CHAPTER 2

ANALYSIS OF BINARY SIGNALING SCHEME
IN FIBER OPTIC CDMA

2.1 INTRODUCTION

Spread spectrum CDMA techniques utilize the excess channel bandwidth in return for multiple user capability and security. The additional complexity involved in the analysis of optical CDMA systems, is the random nature of the APD photodetection process apart from multiple user interference. This photodetection process is governed by a complex probability distribution function which makes exact analysis intractable and approximations are solicited.

In this chapter, the performance analysis of CDMA communications over multimode fiber optic channel is carried out, including the effects of modal noise, signal dependent shot noise, laser leakage currents, APD bulk and surface leakage currents, APD random gain distributions, receiver thermal noise and multiple user interference. For the system model proposed and assumptions made, the moment generating function of the receiver output is derived and different methods to obtain the BER probability from this moment generating function are given. The performance of both OOO and EWO signaling schemes is investigated using Saddlepoint Approximation and Gaussian Approximation analyses for evaluating the BER probability. OOCs, Prime Codes and QCCs are used as the addressing sequences.
2.2 METHODOLOGIES USED FOR THE ANALYSIS

The analysis carried out is based on making a decision about the value of the data bit $b_0^{(1)}$ transmitted in the interval $(0,T)$ by the first user. The decision statistic $Y$ is compared to a threshold $\theta$ to decide if a '0' was sent if $Y < \theta$, or if a '1' was sent if $Y \geq \theta$. The data bits are assumed to be equiprobable. Then the bit error probability $P_{be}$ is given by,

$$P_{be} = \frac{1}{2} [q_0(\theta) + q_1(\theta)]$$  \hspace{1cm} (2.1)

where,

$$q_0(\theta) = \Pr[Y \geq \theta / b_0^{(1)} = 0]$$  \hspace{1cm} (2.2a)

$$q_1(\theta) = \Pr[Y < \theta / b_0^{(1)} = 1]$$  \hspace{1cm} (2.2b)

The probabilities $q_0(\theta)$ and $q_1(\theta)$ are calculated from the conditional moment generating functions $\mu_j(s)$, $j = 0,1$ of the receiver output given by,

$$\mu_j(s) = \mathbb{E}[\exp(sY) / b_0^{(1)} = j ; j = 0,1]$$  \hspace{1cm} (2.3)

This conditional moment generating function is derived in detail for OOO and EWO signaling schemes in section 2.3.

2.2.1 Saddlepoint Approximation Method

The pdf of the decision statistic $Y$ can be written in the form of the contour integral (Helstrom et al., 1992),

$$f_j(Y) = \frac{\mu_j(s) \exp(-isY)}{2\pi i} \frac{ds}{c_j - i\infty} ; c_j < s^*$$  \hspace{1cm} (2.4)
where \( s^* \) is the branch point of the function \( M_g(s) \) and is given by,

\[
s^* = \ln M^* - b \ln[1 + a(M^*-1)]
\]

(2.5)

with

\[
M^* = \frac{1-a}{a(b-1)} > 1
\]

(2.6)

which is derived by setting \( ds/dm = 0 \). The probabilities \( q_0(\theta) \) and \( q_1(\theta) \), calculated using Laplace inversion integrals is given by,

\[
q_0(Y) = \frac{c_0+i\infty}{c_0-i\infty} \int s^{-1} \mu_0(s) \exp(-sY) \frac{ds}{2\pi i} ; 0 < c_0 < s^* \quad (2.7a)
\]

\[
q_1(Y) = -\frac{c_1+i\infty}{c_1-i\infty} \int s^{-1} \mu_1(s) \exp(-sY) \frac{ds}{2\pi i} ; c_1 < 0 \quad (2.7b)
\]

The Saddlepoint method can be conveniently applied to approximate and evaluate these contour integrals. The integrands in Equation (2.7) possess saddlepoints \( c_0 \) and \( c_1 \) respectively on the \( \Re s \) axis, so that the integration passes through the saddlepoint. These integrals are rewritten in the form,

\[
q_j(Y) = \int_{c_j} \exp \{\psi_j(s)\} \frac{ds}{2\pi i} ; \quad j = 0, 1 \quad (2.8)
\]

where,

\[
\psi_j(s) = \ln \mu_j(s) - \ln[-(-1)^j s] -sY \quad ; \quad j = 0, 1 \quad (2.9)
\]
The path of the integration is deformed to pass through the saddlepoint of the integrand.

\[ \psi_j'(s) = \ln \mu_j(s)' - \gamma - s^{-1} \]  \hspace{1cm} (2.10)

By equating the first order derivative of \( \psi_j(s) \) to zero, the saddlepoints are evaluated. This can be easily solved by the Newton's method because the phase of \( \psi_j'(s) \) is convex \( U \) for \( s < 0 \) and for \( s \) in the region \( 0 < s < s^* \) respectively. To eliminate the need to solve for \( M_g(s) \) from \( s \) in Equation (1.38), the saddlepoints can be searched by using its image on the \( M \)-plane. Taking an initial trial value of \( M \), a new one is obtained at each stage by the iteration,

\[ M_g(s) + M_g(s) - \frac{\psi_j'(s)}{\psi_j''(s)} M_g'(s) \]  \hspace{1cm} (2.11)

where,

\[ \psi_j''(s) = \ln \mu_j(s)'' + s^2 \]  \hspace{1cm} (2.12)

To restrict the solution in the region \( M < M^* \) or equivalently \( s < s^* \), the initial values are taken as \( (1+M^*)^2 \) for \( j = 0 \) and 0.9 for \( j = 1 \). The iteration is stopped when the variation of \( M \) falls below \( 10^{-5} \).

However to obtain the saddlepoints for EWO signaling condition \( t \) is required to solve for \( M_g(s) \) from \( s \). A simple iterative procedure is developed and outlined.

\[ [M_g(s)]_{i+1} = \frac{C_1 \exp(s) (1-a)}{1 - a C_1 \exp(s)} \]  \hspace{1cm} (2.13)
where
\[ C_i = \{(1 + a(|Mg(s)|_i - 1))^{b-1}\} \] (2.14)
and
\[ [Mg(s)]_0 = \frac{\exp(s) (1-a)}{1 - a \exp(s)} \] (2.15)

The moment generating function of the APD gain \( g \), calculated using the above procedure is shown in Figures 2.1 to 2.3 for various values of \( G \) and \( K_{eff} \). It is noted that the curves exactly coincide with the actual values.

In the neighborhood of the saddlepoints the integrands of Equation (2.8) can be approximated by Gaussian functions and when these are integrated over \((-\infty, \infty)\) the saddlepoint approximation is obtained (Helstrom et al., 1992).

\[ q_j(Y) = \frac{1}{\sqrt{2\pi\psi_j''(s_j)}} \exp \left\{ \psi_j(s_j) \right\} ; j = 0, 1 \] (2.16)

where \( s_j \) is the saddlepoint.

For a fixed signal strength, the decision level '0' minimizing the BER probability is found by solving the equation,

\[ f_0(\theta) = f_1(\theta) \] (2.17)
or equivalently the equation

\[ R(\theta) = \ln |f_0(\theta)| - \ln |f_1(\theta)| = 0 \] (2.18)
Figure 2.1 $M_g(s)$ versus $s$ calculated using iterative procedure for $G=50$ and $K_{eff}=0.01, 0.02, 0.05$
Figure 2.2  $M_g(s)$ versus $s$ calculated using iterative procedure for $G=100$ and $K_{ef}=0.01, 0.02, 0.05$
Figure 2.3 $M_g(s)$ versus $s$ calculated using iterative procedure for $G=250$ and $K_{eff}=0.01, 0.02, 0.05$
The secant method is applied to solve this equation. At each stage a trial value of $\theta$ is replaced by the iteration,

$$
\theta \rightarrow \theta - \frac{R(\theta)}{R(\theta + \Delta\theta) - R(\theta)} \Delta\theta
$$

(2.19)

where $\Delta\theta$ is of the order of 0.01 $\theta$. An initial trial value of $\theta$ can be chosen at the midpoint between the conditional means $m_{Y|0}$ and $m_{Y|1}$,

$$
\theta = \frac{(m_{Y|0} + m_{Y|1})}{2}
$$

(2.20)

The density functions $f_j(\theta)$ given by Equation (2.4) can be approximated by using the saddlepoint approximation as,

$$
f_j(\theta) = \frac{1}{\sqrt{2\pi} \bar{\psi}_j'(s_j)} \exp \left[ \bar{\psi}_j (s_j) \right] ; \quad j = 0, 1
$$

(2.21)

with the modified phase

$$
\bar{\psi}_j(s) = \ln[\mu_j(s)] - s\theta
$$

(2.22)

2.2.2 Gaussian Approximation Method

In this method the BER probability is evaluated by assuming that the decision statistic $Y$ is approximately gaussian. Though gaussian approximation underestimates the BER, it is expected to be close to the true performance when codes with large duty cycle are used. The pdf of the decision statistic $Y$ can be written as,
\[ f_j(Y) = \frac{1}{\sqrt{2\pi}\sigma_{Yj}} \exp\left(\frac{(y - m_{Yj})^2}{2\sigma_{Yj}^2}\right) \quad ; \quad j = 0, 1 \quad (2.23) \]

The conditional means \( m_{Yj}, j = 0, 1 \) and variances \( \sigma_{Yj}^2, j = 0, 1 \) of \( Y \) are evaluated from the conditional moment generating functions of \( Y \) as,

\[ m_{Yj} = E[Y | b_0^{(1)} = j, j = 0, 1] = [\ln p_j(s)]_{s=0}^{s} \quad (2.24) \]

and

\[ \sigma_{Yj}^2 = \text{Var} [Y | b_0^{(1)} = j, j = 0, 1] = [\ln p_j(s)]''_{s=0} \quad (2.25) \]

The error probabilities \( q_j(\theta), j = 0, 1 \) are then given by

\[ q_j(\theta) = \text{erfc}\left(\frac{\theta - m_{Yj}}{\sigma_{Yj}}\right) \quad (2.26) \]

where

\[ \text{erfc}(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{y^2}{2}\right) \, dy \quad (2.27) \]

The optimum threshold is given by,

\[ \theta = \frac{-k_2 \pm \sqrt{k_2^2 - 4k_1k_3}}{2k_1} \quad (2.28) \]

where,

\[ k_1 = \sigma_{Y1}^2 - \sigma_{Y0}^2 \quad (2.29a) \]
2.3 SADDLEPOINT APPROXIMATION ANALYSIS

The BER probability is evaluated for both binary OOO and EWO signaling schemes by carrying out the analysis of the Saddlepoint Approximation (SA) method under chip synchronous and chip asynchronous conditions.

2.3.1 Binary OOO signaling scheme

In this scheme, the signal term is present at the receiver output only when $b_0^{(1)} = 1$, otherwise only the noise terms are present. The conditional moment generating function of the receiver output defined by Equation (2.3) is given by the product of the moment generating functions of all the terms comprising the decision statistic $Y$.

$$
\mu_j(s) = \mu_j(s) \mu_{bl}(s) \mu_{sl}(s) \mu_{N}(s) \tag{2.30}
$$

where,

$$
\mu_{bl}(s) = E \left[ \exp \left\{ \int_{-\infty}^{\infty} \lambda_{bl} \{ \exp \{ s g h(T-\tau) \} - 1 \} \, d\tau \right\} \right] / b_0^{(1)} = j ; j = 0, 1 \tag{2.31}
$$

$$
\mu_{sl}(s) = E \left\{ \exp \{ s Z_{s_i}(T) \} \right\} = \exp \left\{ \lambda_{sl} w T_c [\exp(s) - 1] \right\} \tag{2.32}
$$

$$
\mu_{N}(s) = E \left\{ \exp[s Z_N(T)] \right\} = \exp \left\{ \frac{s^2 \sigma_N^2}{2} \right\} \tag{2.33}
$$
with
\[
\sigma_N^2 = \frac{N_o w T_c}{2} \quad (2.34)
\]
\[
\mu_j(s) = E\left\{ \exp \left[ \int_{-\infty}^{\infty} \lambda_j(\tau) \left( \exp \left[ \text{sg } h(T-\tau) \right] - 1 \right) d\tau \right] / b_0^{(1)} = j ; j=0,1 \right\}
\]

Since the modal noise \( \gamma \) is independent of APD gain \( g \),
\[
\mu_j(s) = E[\exp (\gamma u) / b_0^{(1)} = j ; j = 0, 1] \quad (2.35)
\]

where,
\[
u = \begin{cases} u_s + u_b + \sum_{k=2}^{K} u_k \\ \end{cases}
\]
\[
u_s = j \lambda'_s w T_c [M_g(s) - 1] \quad (2.37)
\]
\[
u_b = \frac{K \lambda_s}{\sigma_s} w T_c [M_g(s) - 1] \quad (2.38)
\]
\[
u_{k0} = 0 \quad (2.39)
\]

and
\[
u_{k1} = \begin{cases} \lambda'_s w T_c [M_g(s)-1] P_{x,y}(l) & \text{chip synchronous case} \\ \lambda'_s [M_g(s)-1] [P_{x,y}(l) T_c + \{P_{x,y}(l+1) - P_{x,y}(l)\} (\tau_k - lT_c)] & \text{chip asynchronous case} \end{cases}
\]
\[
\quad (2.40)
\]

for \( 0 \leq l \leq N \), \( lT_c \leq \tau_k \leq (l+1)T_c \). For the chip synchronous case, \( \tau_k \) is an integer multiple of \( T_c \). To obtain \( \mu_j(s) \), the variable \( s \) in Equation (1.52) is replaced by the variable \( u \) given by Equation (2.37).
Hence

\[ \mu_j(s) = \exp\left\{ \frac{u_s}{2n_\gamma} \right\} \exp\left\{ \frac{u_b^2}{2n_\gamma} \right\} \]

\[ K \sum_{k=2} \exp \left( \gamma u_{kj} \right) \; j = 0, 1 \quad (2.42) \]

where

\[ E[\exp(\gamma u_{kj})] = \frac{1}{4NT_c} \sum_{x,y \in \{k, \bar{k}\}} \delta_{kj} \left\{ u_{kj} + \frac{u_{kj}^2}{2n_\gamma} \right\} d\tau_k \]

\[ = \frac{1}{4NT_c} \sum_{x,y \in \{k, \bar{k}\}} \delta_{kj} \left\{ u_{kj} + \frac{u_{kj}^2}{2n_\gamma} \right\} d\tau_k \]

\[ = \frac{\exp(-n_\gamma/2)}{4N} \sum_{x,y \in \{k, \bar{k}\}} \sum_{l=0}^{N-1} S_{x,y}(l,s) \quad (2.43) \]

The detailed derivation of Equation (2.43) is given in Appendix 1 for both chip synchronous and chip asynchronous conditions. For the chip synchronous case,

\[ S_{x,y}(l,s) = \exp\left\{ \frac{V_{x,y}(l,s)^2}{2} \right\} \quad (2.44) \]

where

\[ V_{x,y}(l,s) = \frac{P_{x,y}(1) \lambda \cdot T_c \{M_g(s) - 1\}}{\sqrt{n_\gamma}} + \sqrt{n_\gamma} \quad (2.45) \]
For the chip asynchronous case, the integration in Equation (2.43) is carried out by replacing the integrand with the truncated version of its expansion and hence,

\[
S_{x,y}(l,s) = 1 + \frac{1}{6} \left[ \frac{V_{x,y}(l+1,s)^3 - V_{x,y}(l,s)^3}{V_{x,y}(l+1,s) - V_{x,y}(l,s)} \right] + \frac{1}{40} \left[ \frac{V_{x,y}(l+1,s)^5 - V_{x,y}(l,s)^5}{V_{x,y}(l+1,s) - V_{x,y}(l,s)} \right] + \frac{1}{336} \left[ \frac{V_{x,y}(l+1,s)^7 - V_{x,y}(l,s)^7}{V_{x,y}(l+1,s) - V_{x,y}(l,s)} \right]
\]

(2.46)

with \(V_{x,y}(l,s)\) as defined by Equation (2.45). The conditional moment generating function of the decision statistic \(Y\) is then given by,

\[
\mu_j(s) = \exp \left\{ \frac{s^2 \sigma_N^2}{2} + \lambda_{bl} wT_c [M_g(s) - 1] + \lambda_{bl} wT_c [\exp(s)-1] \right\}
\]

\[
\exp \left\{ u_s \frac{u_s^2}{2n_T} + u_b \frac{u_b^2}{2n_T} \right\} [\Gamma(s)]^{(K-1)}
\]

(2.47)

where,

\[
\Gamma(s) = E \left[ \exp \left( \gamma u_k \right) \right]
\]

(2.48)

Therefore,

\[
\ln [\mu_j(s)] = \frac{s^2 \sigma_N^2}{2} + \lambda_{bl} wT_c [M_g(s) - 1] + \lambda_{bl} wT_c [\exp(s)-1]
\]

\[
+ u_s \frac{u_s^2}{2n_T} + u_b \frac{u_b^2}{2n_T} + (K-1) \ln \left[ \Gamma(s) \right]
\]

(2.49)
\[ \ln [\mu_1(s)]' = s\sigma_N^2 + \lambda_t w T_c M_g'(s) + \lambda_s w T_c \exp(s) \]
\[ + u_s' + \frac{u_s u_b'}{n_\gamma} + u_b' + \frac{u_b u_b'}{n_\gamma} + (K-1) \left[ \frac{\Gamma'(s)}{\Gamma(s)} \right] \] (2.50)

and

\[ \ln [\mu_1(s)]'' = \sigma_N^2 + \lambda_t w T_c M_g''(s) + \lambda_s w T_c \exp(s) + u_s'' + \frac{u_s u_s''}{n_\gamma} \]
\[ + \frac{u_s'^2}{n_\gamma} + \frac{u_b u_b''}{n_\gamma} + \frac{u_b''}{n_\gamma} + (K - 1) \left[ \frac{\Gamma''(s)}{\Gamma(s)} - \frac{\Gamma'(s)^2}{\Gamma(s)^2} \right] \] (2.51)

where,

\[ u_s' = j\lambda_s' w T_c M_g'(s) \] (2.52)
\[ u_s'' = j\lambda_s' w T_c M_g''(s) \] (2.53)

\[ u_b' = \frac{K\lambda_s}{\alpha_e} w T_c M_g'(s) \] (2.54)

\[ u_b'' = \frac{K\lambda_s}{\alpha_e} w T_c M_g''(s) \] (2.55)

\[ M_g'(s) = \frac{M_g(s) \{1 + a [M_g(s) - 1]\}}{a(b-1) [M^* - M_g(s)]} \] (2.56)
The derivatives $S_{x,y}'(l,s)$ and $S_{x,y}''(l,s)$ are given in Appendix 2, for the chip synchronous and chip asynchronous conditions. Then using Equations (2.9) through (2.22), the Saddlepoint Approximation for $q_j(0), j = 0,1$ are obtained and the BER probability $P_{be}$ is evaluated using Equation (2.1).

The following default parameter values (Lam et al., 1992) are considered for the numerical analysis.

\begin{align*}
R_b &= \text{Bit rate} = 10 \text{ Mbps} \\
R &= \text{APD load resistance} = 1030 \Omega \\
T_e &= \text{Pre-amplifier equivalent noise temperature} = 1100^\circ\text{K} \\
N_r/2 &= \text{Two sided p.s.d of thermal noise} \\
&= 1.15 \times 10^{15} \text{ J/} \Omega \text{/C}^2 \\
\lambda_{bl} &= \text{APD bulk leakage current photon rate} \\
&= 6.12 \times 10^5 \text{ photons/sec.}
\end{align*}
\[ \lambda_{sl} = \text{APD surface leakage current photon rate} \]
\[ = 7.46 \times 10^{10} \text{ photons/sec.} \]
\[ G = \text{Mean gain of APD} = 250 \]
\[ K_{\text{eff}} = \text{Effective ionization ratio} = 0.01 \]
\[ \eta = \text{APD Quantum efficiency} = 0.75 \]
\[ \alpha = \text{Laser modulation extinction power ratio} = 10000 \]

Low weight OOCs with \( N = 341, w = 5 \), auto- and cross-correlation constraints of unity and Prime codes and QCCs with \( N = 121, w = p = 11 \) are considered for the numerical analysis. The mean signaling rate i.e. the mean signal photon count per bit is given by

\[ \mu_s = \frac{\lambda_s T_s w}{2} \quad (2.60) \]

The BER probability for OOO signaling with OOCs is plotted against the mean signaling rate \( (\mu_s) \) in Figure 2.4 for \( K = 2 \) and 4. The performance curves are shown for different values of the modal noise parameter \( n_\gamma = 100, 200, 400 \). For \( \mu_s = 500, G = 250 \) and \( K = 2 \), \( P_{\text{be}} \) is \( 10^{-13} \) with \( n_\gamma = 400 \). As \( n_\gamma \) decreases to 100, \( P_{\text{be}} \) increases to \( 10^{-7} \). Thus smaller values of \( n_\gamma \) correspond to severe modal noise fluctuations and require higher signaling rates to maintain the same value of \( P_{\text{be}} \). To achieve a \( P_{\text{be}} \) of \( 10^{-8} \), \( \mu_s = 800, 300 \) and 250 are required for \( n_\gamma = 100, 200 \) and 400 respectively. For \( K = 4 \) and \( P_{\text{be}} = 10^{-5}, \mu_s = 300, 200 \) and 160 are required for \( n_\gamma = 100, 200 \) and 400 respectively. Thus modal noise fluctuations significantly affect the BER probability and it is also noted from the figure that the effect of modal noise on \( P_{\text{be}} \) decreases as \( K \) increases. At \( \mu_s = 1000 \) the \( P_{\text{be}} \) achieved for \( K = 2 \) increases by six orders of magnitude \( (10^{-19} \text{ to } 10^{-13}) \) when \( n_\gamma \) decreases from 400 to 200 whereas \( P_{\text{be}} \) for \( K = 4 \) increases by only one order magnitude \( (10^{-7} \text{ to } 10^{-6}) \) for the same variation in \( n_\gamma \). Similar performance curves are obtained for Prime codes and QCCs and are shown in Figures 2.5 and 2.6 respectively.
Figure 2.4 BER probability vs mean signaling rate for OOO signaling with OOCs in the presence of modal noise, \( G=250 \) (SA)
Figure 2.5 BER probability vs mean signaling rate for OOO signaling with Prime codes in the presence of modal noise, G = 250 (SA)
Figure 2.6 BER probability vs mean signaling rate for OOO signaling with QCCs in the presence of modal noise, $G = 250$ (SA)
The three codes are compared at various levels of modal noise in Figures 2.7 and 2.8. It is observed that the $P_{be}$ achieved by these codes with $K = 2$ are similar for $\mu_s < 200$. At higher values of $\mu_s$ (1000 signal photon counts/bit) Prime codes are better than OOCs and QCCs with respect to the BER probability by one order of magnitude. For $K = 4$ the $P_{be}$ achieved is $10^{-10}$, $10^{-8}$ and $10^{-7}$ with Prime codes, QCCs and OOCs respectively, where Prime codes are better than QCCs and OOCs by two and three orders of magnitude respectively. This shows that the performance of these codes differ significantly when $K$ increases. For $K = 4$ and $P_{be} = 10^{-7}$, Prime codes require only 350 signal photon counts/bit, whereas QCCs and OOCs require 750 and 1100 signal photon counts/bit respectively with $n_\gamma = 400$.

The BER probability with OOCs in the presence and absence of modal noise are compared in Figure 2.9 for $K = 2$, $\mu_s = 500$ and $G = 100$. It is noted from the figure that as $n_\gamma > 162$, $P_{be}$ approaches $10^{-9}$ which is the error probability in the absence of modal noise. Similar comparisons are shown for Prime codes and QCCs in Figures 2.10 and 2.11 and the limiting values are found to be $n_\gamma > 330$ with $P_{be} = 10^{-8}$ and $n_\gamma > 152$ with $P_{be} = 10^{-7}$ respectively.

The maximum $K$ the system can support for $P_{be} = 10^{-9}$ with the given system parameter values is shown in Figure 2.12. At $\mu_s = 500$, Prime Codes support 3 active users, whereas OOCs and QCCs support only 2 active users. OOCs are better than QCCs at higher mean signaling rates, but Prime codes show good performance at all signaling rates.

### 2.3.2 Binary EWO signaling scheme

In this scheme, the signal term will be present for both $b_0^{(1)} = 1$ and $b_0^{(1)} = 0$. In the receiver, conceptually the APD output is separately multiplied by $a^{(1)}(t)$ and $\bar{a}^{(1)}(t)$ and integrated for the interval $(0,T)$ to form
Figure 2.7 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for OOO signaling schemes with $K = 2$ and $G = 250$ (SA)
Figure 2.8 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for OOO signaling schemes with $K = 4$ and $G = 250$ (SA)
Figure 2.9 Comparison of BER probability in the presence and absence of modal noise for OOO signaling schemes with OOCs, $\mu_\nu = 500$, $G = 100$ and $K = 2$ (SA)
Figure 2.10 Comparison of BER probability in the presence and absence of modal noise for OOO signaling schemes with Prime codes, $\mu_s = 500$, $G = 100$ and $K = 2$ (SA)
Figure 2.11 Comparison of BER probability in the presence and absence of modal noise for OOO signaling schemes with QCCs, $p_8 = 500$, $G = 100$ and $K = 2$ (SA)
Figure 2.12 Maximum $K$, the system can support for $P_{be} = 1e-9$, employing OOO signaling scheme with OOCs, Prime codes and QCCs (SA)
two sums which represent the number of secondary electrons collected in \( a^{(1)}(t) \) and \( a^{(1)}(t) \) windows respectively. The decision statistic \( Y \) is the difference of these two sums. The conditional moment generating function of the receiver output is then given by,

\[
\mu_j(s) = \mu_I(s) \mu_b(s) \mu_{al}(s) \mu_N(s)
\]  

where,

\[
\mu_b(s) = \exp \left[ \lambda_{bl} wT_c [M_g(s) - 1] \right] \exp \left[ \lambda_{bl} \bar{w}T_c [M_g(-s) - 1] \right]
\]  

\[
\mu_{al}(s) = \exp \left[ \lambda_{al} wT_c [\exp(s) - 1] \right] \exp \left[ \lambda_{al} \bar{w}T_c [\exp(-s) - 1] \right]
\]  

\[
\mu_N(s) = \exp \left\{ \frac{s^2 \sigma_N^2}{2} \right\}
\]  

with

\[
\sigma_N^2 = \frac{N_0 (w+\bar{w})T_c}{2}
\]  

\[
\mu_j(s) = E[\exp (\gamma u) / b_0 (1) = j ; j = 0, 1]
\]  

where,

\[
u = \left\{ u_s + u_b + \sum_{k=2}^{K} u_k \right\}
\]  

\[
u_s = j \lambda_s' wT_c [M_g(s) - 1] + (1 - j) \lambda_s' \bar{w}T_c [M_g(-s) - 1]
\]  

\[
u_b = \frac{K\lambda\beta T_c}{\alpha_v} \left[ w [M_g(s) - 1] + \bar{w}[M_g(-s) - 1] \right]
\]
\[ u_k = \lambda_s T_c \left\{ P_{x,y}(l) \left[ M_g(s) - 1 \right] + \tilde{P}_{x,y}(l) \left[ M_g(-s) - 1 \right] \right\} \] (2.70)

for the chip synchronous case, and

\[ u_k = \lambda_s T_c \left\{ \left[ M_g(s) - 1 \right] P_{x,y}(l) + \left[ M_g(-s) - 1 \right] \tilde{P}_{x,y}(l) \right\} \]

\[ + \lambda_s (\tau_k - l T_c) \left\{ \left[ P_{x,y}(l+1) - P_{x,y}(l) \right] \left[ M_g(s) - 1 \right] + \left[ P_{x,y}(l+1) - \tilde{P}_{x,y}(l) \right] \left[ M_g(-s) - 1 \right] \right\} \] (2.71)

for the chip asynchronous condition, where \( l T_c \leq \tau_k \leq (l+1) T_c \), \( 0 \leq l \leq N \), \( \mu_\gamma(s) \) is obtained by replacing the variable \( s \) in Equation (1.52) by the variable \( u \) given by Equation (2.67).

\[ \mu_\gamma(s) = \exp \left\{ \frac{u_s^2}{2n_\gamma} \right\} \exp \left\{ \frac{u_b^2}{2n_\gamma} \right\} \]

\[ \sum_{k=2}^{K} E \left[ \exp (\gamma u_k) \right] \] (2.72)

where

\[ E[\exp (\gamma u_k)] = \frac{1}{4NT_c} \int_{0}^{T} \sum_{x,y \in \{k, \bar{k}\}} \exp \left\{ u_k + \frac{u_k^2}{2n_\gamma} \right\} d\tau_k \]

\[ = \frac{1}{4NT_c} \sum_{x,y \in \{k, \bar{k}\}} \sum_{l=0}^{N-1} T_c \int_{0}^{T} \exp \left\{ u_k + \frac{u_k^2}{2n_\gamma} \right\} d\tau_k \]

\[ = \frac{\exp (-n_\gamma / 2)}{4N} \sum_{x,y \in \{k, \bar{k}\}} \sum_{l=0}^{N-1} S_{x,y}(l,s) \] (2.73)
$S_{x,y}(l,s)$ is given by Equation (2.44) and (2.46) for the chip synchronous and chip asynchronous conditions respectively, where,

$$V_{x,y}(l,s) = \frac{P_{x,y}(l) \lambda_s T_c [M_g(s) - 1] + \bar{P}_{x,y}(l) \lambda_s T_c [M_g(-s) - 1]}{\sqrt{n_\gamma}} + \sqrt{n_\gamma}$$

(2.74)

The conditional moment generating function of $Y$ is then given by,

$$\mu^2(s) = \exp\left\{ \frac{s^2 \sigma_N^2}{2} + \lambda_b \bar{w} T_c [M_g(s) - 1] + \lambda_s \bar{w} T_c [\exp(s) - 1] + \lambda_b \bar{w} T_c [M_g(-s) - 1] + \lambda_s \bar{w} T_c [\exp(-s) - 1] \right\}$$

$$\exp\left\{ \frac{u_s^2}{2n_\gamma} + u_b + \frac{u_b^2}{2n_\gamma} \right\} \{\Gamma(s)\}^{(R-1)}$$

(2.75)

where,

$$\Gamma(s) = E[\exp(\gamma u_b)]$$

(2.76)

Therefore,

$$\ln[\mu(s)] = \frac{s^2 \sigma_N^2}{2} + \lambda_b \bar{w} T_c [M_g(s) - 1] + \lambda_s \bar{w} T_c [\exp(s) - 1] + \lambda_b \bar{w} T_c [M_g(-s) - 1] + \lambda_s \bar{w} T_c [\exp(-s) - 1]$$

$$+ u_s + \frac{u_s^2}{2n_\gamma} + u_b + \frac{u_b^2}{2n_\gamma} + (K-1) \ln[\Gamma(s)]$$

(2.77)
\[
\ln [\mu(s)'] = s \sigma_N^2 + \lambda_{bl} w T_c M_g'(s) + \lambda_{sl} w T_c \exp(s) \\
- \lambda_{bl} \tilde{w} T_c M_g'(-s) - \lambda_{sl} \tilde{w} T_c \exp(-s) \\
+ u_s' + \frac{u_s u_s'}{n_\gamma} + u_b' + \frac{u_b u_b'}{n_\gamma} + (K-1) \left[ \frac{\Gamma'(s)}{\Gamma(s)} \right] (2.78)
\]

and

\[
\ln [\mu(s)"] = \sigma_N^2 + \lambda_{bl} w T_c M_g''(s) + \lambda_{sl} w T_c \exp(s) + u_s'' + \frac{u_s u_s''}{n_\gamma} \\
+ \lambda_{bl} \tilde{w} T_c M_g''(-s) + \lambda_{sl} \tilde{w} T_c \exp(-s) \\
+ \frac{u_s'^2}{n_\gamma} + \frac{u_b u_b''}{n_\gamma} + \frac{u_b'^2}{n_\gamma} \\
+ (K-1) \left[ \frac{\Gamma''(s)}{\Gamma(s)} - \frac{\Gamma'(s)^2}{\Gamma(s)^2} \right] (2.79)
\]

where,

\[
u_s' = j \lambda_s' w T_c M_g'(s) - (1-j) \lambda_s' \tilde{w} T_c M_g'(-s) (2.80)
\]

\[
u_s'' = j \lambda_s' w T_c M_g''(s) + (1-j) \lambda_s' \tilde{w} T_c M_g''(-s) (2.81)
\]

\[
u_b' = \frac{K \lambda_s}{\alpha_v} w T_c M_g'(s) - \frac{K \lambda_s}{\alpha_v} \tilde{w} T_c M_g'(-s) (2.82)
\]

\[
u_b'' = \frac{K \lambda_s}{\alpha_v} w T_c M_g''(s) + \frac{K \lambda_s}{\alpha_v} \tilde{w} T_c M_g''(-s) (2.83)
\]
\[
M_g'(s) = \frac{M_g(s) \left\{ 1 + a \left[ M_g(s) - 1 \right] \right\}}{a(b-1) \left[ M^* - M_g(s) \right]} 
\]

\[
M_g''(s) = M_g'(s)^3 \left\{ \frac{1}{M_g(s)^2} - \frac{a^2b}{\left\{ 1 + a \left[ M_g(s) - 1 \right] \right\}^2} \right\} 
\]

\[
M_g'(-s) = \frac{M_g(-s) \left\{ 1 + a \left[ M_g(-s) - 1 \right] \right\}}{a(b-1) \left[ M_g(-s) - M^* \right]} 
\]

\[
M_g''(-s) = M_g'(-s)^3 \left\{ \frac{-1}{M_g(-s)^2} + \frac{a^2b}{\left\{ 1 + a \left[ M_g(-s) - 1 \right] \right\}^2} \right\} 
\]

\( \Gamma'(s)/\Gamma(s) \) and \( \Gamma''(s)/\Gamma(s) \) are defined as in Equations (2.58) and (2.59) respectively. Using Equations (2.9) through (2.22), the Saddlepoint Approximation for \( q_j(\theta), j = 0, 1 \) are obtained and the BER probability \( P_{be} \) is evaluated using Equation (2.1).

The same values of parameters considered for OOO signaling analysis are used for the numerical analysis of EWO signaling scheme. The mean APD gain \( G \) is assumed to be 100. The mean signaling rate is given by,

\[
\mu_s = \frac{\lambda_\gamma T_c \left( w + \bar{w} \right)}{2} 
\]

The values of \( P_{be} \) for EWO signaling scheme with OOCs is plotted against \( \mu_s \) in Figure 2.13 for \( K = 2 \) and 4 active users and different values of the \( n_\sigma \) \( n_\gamma = 20, 50 \) and 80 speckle cells. For \( \mu_s = 400 \) and \( K = 2 \), \( P_{be} \) is \( 10^{-9} \) with \( n_\gamma = 80 \) and \( P_{be} \) increases to \( 10^{-5} \) as \( n_\gamma \) decreases to 20. Thus a
Figure 2.13 BER probability versus mean signaling rate for EWO signaling with OOCs in the presence of modal noise, $G = 100$ (SA)
high value of $n_\gamma$ is required to achieve low $P_{be}$. Similar performance curves are shown for Prime codes and QCCs in Figures 2.14 and 2.15 respectively. It is observed from Figure 2.14 that for $n_\gamma = 20$, the increase in $K$ from 2 to 4 increases the $P_{be}$ only at values of $\mu_s > 1000$ signal photon counts/bit. However for $n_\gamma = 80$ the increase in $K$ from 2 to 4 degrades the system for $\mu_s > 550$ and $P_{be}$ increases with increase in $K$ in the system.

OOCs, Prime codes and QCCs are compared at two different levels of modal noise, $n_\gamma = 20$ and 80 in Figures 2.16 and 2.17 for $K = 2$ and 4 respectively. For $K = 2$, $P_{be}$ achieved with OOCs is less than that of Prime codes and QCCs for values of $\mu_s < 500$ ($n_\gamma = 20$) and $\mu_s < 600$ ($n_\gamma = 80$). For $K = 4$, OOCs are better only when $\mu_s < 180$ for both $n_\gamma = 20$ and 80. At higher $\mu_s$, Prime codes show a better performance and QCCs offer higher BER probabilities compared to Prime codes and OOCs. At $\mu_s = 800$, $n_\gamma = 80$ and $K = 2$, the $P_{be}$ achieved with Prime codes is one and two orders of magnitude less than that of OOCs and QCCs respectively. For $K = 4$, Prime codes achieve a $P_{be}$ that is three and four orders of magnitude less than that of OOCs and QCCs. This reveals that the performance of these codes differ significantly as $K$ increases. For $K = 4$ and $P_{be} = 10^{-9}$, Prime codes require $\mu_s = 400$, whereas QCCs and OOCs require $\mu_s = 600$ and 1000 respectively when $n_\gamma = 80$.

The BER probability with OOCs in the presence and absence of modal noise, are compared in Figure 2.18 for $K = 2$, $\mu_s = 500$ and $G = 100$. As $n_\gamma > 62$ the BER probability approaches $10^{-10}$ which is the error probability in the absence of modal noise. From Figures 2.19 and 2.20 the limiting values for Prime codes and QCCs are found to be $n_\gamma > 111$ with $P_{be} = 10^{-10}$ and $n_\gamma > 78$ with $P_{be} = 10^{-8}$.

The maximum $K$ the system can support for a $P_{be}$ of $10^{-9}$ with the given system parameter values is shown in Figure 2.21. At $\mu_s = 1000$, Prime
Figure 2.14 BER probability versus mean signaling rate for EWO signaling with Prime codes in the presence of modal noise, $G = 100$ (SA)
Figure 2.15 BER probability versus mean signaling rate for EWO signaling with QCCs in the presence of modal noise, $G=100$ (SA)
Figure 2.16 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for EWO signaling schemes with $K = 2$ and $G = 100$ (SA)
Figure 2.17 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for EWO signaling schemes with $K = 4$ and $G = 100$ (SA)
Figure 2.18 Comparison of BER probability in the presence and absence of modal noise for EWO signaling schemes with OOCs, $\mu_s=500$, $G=100$ and $K=2$(SA)
Figure 2.19 Comparison of BER probability in the presence and absence of modal noise for EWO signaling schemes with Prime codes, $\mu_s=500$, $G=100$ and $K=2$ (SA)
Figure 2.20 Comparison of BER probability in the presence and absence of modal noise for EWO signaling schemes with QCCs, μ_x=500, G=100 and K=2(SA)
Figure 2.21 Maximum $K$, the system can support for $P_{be}=1e^{-9}$ employing EWO signaling scheme with OOCs, Prime codes and QCCs (SA)
codes support 5 active users, whereas OOCs and QCCs support only 3 active users. At higher mean signaling rates OOCs are better than QCCs.

2.4 GAUSSIAN APPROXIMATION ANALYSIS

In this analysis, $P_{be}$ is evaluated by assuming that the decision statistic $Y$ is Gaussian. Under the Gaussian Approximation (GA) only the mean and variance of $Y$ need to be evaluated and $P_{be}$ is then expressed in terms of simple signal to noise ratios.

2.4.1 Binary OOO signaling scheme

In this scheme, the signal term is present at the receiver output only when $b_0^{(1)} = 1$, otherwise only the noise terms are present. The conditional means $m_{Yj}$, $j=0,1$ and variances $\sigma^2_{Yj}$, $j = 0,1$ of $Y$ are evaluated under binary OOO signaling condition as follows.

$$m_{Yj} = \left[\ln \mu_j(s)\right]_{s=0}^\prime$$

$$= jG\lambda_{s} wT_c + G\lambda_{b0} wT_c + \frac{GK\lambda_{s} wT_c}{\alpha_e} + \lambda_{al} wT_c$$

$$+ \frac{(K-1) G\lambda_{s}T_c}{4N} \sum_{x,y\in\{k,\bar{k}\}} \sum_{l=0}^{N-1} P_{x,y}(l)$$

and

$$\sigma^2_{Yj} = \left[\ln \mu_j(s)\right]_{s=0}''$$

$$= \sigma_N^2 + \lambda_{al} wT_c + \frac{G^2}{n_y} \left[ j\lambda_{s} wT_c + \frac{(K\lambda_{s} wT_c)^2}{\alpha_e^2} \right]$$
Then using Equation (2.1) and (2.26), $P_{be}$ is evaluated.

The numerical analysis is carried out with the same parameters values as in the Saddlepoint Approximation analysis. The BER probability with OOCs is plotted against the mean signaling rate in Figure 2.22 for $K = 2$ and $4$, $n_r = 100, 200$ and $400$. At $n_r = 500$ and $K = 2$, $P_{be}$ is around $10^{-11}, 10^{-9}$ and $10^{-7}$ for $n_r = 400, 200$ and $100$ respectively. The $P_{be}$ decreases for increase in $n_r$. Figures 2.23 and 2.24 show identical performance for Prime codes and QCCs. For $K = 4$, the difference in the $P_{be}$ is within one order of magnitude as $n_r$ decreases from 400 to 100 for all the three codes, and the effect of modal noise is insignificant.

The three codes are compared with $K = 2$ and $4$ in Figures 2.25 and 2.26 respectively, at two different levels of modal noise, $n_r = 100$ and $400$. The BER probabilities achieved are $10^{-12}, 10^{-10}$ and $10^{-6}$ for OOCs, Prime
Figure 2.22 BER probability vs mean signaling rate for OOO signaling with OOCs in the presence of modal noise, G = 250 (GA)
Figure 2.23 BER probability vs mean signaling rate for OOO signaling with Prime codes in the presence of modal noise, $G = 250$ (GA)
Figure 2.24 BER probability vs mean signaling rate for OOO signaling with QCCs in the presence of modal noise, $G = 250$ (GA)
Figure 2.25 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for OOO signaling schemes with $K = 2$ and $G = 250$ (GA)
Figure 2.26 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for OOO signaling schemes with K = 4 and G = 250 (GA)
codes and QCCs respectively at \( \mu_s = 1000, n = 400 \) and \( K = 2 \). For \( K = 4 \) the corresponding BER probabilities are \( 10^{-6}, 10^{-5} \) and \( 10^{-3} \) for OOCs, Prime codes and QCCs respectively. The Gaussian Approximation analysis shows that OOCs are better than Prime codes and QCCs under OOO signaling conditions.

The maximum \( K \) the system can support for \( P_{be} = 10^{-9} \) is shown in Figure 2.27 for the three codes. At \( \mu_s = 1000 \), OOCs support 3 active users and Prime codes support 2 active users. QCCs are found to be unsuitable for multiuser communications. Gaussian Approximation predicts better performance by OOCs for evaluating \( P_{be} \). Since OOCs are low weight codes, this analysis may not be accurate as most of the sequence correlation values are zero.

### 2.4.2 Binary EWO signaling scheme

In this scheme, the signal term will be present for both \( b_0^{(1)} = 1 \) and \( b_0^{(1)} = 0 \). The conditional means \( m_{Y\|j} = 0, 1 \) and variances \( \sigma^2_{Y\|j}, j = 0, 1 \) of \( Y \) are evaluated to be,

\[
m_{Y\|j} = G\lambda_s \left[jw - (1 - j) \bar{w}\right] T_c ; j = 0, 1
\]

and

\[
\sigma^2_{Y\|j} = \sigma_N^2 + \lambda_{bl} \left(w + \bar{w}\right) T_c + \frac{G^2}{n_\gamma} \left[ j\lambda_s \left(w + \bar{w}\right) T_c + \frac{[K\lambda_s \left(w + \bar{w}\right) T_c]^2}{\alpha_e^2} \right] \\
+ G^3 \left(1 - a^2b\right) \left[ \lambda_{bl} \left(w + \bar{w}\right) T_c + j\lambda_s \left(w + \bar{w}\right) T_c + \frac{K\lambda_s \left(w + \bar{w}\right) T_c}{\alpha_e} \right] \\
+ \frac{(K-1) G^3 \left(1 - a^2b\right) \lambda_s T_c}{4N} \sum_{x,y \in \{k, \bar{k}\}} \sum_{l=0}^{N-1} [P_{X,Y}(l) + \bar{P}_{X,Y}(l)]
\]
Figure 2.27 Maximum $K$, the system can support for $P_{be} = 1e^{-9}$, employing OOO signaling scheme with OOCs, Prime codes and QCCs (GA)
\[
+ \frac{(K-1) (G \lambda T c)^2}{4N} \left\{ 1 + \frac{1}{n_\gamma} \right\} \sum_{x,y \in \{k, \bar{k}\}} \sum_{l=0}^{N-1} \left[ P_{x,y}(l) - \tilde{P}_{x,y}(l) \right]^2 \\
- \frac{(K-1) (G \lambda T c)^2}{16N^2} \left[ \sum_{x,y \in \{k, \bar{k}\}} \sum_{l=0}^{N-1} \left[ P_{x,y}(l) - \tilde{P}_{x,y}(l) \right] \right]^2
\]

(2.92)

The BER probability \( P_{be} \) is then evaluated using Equations (2.1) and (2.26) and the numerical analysis is carried out with the same system parameter values as used in the Saddlepoint Approximation analysis. OOCs are first considered and the \( P_{be} \) is plotted against \( \mu_a \) in Figure 2.28 for \( K = 2,4 \) and \( n_\gamma = 20, 50 \) and 80. At \( \mu_a = 400 \) and \( K = 2 \), \( P_{be} \) is \( 10^{-9}, 10^{-7} \) and \( 10^{-4} \) for \( n_\gamma = 80, 50 \) and 20 respectively. For \( K = 4 \), the corresponding values of \( P_{be} \) are \( 10^{-7}, 10^{-6} \) and \( 10^{-4} \) for \( n_\gamma = 80, 50 \) and 20 respectively. As in the previous analyses, effect of modal noise for increase in the BER probability is significant. Similar performance curves for Prime codes and QCCs are shown in Figure 2.29 and 2.30.

The three codes are compared for \( K = 2 \) and 4 in Figures 2.31 and 2.32 respectively, for modal noise parameter \( n_\gamma = 20 \) and 80. For instance, at \( n_\gamma = 80, \mu_a = 400 \) and \( K = 2 \), the BER probabilities achieved by OOCs, Prime codes and QCCs are \( 10^{-9}, 10^{-6} \) and \( 10^{-5} \) respectively, and for \( K = 4 \) these probabilities are \( 10^{-7}, 10^{-5} \) and \( 10^{-4} \).

The maximum number of users the system can support for a BER probability of \( 10^{-9} \) is shown in Figure 2.33. At \( \mu_a = 1000 \), OOCs support 5 active users, Prime codes support 4 active users and QCCs support 2 active users. As seen in the OOO signaling analysis, Gaussian Approximation for evaluating \( P_{be} \) shows superior performance by OOCs than Prime codes and QCCs.
Figure 2.28 BER probability versus mean signaling rate for EWO signaling with OOCs in the presence of modal noise, $G=100$ (GA)
Figure 2.29 BER probability versus mean signaling rate for EWO signaling with Prime codes in the presence of modal noise, $G = 100$ (GA)
Figure 2.30 BER probability versus mean signaling rate for EWO signaling with QCCs in the presence of modal noise, G=100 (GA)
Figure 2.31 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for EWO signaling schemes with $K = 2$ and $G = 100$ (GA)
Figure 2.32 Comparison of OOCs, Prime codes and QCCs in the presence of modal noise for EWO signaling schemes with $K = 4$ and $G = 100$ (GA)
Figure 2.33 Maximum K, the system can support for $P_{be}=1e-9$ employing EWO signaling scheme with OOCs, Prime codes and QCCs (GA)
2.5 RESULTS AND DISCUSSION

The BER probabilities in the absence of modal noise, evaluated using SA and GA are compared for OOO and EWO signaling schemes with OOCs, Prime codes and QCCs in Figures 2.34 to 2.39. Table 2.1 gives comparison of the performances. From the table it is seen that GA predicts better performance by OOCs for evaluating $P_{be}$ under both OOO and EWO signaling conditions. For Prime codes and QCCs, GA yields higher BER floors and SA yields lower values of $P_{be}$.

Table 2.1 Performance Comparison in absence of modal noise

<table>
<thead>
<tr>
<th>Addressing Code</th>
<th>$\mu_s = 500, \ G = 250, \ K = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SA</td>
</tr>
<tr>
<td></td>
<td>OOO</td>
</tr>
<tr>
<td>OOC</td>
<td>1e-10</td>
</tr>
<tr>
<td>Prime Codes</td>
<td>1e-14</td>
</tr>
<tr>
<td>QCC</td>
<td>1e-11</td>
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</table>

The BER probabilities in the presence of modal noise, evaluated using SA and GA analyses are compared for the three codes in Figures 2.40 to 2.42 for OOO signaling and in Figures 2.43 to 2.45 for EWO signaling schemes. Table 2.2 gives the comparison of the performances in the presence of modal noise and reveals that SA is the best method for analyzing OOO and EWO signaling schemes for all the three codes.
Figure 2.34: Comparison of SA and GA methods in the absence of modal noise for OOO signaling with OOCs, $G = 250$
Figure 2.35 Comparison of SA and GA methods in the absence of modal noise for OOO signaling with Prime codes, G=250.
Figure 2.36 Comparison of SA and GA methods in the absence of modal noise for OOO signaling with QCCs, $G = 250$
Figure 2.37 Comparison of SA and GA methods in the absence of modal noise for EWO signaling with OOCs, $G = 250$
Figure 2.38 Comparison of SA and GA methods in the absence of modal noise for EWO signaling with Prime codes, \( G = 250 \)
Figure 2.39 Comparison of SA and GA methods in the absence of modal noise for EWO signaling with QCCs, $G = 250$
Figure 2.40 Comparison of SA and GA methods in the presence of modal noise for OOO signaling with OOCs, $G = 250$
Figure 2.41 Comparison of SA and GA methods in the presence of modal noise for OOO signaling with Prime codes, $G = 250$
Figure 2.42 Comparison of SA and GA methods in the presence of modal noise for OOO signaling with QCCs, $G = 250$
Figure 2.43  Comparison of SA and GA methods in the presence of modal noise for EWO signaling with OOCs, $G = 250$
Figure 2.44 Comparison of SA and GA methods in the presence of modal noise for EWO signaling with Prime codes, $G = 250$
Figure 2.45 Comparison of SA and GA methods in the presence of modal noise for EWO signaling with QCCs, $G = 250$
Table 2.2 Performance Comparison in the presence of modal noise

<table>
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<th>OOO</th>
<th>EWO</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_s = 500$, $G = 250$, $K = 2$, $n_\gamma = 400$</td>
<td>$\mu_s = 800$, $G = 100$, $K = 2$, $n_\gamma = 80$</td>
</tr>
<tr>
<td>SA</td>
<td>$1 \times 10^{-12}$</td>
<td>$1 \times 10^{-13}$</td>
</tr>
<tr>
<td>GA</td>
<td>$1 \times 10^{-10}$</td>
<td>$1 \times 10^{-11}$</td>
</tr>
</tbody>
</table>

At a given $\mu_s$, there is an optimum value for the mean gain of APD yielding $P_{be}$ minimum. This optimum mean APD gain $G_{opt}$ is plotted for various values of $\mu_s$ and are shown in Figures 2.46 to 2.48 for OOCs, Prime Codes and QCCs. It is observed that $G_{opt}$ decreases as $K$ increases. Also, at higher $\mu_s$ the gain required is lower. From Figure 2.49 it is noted that the optimum APD gain requirement is similar for OOCs, Prime codes and QCCs. Table 2.3 gives $G_{opt}$ required for various values of $\mu_s$ and $K$.

Table 2.3 Optimum mean gain of APD

<table>
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<tr>
<th>$K$</th>
<th>$G_{opt}$</th>
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<tbody>
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<td>270</td>
</tr>
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</table>
Figure 2.46 Optimum mean APD gain versus mean signaling rate for OOCs
Figure 2.47 Optimum mean APD gain versus mean signaling rate for Prime codes
Figure 2.48 Optimum mean APD gain versus mean signaling rate for QCCs
Figure 2.49 Comparison of Optimum mean APD gain required for OOCs, Prime codes and QCCs
The modal noise is found to significantly degrade the system for lower values of $n_r$. As $n_r$ increases, $P_{be}$ approaches that of the system where modal noise is absent. Table 2.4 shows the limiting value of $n_r$ below which modal noise is significant. It is noted from the table that as the number of simultaneous users $K$ increases, the limiting value of $n_r$ decreases. This shows that the effect of modal noise is less when there are more number of active users in the system or vice versa. It is also observed that the minimum $n_r$ is less for EWO signaling systems than for OOO signaling systems. Hence, the effect of modal noise is less pronounced in EWO signaling systems.

Table 2.4 Limiting values of the modal noise parameter

<table>
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<tr>
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<th>Prime code</th>
<th>QCC</th>
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</tr>
<tr>
<td>4</td>
<td>129</td>
<td>47</td>
<td>56</td>
</tr>
</tbody>
</table>

The capacity of the Fiber Optic CDMA network i.e. the maximum $K$ the system can support for $P_{be}$ less than $10^{-9}$ is compared in Table 2.5 and Table 2.6.

Table 2.5 Comparison of maximum $K$ for $P_{be} = 1e^{-9}$ at $\mu_n = 1000$, $G = 250$

<table>
<thead>
<tr>
<th>Code</th>
<th>GA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OOO</td>
<td>EWO</td>
</tr>
<tr>
<td>OOC</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Prime Code</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>QCC</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 2.6 Comparison of maximum K for $P_{be} = 1e^{-9}$

at $\mu_a = 500$, $G = 250$

<table>
<thead>
<tr>
<th>Code</th>
<th>GA</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OOO</td>
<td>EWO</td>
</tr>
</tbody>
</table>
| OOC
N=341, w=5  | 2  | 4  | 2  | 2   |
| Prime Code
N=121, w=11 | 2  | 3  | 2  | 4   |
| QCC
N=121, w=11 | 1  | 2  | 2  | 2   |

The Saddlepoint Approximation analysis being more accurate, it is observed that Prime codes with EWO signaling scheme supports a maximum number of 5 simultaneous users. The total number of users in the system is then 25 if it is assumed that only 20% of them are active at any instant of time. The effect of modal noise on the capacity of the system is shown in Figure 2.50. It is noted that the modal noise does not affect maximum K beyond a certain level. These results show that a Fiber Optic CDMA system employing EWO signaling scheme and Prime codes is most preferrable with respect to the network throughput.
Figure 2.50 Effect of modal noise on maximum $K$ with $P_{bc} = 1e^{-9}$