CHAPTER 4

DEVELOPMENT OF GROUNDWATER MODEL

4.1 GENERAL

As the main objective of this dissertation work is to develop a methodology for assessing the sustainability status of a watershed using Remote sensing data and GIS techniques, by developing various sub models. In this chapter a groundwater model is developed to assess the groundwater potential in the watershed and also to predict the response of the hydrological system for the change in landuse. The study area has been divided into a grid pattern and a distributed parameter (Mathematical) model is developed for the aquifer system. The various inflows and outflows to the system are established and the net discharge or recharge are assessed and simulated at each node.

The basic aquifer simulation programme (PRICKET and LONNQUIST) is used to analyse the groundwater flow problem. The initial aquifer parameters required are estimated using double porosity techniques and the final aquifer parameters are estimated using ‘PEST’ module of ‘MODFLOW’ version 4.2. These parameters are used in the Basic Aquifer Simulation Programme and the model is run. The model is then calibrated with the data for the period 1983-88 and proved by using the data for the period 1989-92.

For the purpose of this research work, the entire study area are divided into 81 microwatersheds and the quantity of surface water that can be harnessed and the groundwater that can be extracted from each
microwatershed have been computed. Then the total quantity of water available in each microwatershed was computed and this quantity is used for assessing the water availability status of the microwatershed and computing the sustainable landuse.

4.2 STRUCTURE OF THE MODEL

The study area has been divided into 1km x 1km grid as shown in Fig.4.1. (finite difference lattice) Then the various inflows and outflows to each node have been assessed and the changes in storage (Net Q) have been simulated at each grid point (Node point).

4.2.1 Inflows to Each Grid Square

The following are the various inflow components.

a. Rainfall Recharge
b. Riverbed Recharge
c. Returnflow from Irrigation and
d. Subsurface Inflow.

The computation of the above components have been discussed in para 3.6 of chapter 3.

4.2.2 Outflows from Each Grid Square

The various outflows taking place are:

a. Extraction for human and animal population
b. Agricultural extraction
c. Loss due to evapotranspiration of natural vegetation and
d. Subsurface outflow.
Fig 4.1 Finite difference lattice
The computation of the above components have been discussed in para 3.6 of chapter 3.

4.2.3 Computation of Net Discharge or Recharge

After computation of various inflows and outflows to each grid square, the net discharge or recharge to each grid square is computed using the following continuity equation.

\[ Q_{\text{net}} = I_f - Q_f \]  

(4.1)

where
- \( Q_{\text{net}} \) = the net changes in storage of all inflows and outflows.
- \( I_f \) = the total volume of inflow components.
- \( Q_f \) = the total volume of outflow components.

The various entities involved in the operation are presented in the schematic diagram shown in Fig.4.2.

4.3 FORMULATION OF MATHEMATICAL MODEL

Any artifact that can duplicate the working of a system is termed as a model. The operation of the model and the analysis of the results thereof are very essential for successful functioning of any engineering structure. The behaviour of the physical systems can be expressed in terms of algebraic or differential or integral equations in the mathematical formulation. The mathematical model, being highly flexible, the effect of modifications in the values of the parameters can be studied with ease. The other advantage include low cost, absence of scaling problem and versatility.
### 4.3.1 Mathematical Background

The partial differential equations (Bittinger et al) governing non-steady state two-dimensional flow of groundwater in a homogeneous and anisotropic aquifer are given by:

\[
\frac{\partial}{\partial x} \left[ T \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y} \left[ T \frac{\partial h}{\partial y} \right] = \frac{S}{T} \frac{\partial h}{\partial t} + \frac{Q}{T}
\]

Where:
- \( T \) = aquifer transmissivity in \( \text{m}^2/\text{day} \)
- \( h \) = head, metres
- \( t \) = time in days
- \( S \) = aquifer storage coefficient
- \( x, y \) = rectangular co-ordinates
- \( Q_w \) = Net constant withdrawal.

There is no general solution available for Equation 4 for complex geometry. However, a numerical solution for that equation is obtained through the finite difference approach. This approach first involves replacing the continuous aquifer system by an equivalent set of discrete cells. Both the space and time variables are treated as discrete parameters. Secondly, the equation governing the flow of groundwater for the discrete lattice is written in finite difference form. Finally, the resulting equations are solved numerically usually with the aid of digital computers. The finite difference grid adopted and vector volume concept is shown in Fig. 4.3.

The differentials \( \partial x \) and \( \partial y \) are approximated by the finite differences \( \Delta x \) and \( \Delta y \) respectively. The area \( \Delta x \times \Delta y \) should be small compared to the total area of the aquifer to the extent that the discrete model is a representation of the continuous system.
Fig. 4.3 Finite difference grid and vector volume concept

a. Finite difference grid

b. Vector volume concept
The continuity condition relating the flow rates entering and leaving the node \(i,j\) is given below:

\[ Q_1 + Q_3 + Q_n = Q_2 + Q_4 + Q_5 + Q_6 \]  

Where:

- \(Q_1, Q_2, Q_3, Q_4\) = node to node water transfer rate with arbitrarily assigned flow directions.
- \(Q_5\) = amount of water taken into or released from storage per unit time increment.
- \(Q_6\) = net constant withdrawal rate.
- \(Q_n\) = quantum of leakage, induced infiltration, evapotranspiration etc.

Determination of the values of the flow rate terms in the Equation 4.3 involves three considerations. First, it is necessary to define what portion of the aquifer is represented by every individual term. Secondly it should be noted that although the flow rates may take place in any direction in the aquifer, they are restricted to the \(x\) and \(y\) directions in the finite difference approach. The portions of the aquifer included in the flow rate terms then may be referred to as "vector volumes" to emphasise that not only a volume but also the direction of flow is being considered. Finally, since it is discretised, Equation 4.3 represents an instantaneous balance at the end of a time increment.

Applying Darcy's law to the flow rate terms \(Q_1\) to \(Q_4\):

\[ Q_1 = T_{i-1,j,2} (h_{i-1,j} - h_{ij}) \Delta y/\Delta x \]  \hspace{1cm} (4.4)

\[ Q_2 = T_{i,j-1,2} (h_{ij} - h_{i+1,j}) \Delta y/\Delta x \]  \hspace{1cm} (4.5)
where $T_{ij,1}=\text{aquifer transmissivity within the vector volumes between nodes, } i,j \text{ and } i,j+1$.

\[
Q_3 = T_{ij,1} (h_{i,j+1} - h_{i,j}) \Delta x/\Delta y
\]

\[
Q_4 = T_{ij,1} (h_{i,j} - h_{i,j-1}) \Delta x/\Delta y
\]

\[
T_{ij,2} = \text{aquifer transmissivity within the vector volumes between nodes } i,j \text{ and } i+1,j
\]

\[
h_{ij} = \text{calculated heat at the end of a time increment.}
\]

\[
Q_5 = S \Delta x \Delta y (h_{ij} - h_{\phi ij})/\Delta t
\]

\[
h_{\phi ij} = \text{calculated head at node } i,j \text{ at the end of previous time increment, } \Delta t.
\]

\[
\Delta t = \text{time increment elapsed since the last calculation of heads.}
\]

\[
Q_6 = Q_{ij}
\]

\[
Q_n = Q_n
\]

Substituting the above values in Equation 4.3.

\[
Q_n + T_{i-1,j} (h_{i-1,j} - h_{i,j}) \Delta y/\Delta x + T_{ij,1} (h_{i,j+1} - h_{i,j}) \Delta x/\Delta y
\]

\[
= T_{ij,2} (h_{ij} - h_{i+1,j}) \Delta y/\Delta x + T_{ij,1} (h_{ij} - h_{ij-1}) \Delta x/\Delta y +
\]

\[
S \Delta x \Delta y (h_{ij} - h_{\phi ij}) / \Delta t - Q_{ij}
\]

Dividing both sides of the equations by the product of the $\Delta x \Delta y$ and rearranging terms yields.

\[
T_{i-1,j} (h_{i-1,j} - h_{i,j}) / \Delta x^2 + T_{ij,2} (h_{i+1,j} - h_{i,j}) / \Delta x^2
\]

\[
+ T_{ij,1} (h_{ij+1} - h_{ij}) / \Delta y^2 + T_{ij,1} (h_{ij} - h_{ij-1}) / \Delta y^2
\]

\[
= S(h_{ij} - h_{\phi ij})/\Delta t + Q_{ij} / \Delta x \Delta y - Q_n / \Delta x \Delta y
\]
The resulting set of simultaneous equations are solved by the Modified form of Iterative Alternating Direction Implicit method (MIADI).

### 4.3.2 Modified Iterative Alternating Direction Implicit Method

The Iterative Alternating Direction Implicit Method involves, reducing a large set of simultaneous equation down to a number of small sets for a given time increment. The set of column equation is then implicit in the direction along the column and explicit in the direction orthogonal to the column alignment. The above process is repeated a significant number of times to achieve convergence and this completes the calculations for the given time increment. Peaceman and Rachford (1955) point out that this technique is unconditionally stable regardless of the size of the time increment. It solves the equations first column by column and then row by row using Gauss seidel iterative procedure.

The above equations are modified and rearranged assuming \( \Delta y = \Delta x \) (equal grid) and multiplying both sides by \( \Delta x \).

\[
\begin{align*}
T_{i+1,j,2} (h_{i-1,j} - h_{ij}) + T_{ij,2} (h_{i+1,j} - h_{ij}) + T_{ij,1} (h_{ij+1} - h_{ij}) + T_{ij,-2} (h_{ij} - h_{ij}) = S \Delta x / \Delta t (h_{ij} - h_{ij}) + Q_y - Q_n \\
T_{ij,1} (h_{ij-1} - h_{ij}) = S \Delta x / \Delta t (h_{ij} - h_{ij}) + Q_y - Q_n
\end{align*}
\]  

(4.10)

The above equation is expanded, signs are reversed and terms of \( h_{ij} \) are grouped together to yield -

\[
\begin{align*}
h_{ij} (T_{i+1,j,2} + T_{ij,2} + T_{ij,1} + T_{ij,-1,1} + S_{ij} \Delta x / \Delta y) \\
- (T_{i-1,j,2} h_{i-1,j} - T_{ij,2} h_{i+1,j} - T_{ij,1} h_{ij+1} - T_{ij,1} h_{ij} + T_{ij,-2} h_{ij} - T_{ij,1} h_{ij-1} - T_{ij,1} h_{ij+1} = (S_{ij} \Delta x / \Delta t) h_{ij} - Q_y + Q_n
\end{align*}
\]

(4.11)
The above equation is then written in two forms, one for solving the node equations by columns and the other for solving the node equations by rows.

For calculation by columns the equation is rearranged as follows.

\[ -T_{ij-1,1} (h_{ij-1}) + h_{ij} (T_{i-1,j,2} + T_{i,j,1} + T_{i,j-1,1} + S_{ij} \Delta x^2/\Delta t) 
   - (T_{ij,1} h_{ij+1}) = S(\Delta x^2/\Delta t) h_{ij} - Q_{ij} + T_{i-1,j,2} h_{i-1,j} 
   + T_{ij,2} h_{i+1,j} + Q_n \]  

(4.12)

The above equation is of the form.

\[ AA_j h_{i,j-1} + BB_j h_{ij} + CC_j h_{i,j+1} = DD_j \]  

(4.13)

where the constant terms are:

\[ AA_j = -T_{ij-1,1} \]  

(4.14)

\[ BB_j = T_{i-1,j,2} + T_{i,j,2} + T_{i,j-1,1} + S \Delta x^2/\Delta t \]  

(4.15)

\[ CC_j = -T_{ij,1} \]  

(4.16)

\[ DD_j = (S \Delta x^2/\Delta t) h_{ij} + T_{i-1,j,2} h_{i-1,j} + T_{i,j,2} h_{i+1,j} + Q_n \]  

(4.17)

A similar set of equations can be written for calculation by rows.

Eventhough several packages like 'MODFLOW' are available for the analysis of groundwater flow problem, in this study the Basic Aquifer Simulation Programme published by PRICKET and LONNQUEST has been used, because the source codes are available for any modification. Moreover in this study, a package of programme has been developed comprising of various surface activities like precipitation, runoff, evapotranspiration and recharge/discharge and subsurface problem, GIS analysis for integration,
linear programming model for optimization etc. In order to have a single package to carry out the sequences of operations and also because of its simplicity, the well documented computer programme developed by Pricket and Lonnquist has been adopted in this study for analysis of groundwater flow problems. However 'MODFLOW' version 4.2 has been used for estimating the aquifer parameters using the 'PEST' module which will be described latter.

4.4 DETAILED STUDY OF THE AQUIFER

The configuration of the aquifer under study is shown in Fig. 4.1. The aquifer area is divided into 1 km x 1 km grids both along x and y directions. There are 50 columns and 20 rows in the grid. The total area of the aquifer is around 650 Sq.km. The aquifer is treated as non-homogeneous anisotropic.

4.4.1 Boundary Conditions

It is well known that the specification of proper boundary conditions is an imperative and important step in the application of numerical models. Most of the present day models have distinct segments for introduction of boundary conditions. They clearly distinguish between the two types of boundary conditions the dirichetlary conditions, the function specified and noiman conditions with the gradient of the function specified. For the groundwater flow problems, this corresponds to the specifications of groundwater levels and flow rates. The boundary conditions are incorporated explicitly in the algebraic equation developed in the numerical model and the solution of the same follows as the next step. However in the Basic Aquifer Simulation Programme, the value of Transmissivity and Storage coefficient are suitably adjusted for boundary nodes depending upon the boundary conditions. The manner in which the boundary nodes are dealt with in the present case is explained below:
The northern boundary is treated as no flow boundary based on the lithological characters of the subsurface formations. The western boundary is considered as flow boundary along with the flow rates as specified. The eastern boundary is the sea through which some meagre subsurface underflow, occurs. In the absence of satisfactory data on the head or flow conditions along the southern boundary it is treated as flow boundary with unknown head and unknown flows along the boundary.

4.4.2 Estimation of Aquifer Parameters

Pump tests have been conducted in four wells spread over the entire area of aquifer by State Surface and Groundwater Data Centre, Water Resources Organisation (WRO), Public Works Development. The details of wells are given in para 3.4.4. The data are analysed using the double porosity techniques, since most part of the aquifers are heterogeneous formations with two different permeable media. Even though other methods, such as numerical methods are available to estimate the aquifer parameters, this method is used, because initial approximate values are used in the computation. These approximate values are subsequently modified using the 'PEST' module in the MODFLOW version 4.2 package.

4.4.2.1 Parameter Estimation using Double Porosity Techniques

A generalised concept of fissured rock, as described by Barenblatt and Zheltov (1960) and Barenblatt et al. (1960) considers the rock mass broken up into blocks of irregular size and shape by fissures (Fig. 4.4). To apply the method of analysis of infinitesimals to the fissured rock medium, Barenblatt et al. (1960) assumed that any infinitely small volume of rock consists of a large number of porous blocks as well as large number of randomly distributed, sized and oriented fissures. With this approach, the drawdown in hydraulic head for any distance ‘r’ from the abstraction well and the time ‘t’ in the fissure, denoted by $s_1$, and the corresponding
drawdown, averaged over the thickness of the porous block, denoted by $s_2$, were regarded as the two values of drawdown at the point $r$ of the double porosity medium. In the following the "porous block" will be referred to as the "block" and in the mathematical symbols used, the subscript 1 will denote the fissure and the subscript 2 the block.

For an unconfined aquifer with a free surface at $Z = L$ (Fig. 4.5) the differential equation when the rock is not fractured is given by Boulton (1973).

$$T_1 \left( \frac{\partial^2 s'_1}{\partial r^2} + \frac{1}{r} \frac{\partial s'_1}{\partial r} \right) = S_1 \frac{\partial s'_1}{\partial t} + \alpha_2 S' \int_0^t \frac{\partial s'}{\partial t_{t=\tau}} e^{-\alpha_2(t-\tau)} d\tau \tag{4.18}$$

where $T = K_1 L, \alpha_2 = 3K_1/S'L$, $S'$ is the specific yield from storage and $s'_1$ is the mean drawdown over the thickness $L$ of the aquifer.

For the confined aquifer consisting of fractured, with impervious horizontal layers at $Z = 0$ and $Z = L$, the differential equation may be written as

$$T_1 \left( \frac{\partial^2 s_1''}{\partial r^2} + \frac{1}{r} \frac{\partial s_1''}{\partial r} \right) = S_1 \frac{\partial s_1''}{\partial t} + \alpha_1 S' \int_0^\infty \frac{\partial s''}{\partial t_{t=\tau}} e^{-\alpha_1(t-\tau)} d\tau \tag{4.19}$$

where $s_1''$ is the drawdown which is constant over the thickness $L$ of the aquifer and $\alpha_1 = \pi^2K_2/4H^2$

Let $s_1$ be the drawdown averaged over the thickness $L$ of the aquifer, when there is a free surface and fractured rock. Then

$$s_1 = s'_1 + s_1'' \tag{4.20}$$
Adding each side of Equations (4.18) and (4.19)

\[ T_1 \left( \frac{\partial^2 s_1'}{\partial r^2} + \frac{1}{r} \frac{\partial s_1'}{\partial r} \right) = S_1 \frac{\partial s_1''}{\partial t} + \alpha_1 S_2 \int_0^{t \cdot \tau_{t=x}} e^{-\alpha_1(t-t)} \right) + \alpha_2 S' \int_0^{t \cdot \tau_{t=x}} e^{-\alpha_2(t-t)} \right) \]

From Equation 4.20 it follows that

\[ \frac{\partial s_1}{\partial t} = \frac{\partial s_1'}{\partial t} + \frac{\partial s_1''}{\partial t} \tag{4.22} \]

The time rate of mean drawdown due to presence of the free surface \( \frac{\partial s_1'}{\partial t} \) is small compared with the drawdown due to compressibility of the block \( \frac{\partial s_1''}{\partial t} \). Hence from equation (4.22)

\[ \frac{\partial s_1''}{\partial t} = \frac{\partial s_1}{\partial t} \tag{4.23} \]

For large values of time, however the influence of the free surface is larger as compared with that of the blocks and \( \frac{\partial s_1''}{\partial t} \) is negligible as compared with \( \frac{\partial s_1'}{\partial t} \). In this case

\[ \frac{\partial s_1'}{\partial t} = \frac{\partial s_1}{\partial t} \tag{4.24} \]

Using Equations (4.23) and (4.24), the general differential equation for the flow in a fissured unconfined formation becomes,
where $S_j$ is the mean drawdown over the thickness $L$ of a fissured unconfined aquifer

$$
\alpha_1 = \pi^2 K_2/4H^2 \text{ and } \alpha_2 = 3K_1/S' L
$$

The subsidiary equation to equation 4.25

$$
\frac{d^2 s_1}{dr^2} + \frac{1}{r} \frac{ds_1}{dr} - q^2 s_1 = 0 \quad (4.26)
$$

where

$$
q^2 = \frac{pS_1}{T_1} + \frac{\alpha_1 S_2}{T_1} + \frac{p}{p + \alpha_1} + \frac{\alpha_2 S'}{T_1} + \frac{p}{p + \alpha_2}
$$

and $p$ is a constant in Laplace transformation

The required solution for the drawdown distribution $s_1$ is.

$$
s_1 = \frac{Q}{2\pi T_1} \frac{1}{p} K_0 (qr) \quad (4.27)
$$

where $K_0$ is the modified Bessel function of second kind of zero order.

Applying the inversion theorem for the laplace transformation to (4.27) the latter becomes

$$
\frac{\partial s_1}{\partial t} = \frac{Q}{2\pi T_1} \frac{1}{2\pi i} \int_{r^{-\infty}}^{r^{+\infty}} e^{i\lambda} K_0(\nu r) d\lambda \quad (4.28)
$$
where $\lambda$ written in place of $p$ is now a complex variable and

$$
\theta^2 = \frac{\lambda S_1}{T_1} + \frac{\alpha_1 S_2}{T_1} \frac{\lambda}{\lambda + \alpha_1} + \frac{\alpha_2}{T} \frac{\lambda}{\lambda + \alpha_2}
$$

(4.29)

using

$$
K(t, y) = \int_0^\infty \frac{x J_0(\gamma x)}{\sqrt{\theta^2 + x^2}} \, dx
$$

(4.30)

Eq (4.25) becomes

$$
\frac{\partial s_1}{\partial t} = \frac{Q}{2\pi T_1} \int_0^\infty \frac{x J_0(\gamma x)}{r - i\infty} \frac{e^{\lambda t}}{\sqrt{\theta^2 + x^2}} \, dx
$$

(4.31)

Equation (4.27) becomes

$$
S_1 = \frac{Q}{2\pi T_1} \int_0^\infty \frac{x J_0(\gamma x)}{r - i\infty} \frac{e^{\lambda t} - 1}{\sqrt{\theta^2 + x^2}} \, dx
$$

(4.32)

The solution of equation (4.32) is the drawdown equation for the fissure flow in a fractured unconfined aquifer (Boulton and Streltsova, 1978a) is

$$
S_1 = \frac{Q}{2\pi T_1} \int_0^\infty x J_0(\gamma x) \left\{ \sum_{n=1}^\infty \frac{1 - \exp(-\alpha_1 p_n t)}{p_n x_n} \right\} \, dx
$$

(4.33)

where

$$
X_n = C_0 + \frac{C_1}{\alpha_1 p_n} + \frac{C_2}{\alpha_2 p_n} + \frac{C_1 p_n}{(\alpha_1 p_n)^2} + \frac{C_2 p_n}{(\alpha_2 p_n)^2}
$$

(4.34)
X1, X2 and X3 being obtained by putting
p1, p2 and p3 denoted by pn, in turn
p1, p2 and p3 are the roots of the cubic equation.

\[ p^3 - Cp^2 + Dp - E = 0 \]  \hspace{1cm} (4.35)

where

\[
C = \frac{\left( x^2 + (\alpha_1 + \alpha_2) x + \alpha_1 \alpha_2 \right) C_0 + C_1 + C_2}{C_0}
\]
\[
D = \frac{\left( (\alpha_1 + \alpha_2) x^2 + \alpha_1 \alpha_2 x + C_1 + \alpha_1 C_2 + \alpha_2 C_1 \right)}{C_0}
\]
\[
E = \frac{\alpha_1 \alpha_2 x^2}{C_0}
\]

and

\[
C_0 = \frac{S_1 T_1}{C_0}, \quad C_1 = \alpha_1 S_2 T_1 \quad \text{and} \quad C_2 = \alpha_2 S_1 T_1
\]

The roots \( p_n \) of eq (4.35) are all real and positive. Quantities C, D and E are positive.

The drawdown function \( W_1 = 4 \pi T_1 S_1 / Q \) [calculated from equations 4.33 and 4.35 for assumed values of parameters \( \alpha_1, \alpha_2, S_1, S_2 \) and \( S' \) at various distances from the abstraction well 'r' and dimensionless time \( \theta' = 4T_1 \delta S'r^2 \)] are shown plotted on logarithmic graph paper in Fig.4.6 (Type curve). Values of integral \( W_1 = 4 \pi T_1 S_1 / Q \) calculated for \( \alpha_1 = 1000, \alpha_2 = 0.1, \delta S_1 = 0.0001, S_2 = 0.001, S' = 0.1 \) and the various values of \( \theta' = 4T_1 \delta S'r^2 \) are given in Table 4.1.

Using the above equations, based on the pump test data, the drawdown function (W) and the dimensionless time factor (θ) are derived together with the aid of the established logarithmic plot using the well function

\[ W = 4 \pi T_1 S_1 / Q \]  \hspace{1cm} (4.36)

and the dimensionless time factor

\[ \theta = 4T_1 \delta S'r^2 \]  \hspace{1cm} (4.37)
Fig 4.4 Rock mass broken up into blocks of irregular size by fissures

Fig 4.5 Unconfined flow to a well in a fractured aquifer

Fig 4.6 Type curve for the drawdown function
Table 4.1 Values of the drawdown function $W_1 = 4\pi T_1 S_1 / \theta$

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<th>$\theta / \pi$</th>
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<th>2</th>
<th>3</th>
<th>5</th>
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Note: $X_1 = 1000$, $X_2 = 0.1$, $S_1 = 0.001$, $S_2 = 0.1$
The Transmissivity values are estimated adopting the following procedure:

First a logarithmic plot of drawdown $s$ in $y$ axis verses $ut/r^2$ in $x$ axis is drawn (Fig. 4.7) and superimposed on the type curve (Fig. 4.6) with respect to "$r$". The well function ($W$) value and dimensionless time factor ($\theta$) values are obtained for the matching points with reference to 's' and 'ut/r^2' from the plot made from pump test data. The Equation 4.36 and 4.37 are then solved to obtain Transmissivity ($T$)

$$T = \frac{(W)Q}{4\pi s_1} \quad (4.38)$$

The above parameter is estimated for the four wells spreading over the entire cross section of the aquifer. The above aquifer parameter is subsequently modified using the 'PEST' module of MODFLOW version 4.2 in the model calibration process. The computed values of transmissivity are presented in Table 6.5 and 6.6.

4.4.3 Parameter Values for the Boundary Cells

Values of the aquifer parameters to be assigned for the boundary nodes depend on the nature of the boundary namely whether it is of the barrier or recharge type. These situations are illustrated in Fig. 4.8.

4.4.3.1 Barrier boundary

A barrier boundary being one across which there is no flow of water is handled in the programme either by assigning zero transmissivity outside the boundary of interest or by using a modified value for the transmissivity based on vector volume concept. This works out as half the value it would have as an interior node.
Fig. 4.7 Matching curve showing 's' drawdown Vs 'ur^2'.

Well No. 97065
Calculation

\[ T = \frac{Q}{S} \]

where:
- \( Q \): Gradients in \( \text{m}^3/\text{day} \)
- \( S \): Gradients in \( \text{m}^2/\text{day} \)
- \( T \): Time drawdown

\( T = 42 \times 3 \times 10^{-4} \text{ sec} \)

0.0296 \( \text{m}^3/\text{day} \) (for)

0.01 \times 10^{-4} \text{ m}^3/\text{sec}
Fig 4.8 Boundary conditions for groundwater model
The Storage factor $SF_{i,j} = S \Delta x \Delta y$

$SF_{i,j}$ = Storage factor for node located at nodal co-ordinates $i, j$, Cubm/M

$S$ = aquifer storage co-efficient

$\Delta x, \Delta y$ = finite difference grid interval.

The Storage factor outside the barrier boundary should not be set equal to zero, while running the programme as it would cause a zero divide check error. Hence one half of the values of storage factor is assigned for the boundary nodes and full values to the interior nodes.

### 4.4.3.2 Recharge boundary

A recharge boundary is considered as one along which there is no drawdown or change in water level. A very large value for storage factor, namely $1.52 \times 10^{18} \text{ M}^3/\text{M}$ has been provided in the simulation programme. The value of transmissivity along the recharge boundary will be half the value as per vector volume concept. All the interior nodes are assigned the full value of Transmissivity. If the boundary node happens to be a corner node, the factor would work out as $1/4$.

### 4.4.4 Simulation Programme

The flow chart for the Basic Aquifer Simulation Programme is shown in Fig. 4.9. This Basic Aquifer Simulation Programme is intended for use when analyzing cause and effect relationship involving drawdowns and heads in a non-steady state, non homogeneous and/or homogeneous, isotropic aquifer systems. Under these conditions it is possible to include special boundaries such as irregular, barrier or recharge boundaries (constant head) and constant withdrawal or recharge rates.
Fig 4.9 Flow chart for Basic Aquifer Simulation Program
4.4.4.1 Input Data for the Groundwater Model

The magnitudes of the various inputs and outputs and net changes in storage (NQ) at each nodal point of the aquifer are derived for a time interval of a fortnight from the mass balance equation estimated for considering both the surface and subsurface flow components.

The finite difference lattice adopted leads to 796 nodal points for the aquifer considered for this study.

4.4.4.2 Input Data for Each Node

Initially each node is assigned default values for the following.

i) Column number = I

ii) Row number = J

iii) Aquifer transmissivity between I,J and I,J+1 = T_{(I,J,1)}

iv) Aquifer transmissivity between I,J and I+1,J = T_{(I,J,2)}

v) Storage factor which is kept constant in the solution = SF_{1(I,J)}

vi) Heads at the node = H_{(I,J)}

vii) Constant withdrawal (or) net Q = Q_{(I,J)}

The basic programme has three main sections. The first section handles the input data. The second section carries out the results and the last section controls the printing of the results. The various operations carried out in the groundwater distributed model is shown in Fig.4.10.
Fig 4.10 Flow chart for groundwater distributed model
4.5 MODEL CALIBRATION WITH ‘PEST’ OF ‘MODFLOW’

Groundwater models are usually applied to conceptualize and understand a hydrologic system or to predict the outcome of a future change to the system. In order to provide some assurance that the model reflects the behaviour or appearance of the flow system, it must be calibrated prior to use as a predictive tool. Calibration is accomplished by finding a set of parameters, boundary conditions and excitations or stresses that produce simulated heads (or drawdowns) and flux that match measurement values within an acceptable range of error. Model calibration can be performed by the hand-operated trial-and-error adjustment of aquifer parameter or by automated calibration programs such as PEST (supported by PMWIN), MODINV (Doherty 1990) or MODFLOW/P (Hill 1992).

The above module is applied to this study area and the model calibration is carried out. The study area with the location of observation bore wells is shown in the Fig.4.11. The flow system is a small unconfined aquifer, which is controlled by the river, which across it. The aquifer is approximately 15m thick and is composed of charnockites with soil in the top layer. The river is having hydraulic connection with the unconfined aquifer.

The required parameters for this study area as calculated using the estimation menu provides an interface between PMWIN, the flow model MODFLOW and the parameter estimation program PEST. Using PMWIN and PEST, the required parameters can be calibrated. An estimated parameter (eg. Transmissivity) is defined by using the zone input method of data editor. The bore co-ordinates and the observation values are given in the bores and observations. Using PEST, model parameters and/or excitations can be adjusted until model generated numbers fit the observation values as closely as possible. PEST searches an optimal parameter set for which the sum of squared deviation between
model generated observations and experimental observations is to a minimum.

The control data and execution/output options in the used to set the internal array dimensions of PEST and to optimisation algorithm to the problem at hand. Further, then some data output options. The PEST control data dialogue box Fig.4.12.

The various parameters used in the optimization process shown in the parameter list dialogue box (vide Fig.4.13) use parameters estimation. The bores and observations used in this presented in the bores and observation dialogue box in Fig.4.14. The bores and the bore number used in the simulation are presented in Fig.No.4.15 for illustration. The sequences of operations carried out parameter estimation is shown in Fig.4.16.

4.6 COMPUTATION OF GROUND WATER AVAILABLE

After calibrating the model, it is used for assessing the available groundwater. The model is run for every fortnight and level at each grid point are estimated. The microwatershed bores superimposed over the finite difference lattice. The spatial distribution of the grid points for each microwatershed is assessed. The groundwater level in the grid points have been taken and the groundwater level for the microwatershed is assessed.

In order to compute the available groundwater that is extracted the changes in groundwater level (Ah) between the minimum and maximum water level is computed. Then considering the porosity extent of the aquifer below the microwatershed, the possible quantity of groundwater available is estimated using the following relations:
Fig 4.12 Pest control data dialog box

Fig 4.13 Parameter list dialog box
Fig 4.14 Bores and observations dialog box

Fig.4.15 Bores and the bore numbers (sample)
Similar analysis is made for all the 81 microwatershed; quantity of groundwater that can be extracted in each microwatershed estimated.

4.7 COMPUTATION OF SURFACE WATER AVAILABLE EACH MICROWATERSHED

The surface runoff or overland flow that can be harnessed each microwatershed by constructing surface pond or farm pond is assessed from the computed runoff at each grid-point. The average runoff or overland flow in each microwatershed is computed adopting the same method as explained in para 4.8. In this case, instead of changes in groundwater level, here the computed depth of surface runoff as discussed in para 3.6.1.1 is used. The quantity of overland flow is estimated by using the following relation.

\[ q_2 = h_{av} \times A \]

where \( q_2 \) = available surface runoff or overland flow
\( h_{av} \) = average depth of runoff in each microwatershed
\( A \) = area of each microwatershed.

75 percent of the dependable yield is taken for analysis.
4.8 COMPUTATION OF TOTAL QUANTITY OF WATER

In order to assess the water availability in each of the microwatershed with respect to water resources, the quantity of surface water plus the quantity of groundwater that can be extracted is assessed as discussed. Then the total quantity of available water for the use of the various activities taking place in each microwatershed is assessed and used in the subsequent analysis.

\[ Q_{aw} = q_1 + q_2 \]  

(4.41)

Where

- \( Q_{aw} \) = total quantity of water available in each microwatershed
- \( q_1 \) = quantity of groundwater that can be extracted
- \( q_2 \) = quantity of surface water that can be harnessed.