CHP4TER 3

STEADY-STATE MODELING AND SIMULATION OF THE PMBLDC MOTOR

3.1 INTRODUCTION

PMBLDC motor is an alternative to the conventional DC motor and it acts like a DC machine when viewed from the voltage source. The machine usually consists of surface mounted or buried permanent magnets in the rotor, a three-phase distributed winding on the stator and rotor position sensors to control the switching sequence of the MOSFETs/IGBTs of the inverter placed between the stator and the DC voltage source. Several papers (Fardoun et al. 1994, Sudhoff and Krause 1990a, 1990b) have been published on the modeling and simulation of permanent magnet machine drives. They focus only on the machine operation in the full-on mode at rated speed and discuss the magnitude of the torque of the system as a function of the phase shift of the induced voltage. Some steady-state analysis (Fardoun et al. 1994, Sudhoff and Krause 1990a, 1990b) has been done and they lack experimental verification and simulation detail.

Hence the permanent magnet machine is modeled using Saber software in this work. The Saber model is developed using the analytical work reported in the literature (Pragasen Pillay and Ramu Krishnan, 1989). The performance of the machine is evaluated neglecting friction, windage and core losses in both the PWM and the full-on modes. The simulation results are discussed.
3.2 SIMULATION PACKAGE

Saber, a general purpose simulator now available on Windows NT platform, provides user friendly and user customizable environment. It is a comprehensive mixed technology simulator spanning analog and digital domains and is capable of simulating systems described by a mixture of models at the primitive, functional and behavioral levels. It is also a mixed technology (electrical, mechanical, optical, etc) simulator. The package consists of three major tools: SaberSketch, SaberGuide and Saberscope. A schematic representation of the circuit to be simulated is obtained pictorially using the SaberSketch. Symbol libraries available in the package are used in this process. Simulation is done using the SaberGuide. The SaberScope, which is the graphical waveform analyzer or waveform post processor, provides enhanced facility for easy handling of simulation data. There is also a waveform calculator for easy and flexible manipulation of waveforms.

3.3 MODELING OF THE PMBLDC MOTOR

The circuit equations of the three windings of the PMBLDC motor in phase variable form are

\[
\begin{align*}
  v_a & = R_{ia} + \frac{d}{dt} (L_{aia} + L_{abi} + L_{acic}) + e_a \\
  v_b & = R_{ib} + \frac{d}{dt} (L_{baia} + L_{bib} + L_{bic}) + e_b \\
  v_c & = R_{ic} + \frac{d}{dt} (L_{caia} + L_{cbib} + L_{acic}) + e_c
\end{align*}
\]

where

- \( v_a, v_b, v_c \) are the applied phase voltages
- \( e_a, e_b, e_c \) are the induced phase voltages
- \( i_a, i_b, i_c \) are the phase currents
- \( L_{ia}, L_{ib}, L_{ic} \) are the self-inductances of the phases
- \( L_{ab}, L_{ba}, L_{ac}, L_{ca}, L_{bc}, L_{cb} \) are the mutual inductances between the various phases and

\( R \) is the resistance of each phase.
The airgap in the prototype motor is uniform and hence the self-inductances are all equal to $L_4$ and the mutual inductances are equal to $M$. The equations (3.1) to (3.3) are simplified as

\begin{align*}
  v_a &= R_{i_a} + \frac{d}{dt}(L_{i_a}) + e_a \\
  v_b &= R_{i_b} + \frac{d}{dt}(L_{i_b}) + e_b \\
  v_c &= R_{i_c} + \frac{d}{dt}(L_{i_c}) + e_c
\end{align*}

where $L = L_4 - M$ is the leakage inductance of each phase.

The simplified schematic diagram of the PMBLDC machine with voltage source inverter is shown in Figure 3.1. The inverter is operated in the 120° mode. For a given speed, each phase of the motor is modeled as speed-emf in series with resistance and leakage inductance of the winding (Pragasen Pillay and Ramu Krishnan 1989). The source and the lead inductances are also taken into consideration in the simulation. The torque developed $T_d$ can be expressed in terms of the induced phase voltages and the phase currents as given in equation (3.7).

\[ T_d = \frac{(e_{a,i_a} + e_{b,i_b} + e_{c,i_c})}{\omega} \]  

(3.7)

where $\omega$ is the angular speed of the motor.

Figure 3.1 Steady-state model of the PMBLDC motor
Alternatively, the torque developed can be written in terms of armature reaction flux $\phi_a$, rotor flux $\phi_r$ and the space phase angle $\beta$ as in equation (3.8).

$$ T_d \propto \phi_a \phi_r \sin \beta \quad (3.8) $$

For a given rotor flux and a fixed space phase angle, the torque developed is proportional to the current as in conventional DC motor and is expressed as

$$ T_d = K I_d \quad (3.9) $$

where $I_d$ is the average source current. For generation of gate pulses to the MOSFETs, the time period of the speed-emf is calculated at a given speed and is used to generate a train of six firing pulses each of 120° duration for every cycle. The current pulse applied to the phases are delayed with respect to the zero crossing of the corresponding phase emfs to control the phase shift between the armature reaction and field fluxes. The induced phase voltages are assumed to be sinusoidal in simulation since the prototype motor has sinusoidal back-emfs.

### 3.4 OPERATION IN THE FULL-ON MODE

In the full-on mode, a MOSFET switch is allowed to conduct for 120° and a pair of MOSFET switches conduct for a full 60° period. The rotor and the stator fluxes are displaced by 90° by delaying the rectangular firing pulse by 30° with respect to the zero crossing of the corresponding back-emf. The firing sequence for the MOSFETs over a cycle of the inverter output is 12, 23, 34, 45, 56, 61. The firing pulses are shown in Figure 3.2

Figure 3.3 shows the waveforms of source current, MOSFET 1 drain current, phase-a current and the commutation overlap between the phase-a and phase-b currents. The dip in the phase currents is due to the leakage inductance of the stator winding which does not allow instantaneous current transfer. The inherent turn-on time of the MOSFET has negligible effect compared to the role of the leakage inductance. The commutation overlap is due to the presence of source inductance and the inherent
Figure 3.2 Firing pulses for MOSFETs of the inverter

Figure 3.3 Waveforms of source current, device current, phase-a current and commutation overlap
turn-on and turn-off times of the MOSFET. The transient switch current has very high spikes during the turn-on and turn-off intervals. The source current is oscillatory in nature and imposes harmonic burden on the source. Thus filtering is required in the source side.

Figure 3.4 shows the phase-a current and the current through a diode D5. It is observed that there is a current spike of about 60A in D5 when rated phase-a current commutates from M3 to M5 with M4 on. It is therefore necessary to use power diodes with high repetitive peak current though the rated current of the diode may be low. MOSFET 1 current and the corresponding drain to source voltage are shown in Figure 3.5. It is observed that there are spikes during turn-on and turn-off of MOSFET 1. Further it can be seen that the MOSFET 1 voltage abruptly rises to DC supply voltage and immediately falls to a low value and then builds up gradually to the same supply voltage when phase-a current commutates from M1 to M3.

Figure 3.6 and 3.7 show the magnified MOSFET 1 turn-on current and reverse recovery current with high di/dt which is destructive. It is observed from Figure 3.6 that a positive peak of 15 A and a negative peak of 20 A occur in the turn-on current waveform of the MOSFET model used from the Saber library. During turn-off, the MOSFET 1 current is highly oscillatory in nature with a positive peak of 20 times the rated current. Suitable protection circuit is to be implemented. Figure 3.8 shows the phase-a, phase-b and phase-c currents along with respective back-emfs. At every crossing of the back-emfs, there are dips in the phase currents representing the commutation in the other two phases.

Figure 3.9 shows the phase-a, phase-b and phase-c currents along with the line to line back-emfs. It can be seen that the firing of MOSFET has been done when the line to line back-emf advances by 30° to obtain the maximum torque with maximum efficiency. Figure 3.10 shows the phase-a current and the line to line machine voltage. It is seen that the line to line voltage is trapezoidal with spikes and dips associated with the rise and fall of phase-a current.
Figure 3.4 Waveforms of phase-a current and diode D5 current

Figure 3.5 Waveforms of current and drain to source voltage of MOSFET 1
Figure 3.6 Waveform of turn-on current of MOSFET 1

Figure 3.7 Waveform of reverse recovery current of MOSFET 1
Figure 3.8 Waveforms of phase currents and phase voltages

Figure 3.9 Waveforms of phase currents and line voltages
Figure 3.10 Waveforms of a line current and a line voltage

Figure 3.11 shows the waveforms of input and output power. The input power is oscillatory in nature like the input current and the output power fluctuates with pronounced dips at the commutating points. The output torque of the PMBLDC motor is shown in Figure 3.12. The torque dip is due to the leakage inductance of the machine. If the leakage inductance is reduced, the torque ripple can be reduced.

Figure 3.13 shows the phase-a current for shift angles 10°, 30° and 50°. It is seen that the shape as well as the magnitude of the phase current changes as the firing angle is shifted with respect to line to line back-emf. The source current for shift angles 10°, 30° and 50° are shown in Figure 3.14. The shape as well as the magnitude of the source current change and maximum positive spike occurs at 50° shift.
Figure 3.11 Input and output power waveforms

Figure 3.12 Output torque waveform
Figure 3.13 Phase current waveforms for different shift angles

3.14 Source current waveforms for different shift angles
The input power for 10°, 30°, 50° shift angles and the corresponding output powers are seen in Figure 3.15. The output torque for 10°, 30°, 50° shift angles are shown in Figure 3.16. The shape of the torque waveform changes drastically when the shift angle deviates from 30°.

3.5 OPERATION IN THE PWM MODE

PWM operation is carried out by using timer circuits for the bottom MOSFETs only. The duration and interval of the rectangular pulses applied to the input of the timers are calculated based on the assumed speed of the motor. The outputs of the timers are fed to the gates of the bottom MOSFETs.

Figure 3.17 shows the phase-a current in the full-on mode and the PWM modes. Shape of both the waveforms is similar. The magnitude of the PWM mode current is slightly increased. It is because the voltage actually applied in the PWM mode is greater than that of the full-on mode even though the effective voltage is same in both the cases.

Figure 3.18 shows the input current in the PWM and the full-on modes. It is clear that the PWM mode of operation injects more harmonics into the supply and increases the cost of filtering requirements.

Figure 3.19 shows the input as well as the output powers. Oscillations exist in the input power due to the frequent switching of the bottom MOSFETs. Output power has dips, which can be reduced if leakage inductance of the stator is reduced.

The output torque in the PWM mode and the full-on mode are seen in Figure 3.20. The output torque in the PWM mode is greater than that of the full-on mode because the voltage actually applied is more in the PWM mode.
Figure 3.15 Input and output power waveforms for different shift angles

Figure 3.16 Output torque waveforms for different shift angles
Figure 3.17 Phase-a current in the full-on and the PWM modes

Figure 3.18 Source current waveforms in the full-on and the PWM modes
Figure 3.19 Input and output power waveforms in the PWM mode

Figure 3.20 Output torque waveforms in the full-on and PWM modes
3.6 COMPARISON BETWEEN THE FULL-ON AND THE PWM MODES

The harmonic content increases above and below 30° phase shift resulting in less efficiency. In order to compare the output torque in the full-on and the PWM modes, the source voltage and the duty ratio in the PWM mode are adjusted so that the average voltage applied will be the same for both cases. It is seen that the torque output of the PWM mode is greater than that of the full-on mode. Figure 3.21 shows the relationship between the developed torque and the input current as obtained by simulation and this confirms the torque equation (3.9). The harmonic profile at different shift angles are shown in Figure 3.22 and it shows that the harmonics are minimum at 30° shift.

The performance of the PMBLDC motor in the full-on mode for different speeds has been presented in Table 3.1. From the table, it can be seen that the machine delivers rated power at 1500 rpm. The efficiency can go upto 97.94 % at light loads.

Table 3.2 shows the performance of the PMBLDC motor in the PWM mode. The modulating frequency is 24 kHz. It is seen that the efficiency of the PWM mode is inferior to that of the full-on mode. For the same speed, motor draws more power from the source in the PWM mode than the full-on mode and delivers more power. The input current as well as the phase current is larger in the PWM mode. For the same speed, the torque in the PWM mode is increased because of the increased output power.

The simulation results and waveforms are in good agreement with those obtained experimentally and reported in the literature (Natarajan et al 2000a).
Figure 3.21 Output torque Vs input current

Figure 3.22 Harmonic profile for different shift angles
Table 3.1 Performance of the PMBLDC motor in the full-on mode

<table>
<thead>
<tr>
<th>Speed rpm</th>
<th>Back emf V</th>
<th>Input current A</th>
<th>Input W</th>
<th>Phase current A</th>
<th>Output W</th>
<th>Torque Nm</th>
<th>Efficiency %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>114.312</td>
<td>2.265</td>
<td>498.22</td>
<td>1.905</td>
<td>439.95</td>
<td>2.8</td>
<td>88.3</td>
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<tr>
<td>1590</td>
<td>121.165</td>
<td>1.422</td>
<td>312.82</td>
<td>1.18</td>
<td>289.4</td>
<td>1.7381</td>
<td>92.51</td>
</tr>
<tr>
<td>1620</td>
<td>123.45</td>
<td>1.146</td>
<td>252.03</td>
<td>0.973</td>
<td>237.23</td>
<td>1.3984</td>
<td>94.13</td>
</tr>
<tr>
<td>1650</td>
<td>125.74</td>
<td>0.864</td>
<td>190.08</td>
<td>0.745</td>
<td>181.83</td>
<td>1.0523</td>
<td>95.55</td>
</tr>
<tr>
<td>1680</td>
<td>128.024</td>
<td>0.586</td>
<td>128.82</td>
<td>0.531</td>
<td>124.97</td>
<td>0.7104</td>
<td>97.01</td>
</tr>
<tr>
<td>1710</td>
<td>130.31</td>
<td>0.302</td>
<td>66.396</td>
<td>0.315</td>
<td>65.03</td>
<td>0.3631</td>
<td>97.94</td>
</tr>
</tbody>
</table>

Table 3.2 Performance of the PMBLDC motor in the PWM mode

<table>
<thead>
<tr>
<th>Speed rpm</th>
<th>Back emf V</th>
<th>Input W</th>
<th>Output W</th>
<th>Torque Nm</th>
<th>Efficiency %</th>
<th>Phase current A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500</td>
<td>114.312</td>
<td>584.94</td>
<td>486.71</td>
<td>3.0985</td>
<td>83.21</td>
<td>2.1049</td>
</tr>
<tr>
<td>1590</td>
<td>121.165</td>
<td>418.3</td>
<td>360.62</td>
<td>2.166</td>
<td>86.21</td>
<td>1.4723</td>
</tr>
<tr>
<td>1620</td>
<td>123.45</td>
<td>376.83</td>
<td>329.61</td>
<td>1.943</td>
<td>87.47</td>
<td>1.3407</td>
</tr>
<tr>
<td>1650</td>
<td>125.74</td>
<td>314.67</td>
<td>276.19</td>
<td>1.598</td>
<td>87.77</td>
<td>1.0904</td>
</tr>
<tr>
<td>1680</td>
<td>128.024</td>
<td>271.1</td>
<td>238.8</td>
<td>1.357</td>
<td>88.09</td>
<td>0.9414</td>
</tr>
<tr>
<td>1710</td>
<td>130.31</td>
<td>226.8</td>
<td>200.89</td>
<td>1.122</td>
<td>88.78</td>
<td>0.7888</td>
</tr>
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</table>
3.7 CONCLUSION

The various modules of the PMBLDC drive namely the PMBLDC motor, the three-phase inverter bridge and the corresponding firing circuit and load are modeled individually using the Saber package and then integrated to provide the overall model of the drive. Modeling is carried out for both the full-on mode and the PWM mode of operation of the three-phase inverter. The performance of the PMBLDC motor is evaluated both for the full-on mode and the PWM mode for different shift angles and different speeds. The performance characteristics clearly show that the PMBLDC motor behaves like a DC shunt motor.