CHAPTER 3

MODEL BUILDING AND REPRESENTATION

This chapter discusses about the model that has been developed to solve open shop scheduling problem. It also describes about the assumptions used in the models for investigation. It explains the measure of performance used in the investigation.

3.1 ORIGIN OF THE PROBLEM

Consider the problem of testing components of an electronic system or a large automotive garage with specialized shops as illustrated in the first chapter. Gonzalez and Sahni (1976) have taken this type of problem for their investigation and identified these problems as open shop scheduling problems, with no two tasks that could be taken up simultaneously.

Consider a large automotive garage with specialized shop illustrated in chapter one. A car may require the following works: replacement of exhaust system, alignment and tune up. These three tasks may be carried out in any order, assuming that each of these three tasks are performed independently in three separate places in the workshop, rendering not possible to perform two tasks simultaneously. Here pre-emptions are not allowed. In this problem any operation can be carried out first followed by the other two. For example, one could choose alignment work first followed by tune up and then exhaust
system. This constitutes one combination to carry out the job. Likewise there are six different combinations possible to schedule. It indicates that one job with three operations can be carried out by six different combinations i.e., n!, where ‘n’ is the number of operations to finish the job. A problem with 10 jobs and each job require 20 such operation, then there will be about (20!)^{10} possible combinations of scheduling of jobs. A priority-based approach is more suitable to satisfy the objective function based on one or other the measures of performance.

Even though many objective functions are available, there are several reasons why makespan minimization is considered first here. This criterion has much historical significance with considerable research in the early sixties to the so called simple problems which later found to be combinatorial exploding. During the sixties, the researcher focuses on one machine and two-machine problem only. A six-job six-machine problem was not solved till 1989.

A scheduling problem with the objective to minimize the makespan can be considered analogous to the well-known Travelling Salesman Problem, which is in a real sense a hypothetical problem as mentioned by Jain (1998). However solving either the Travelling Salesman Problem or open shop scheduling problem will provide an insight into finding solutions to more practical instances, which have non-regular performance measures. Also the makespan minimization problem has earned a reputation for being difficult to solve. It is probably the most studied and well developed model in deterministic scheduling theory, acting as a comparative test-bed for different solution techniques old and new, and as it is also strongly motivated by practical requirements.
Suppose there are 'n' jobs and further suppose that each machine must perform 'm' operations, so there are m! possible sequences to finish the job. If these m! permutations are independently chosen for each job, then there will be (m!)^n possible combinations of scheduling the jobs. Complete enumeration of all the feasible solutions to identify the optimal one is not practical. Because the number of possible combinations astronomically increases as 'n' and 'm' are increased, this type of problem in literature is known as NP-hard problem.

3.2 EXISTING METHODS AND ALGORITHMS

Gonzalez and Sahni (1976) developed an algorithm for optimal finish time for two machines open shop scheduling problem. 'Denote t_{i,j} the task time on processor one, by a_i and t_{2,i} by b_i where 1 <= i <= n. Basically, the algorithm proceeds by dividing the jobs into two groups named: A and B. The jobs in A have a_i >= b_i while those in B have a_i < b_i, where a_i's are the processing time taken on machine one for each job and b_i's are the processing time taken on machine two for each and every job. The schedule is built from the middle with jobs from A added on at the right and those from B at the left. The schedule from the jobs in A is such that there is no idle time on machine one. The part of the schedule made up with jobs in B is such that the only idle time on machine two is at the beginning. In addition, the processing of a job on machine one can be started such that its processing on machine two can be carried out immediately after completion on machine one. Finally, some finishing touches involving only the first and the last jobs in the schedule are made. This guarantees an optimal schedule' as given in Gonzalez and Sahni (1976).
This algorithm is effective for the two-machine case with the objective of minimizing the makespan. But, for more than two jobs, with non-linear time (the jobs total processing time are not the same for all the jobs), the algorithm given by them cannot easily find sometimes even feasible schedule. An algorithm is necessary to solve not only for the 2-machine case, but also for the ‘m’ machine case in general when the processing times are arbitrary chosen. Moreover, the literature review indicates that, no one has made so far such an attempt to solve the open shop scheduling problem with arbitrary processing time. An investigation is required to solve this problem with the objective of minimizing the makespan, when the processing times are arbitrary chosen. It is worthwhile with scope for applications in many real life problems. So the investigation primarily focuses on minimizing the makespan when the processing times are being arbitrary chosen, this is due to the fact that in reality cases such as some operations may require higher processing time and some other operations may require lower processing time in real-life situations.

3.3 OPEN SHOP SCHEDULING PROBLEM

In the event of having ‘n’ jobs available for processing through ‘m’ machines. Each job can be processed in any order. The processing of a given job on a given machine is called an operation. We shall denote the operation on the i<sup>th</sup> job on the j<sup>th</sup> machine by (i,j). The processing times are denoted by ‘t<sub>ij</sub>’. Each operation (i,j) takes certain length of time, the processing time to perform. We denote this by ‘t<sub>ij</sub>’. Let us assume that the processing times are arbitrarily chosen. The general problem is to find an optimal schedule on the basis of certain measure of performance. In the area of open shop scheduling the objective are being decided using various factors like quality, promptness, customer satisfaction, minimizing makespan, utilization of the resources in
optimally, etc. The two most common measure of performance that was found in the literature is due date related criterion and flow time based criterion. The author focuses on flow time based criterion only.

3.4 ASSUMPTIONS

The author makes a number of assumptions about the structure of the open shop scheduling problem. Some were mentioned explicitly above; otherwise implicit. Here we shall list all the assumptions both for further emphasis and for easy references in the remaining chapters.

1. There are ‘n’ jobs ready for processing at time t=0
2. There are ‘m’ machine ready for processing at time: t=0 and they never breakdown during processing. No maintenance operations are considered and manpower of uniform ability is always available.
3. Pre-emption not allowed. Each job once started must be completed before another operation may be started in that machine.
4. Each job has ‘m’ distinct operations, one on each machine.
5. A machine also may be idling for want of jobs.
6. The jobs can be processed in any order, i.e. there is no technological constraint.
7. No machine may process more than one job at a time.
8. The setup time and transportation time between the machines are included in the processing time, i.e., the transfer time between machine is considered negligible compared with the processing time.
9. There is no randomness. In particular,
   a. The number of jobs are known and fixed.
   b. The number of machines are known and fixed.
c. The processing time are known and fixed.
d. The ready times are known and fixed.

10. Processing times are arbitrary chosen from a standard distribution.

11. There is no parallel processing (i.e., a job may not be processed simultaneously).

12. Machine idle time alone is considered in this thesis.

13. The monetary value of the unit machine idle time is considered same for all the machines.

The author believes that a highly idealized construction is necessary before solving the Open Shop Scheduling Problem. In the area of Open Shop Scheduling such a structural investigation can be used for the development of effective heuristics Brasel et al. It is not easy to state our objectives in scheduling. They are numerous, complex and conflicting. Instead we shall indicate in general terms a few of the criteria by which we might judge our success. These criteria will be sufficient to motivate the mathematical definition of the performance measures that we shall use.

3.5 MODEL REPRESENTATION

A schedule is defined by a complete and feasible ordering of operations to be processed on each machine and in open shop scheduling, there are two main ways of graphically representing such an ordering:

- Gantt Chart
- Disjunctive Graph

The thesis uses Gantt Chart representation only for the job schedule.