CHAPTER 1

INTRODUCTION

1.1 OUTLINE OF THESIS

The problem of realizing a given real symmetric \((nxn)\) matrix as the short-circuit admittance matrix \(Y\) or open-circuit impedance matrix \(Z\) of an \(n\)-port network on more than \((n+1)\)-nodes and specified port configuration is not yet solved. One has to get the necessary and sufficient conditions which the given \((nxn)\) matrix should satisfy for it to be realizable as \(Y\)- or \(Z\)-matrix of an \(n\)-port network containing more than \((n+1)\)-nodes. Necessary and sufficient conditions are known for the given real symmetric \((nxn)\) short-circuit admittance matrix \(Y\) to be realizable with exactly \((n+1)\)-nodes. For more than \((n+1)\)-nodes known results are the sufficient condition of dominance and necessary condition of paramountcy neither of which are both necessary and sufficient conditions.

Biorci (1962) made an important conjecture in a panel discussion, that for the realization of a real symmetric \((nxn)\) short-circuit admittance \(Y\)-matrix with \((n+p)\)-nodes, \((p \geq 2)\) one may require only \(n(n+1)/2\) number of positive edge conductances which is exactly the number of independent entries in the matrix. The solution to this problem is applicable in the

- Synthesis of RLC networks from residue matrices
- Synthesis of single element kind \(n\)-port networks
- Design of multiport networks and systems.

In this thesis, Biorci's conjecture is proved. Besides, it deals with synthesis of resistive \(n\)-port network on more than \((n+1)\)-nodes.
1.2 REVIEW OF LITERATURE

Cederbaum (1958) proved that paramountcy is a necessary condition for a real symmetric matrix to represent the short-circuit admittance matrix or open circuit impedance matrix of a resistive n-port network.

Guillemin (1960) proved that a real symmetric matrix can be realized as the short-circuit admittance matrix of a resistive n-port network with \((n+1)\)-nodes with a port configuration in the form of a linear tree if and only if the matrix is uniformly tapered.

It has been shown by Weinberg (1962)

- that paramountcy is a necessary and sufficient condition for the realization of a real symmetric matrix as the short-circuit admittance matrix of a 3-port network.
- that dominance is a sufficient condition for the realization of a real symmetric matrix as the short-circuit admittance matrix.

Subsequently several methods have been given for the realization of the short-circuit admittance matrix \(Y\) of a resistive n-port network with \((n+1)\)-nodes. Each of these methods is based on one of the following two approaches. The first approach requires the decomposition of matrix \(Y\) into triple product \(Y = C_0 \cdot G \cdot C_0^T\), where \(C_0\) is a matrix whose elements are \(-1, 1, 0\) and \(G\) a diagonal matrix with non-negative elements representing the edge conductances. The realization of \(Y\) is accomplished if \(C_0\) can be realized as the fundamental cut-set matrix of a connected linear graph. Cederbaum (1959) uses this approach. The second approach involves the determination of port-tree structure (if one exists) and if a congruent transformation on \(Y\) results in a dominant matrix, the realization can be accomplished by inspection. Guillemin (1960), Biorci and Civalleri (1961) and Boesch and Youla (1965) use this approach. Cederbaum’s (1965) decomposition algorithm and Bapeswara Rao’s (1970) algorithm for realizing a
As regards the synthesis of resistive n-port networks with more than \((n+1)\)-nodes from a given matrix \(Y\), two general methods have been suggested by Guillemin and Cederbaum. Guillemin's (1961) method is based on the parallel connection of two networks. The port configuration of these networks forms the subgraph of a linear tree. The combination is such that an appropriate n-port network derived from it has a short-circuit admittance matrix equal to the given matrix \(Y\). Cederbaum's (1965) method attempts to obtain through equivalent network approach, a network containing no negative conductances, starting from a given realization which may perhaps include some negative conductances.

Some necessary and some sufficient conditions were obtained by Frisch and Swaminathan (1965) for the case on \((n+2)\)-node, n-port resistive networks. It is proved that \textit{supremacy} is a necessary condition for the realization of a given real symmetric matrix of an \((n+2)\)-node n-port resistive network.

Following Guillemin's approach, introducing the concepts of network of departure and padding n-port network, some work had been done mostly for realizing \((n+2)\)-node, n-port networks by Subbarami Reddy (1972). Using potential factors of an n-port network, originally defined by Murti and Thulasiraman (1967), formulae have been derived for the edge conductances of padding n-port network with \((n+p)\), \(p \geq 2\) nodes and a general method was suggested by Subbarami Reddy (1972) using these formulae and the network of departure to realize an \((n+p)\)-node n-port network.

In addition, Subbarami Reddy and Thulasiraman (1972) have dealt with the case of \((n+2)\)-node n-port resistive network realization when both \(Y\) and \(K\)-matrices are specified.
As in the case of realization of (n+1)-node n-port network from an (nxn) short-circuit conductance matrix, Biorci (1962) conjectured in a panel discussion that if at all an (nxn) short-circuit admittance matrix Y is realizable on (n+p)-nodes, p ≥ 2 nodes without ideal transformers only a maximum of n(n+1)/2 positive edge conductances will be required. This conjecture is neither proved nor disproved till now though some attempts in this direction were made by Biorci (1966) himself.

1.3 SCOPE OF THE PRESENT WORK

The following will be the main topics constituting the scope of the present work.

- Definition of the concepts of network of departure and padding n-port networks.
- Generation of padding n-port networks using the concept of potential factors.
- Proof of Biorci's conjecture on the maximum edge conductances required to realize a given (nxn) real symmetric matrix as the short-circuit conductance matrix of an n-port network on more than (n+1)-nodes.
- Development of a new algorithm to realize a (nxn)-real symmetric Y-matrix on specified port structure of (n+2)-nodes and hence deduction of a necessary and sufficient condition for such a realization.
- Biorci's matrix made realizable in accordance with his conjecture.
Chapter 1 introduces the general class of resistance n-port network problems and the issues that need to be attended. Then confining to the following two topics,

- Y-matrices of n-port networks on \((n+1)\)-nodes, that is, n-port networks of rank \(n\) and
- Y-matrices of n-port networks on more than \((n+1)\)-nodes,

a detailed literature survey discussing the existing methods of solution and their drawbacks is presented. The desirable features that the proposed methods of solution must possess, which remain unfulfilled in the existing methods, are also identified and listed. Biorci’s conjecture on the realization of short-circuit conductance matrix is stated and the scope of the present work is pointed out.

The concept of departure and padding n-port network is reviewed in Chapter 2. Several important properties of these networks are examined. These networks form the basic tools in the synthesis procedures given in the chapters to follow. Subbarami Reddy (1972) has reported the material discussed in this chapter. The modified cutset matrix \(C\) introduced by Cederbaum (1965) and the potential factor matrix \(K\) which has been defined by Murti and Thulasiraman (1967) in connection with establishing a criterion for the proper parallel connection of n-port networks are examined. The relationship between the \(C\)- and \(K\)-matrices is reviewed. The formulae for the edge conductances derived by Subbarami Reddy (1972) of a padding n-port network with \((n+p)\), \(p \geq 2\) nodes are presented for the sake of completeness and good understanding.

In Chapter 3 the general procedure given by Subbarami Reddy (1972) for the realization of a short-circuit conductance matrix is mentioned briefly. Using this result the Y-matrix synthesis problem is converted into one of optimization problem and Biorci’s conjecture on the maximum number of edge conductances required to realize a given \((nxn)\) real symmetric matrix as the short-circuit conductance matrix of an n-port network is proved. Lagrange multiplier method of optimization is applied and it was concluded that utmost \(n(n+1)/2\) edge
conductances are required to effect such a realization. While the realization of the $Y$-matrix of a resistive $n$-port network having a specified port configuration looks formidable requiring the solution of nonlinear equations, synthesis of a resistive $n$-port having prescribed $K$- and $Y$-matrices is straightforward requiring the solution of a linear program. A general procedure for the simultaneous realization of the $K$- and $Y$-matrices of a resistive $n$-port network is given. Maximum number of edge conductances required in this case is proved to be $n(n+1)/2+n(p-1)$.

Chapter 4 addresses the problem of deriving the necessary and sufficient conditions for the realization of a given real symmetric $(nxn)$ matrix as the short-circuit conductance matrix of resistive $n$-port network without ideal transformers. Biorci’s conjecture on the maximum number of conductance required which has been proved in Chapter 3 plays a very important role in arriving at the results presented in this chapter.

In Chapter 5, the main contributions of this thesis are listed.