.8.
### TABLE 7.2.1
**Variations in α and β**

<table>
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<tr>
<th>α</th>
<th>0.2</th>
<th>0.4</th>
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<td>P</td>
<td>NP</td>
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</tr>
<tr>
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<td>45294</td>
<td></td>
</tr>
<tr>
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<td>105</td>
<td>45292</td>
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</tr>
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<td>0.80</td>
<td>105</td>
<td>45358</td>
<td></td>
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<td>105</td>
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</tr>
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<td>0.80</td>
<td>105</td>
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<td>45786</td>
<td></td>
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<tr>
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<td></td>
<td>0.80</td>
<td>105</td>
<td>44464</td>
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**Observations:**

1. As α - the part of the purchase cost to be paid at T₁ increases, there is decrease in the net profit but increase in α doesn’t make any effect on cycle time T and selling price P.

2. As β increases, i.e. as we make our part payment late, there is increase in the net profit but increase in β doesn’t make any effect on cycle time T and selling price P.
TABLE 7.2.2
Variations in $I_c$ and $I_e$

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<tr>
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<th>0.16</th>
<th>0.17</th>
<th>0.18</th>
<th>0.19</th>
<th>0.20</th>
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<td>$I_e$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td>T</td>
<td>0.79</td>
<td>0.77</td>
<td>0.76</td>
<td>0.75</td>
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<td>P</td>
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<td>105.32</td>
<td>105.32</td>
<td>105.32</td>
</tr>
<tr>
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<td>NP</td>
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<td>0.77</td>
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</tr>
<tr>
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<td>105.32</td>
<td>105.33</td>
<td>105.33</td>
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<td>0.79</td>
<td>0.77</td>
</tr>
<tr>
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<td>105.33</td>
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</tr>
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<td>NP</td>
<td>45658</td>
<td>45649</td>
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<td>45630</td>
</tr>
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</table>

Observations:

1. As interest charge $I_c$ increases there is slight decrease in the cycle time $T$ and net profit but it doesn't make any remarkable effect on selling price $P$.
2. As interest earned $I_e$ increases there is slight increase in the cycle time $T$ and net profit but it doesn't make any remarkable effect on selling price $P$. 
### Table 7.2.3
Variations in a and b

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>T</th>
<th>P</th>
<th>NP</th>
</tr>
</thead>
<tbody>
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</tr>
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<td>1000</td>
<td>0.89</td>
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<td>79605</td>
</tr>
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<td>1200</td>
<td>0.89</td>
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<td>117.86</td>
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<td>0.76</td>
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<td>91.00</td>
<td>30979</td>
</tr>
</tbody>
</table>

**Observations:**

1. As a (indirectly the demand) increases there is increase in the selling price P as well as the net profit NP but decrease in the cycle time T.

2. As b increases (i.e. indirectly the demand decreases) there is decrease in the selling price P, cycle time T and the net profit NP.
**TABLE 7.2.4**

Variations in unit cost $C$

<table>
<thead>
<tr>
<th>C</th>
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<th>P</th>
<th>NP</th>
</tr>
</thead>
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<td>10</td>
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<td>20</td>
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**Observation:**

Increase in unit cost $C$ decreases the cycle time $T$ and the net profit $NP$ but doesn’t make any remarkable effect on selling price $P$.

**TABLE 7.2.5**

Variations in $T^*$

<table>
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<th>$T^*$</th>
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<th>NP</th>
</tr>
</thead>
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<td>0.42</td>
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**Observation:**

Increase in the credit period $T^*$ increases the cycle time $T$ and the net profit $NP$ but doesn’t make any remarkable effect on selling price $P$. 
Fig : 7.2.1 : EFFECT OF $\alpha$ ON NET PROFIT

Fig : 7.2.2 : EFFECT OF $\beta$ ON NET PROFIT

Fig : 7.2.3 : EFFECT OF $I_c$ ON NET PROFIT
Fig 7.2.4: EFFECT OF le ON NET PROFIT

Fig 7.2.5: EFFECT OF DEMAND a ON NET PROFIT

Fig 7.2.6: EFFECT OF b ON NET PROFIT