APPENDIX 2

FREQUENCY DEPENDENCE OF RESISTANCE

The key issue as far as the estimation of resonant overvoltages in transformers are concerned lies in the determination of losses. It is known that in coils skin and proximity effects tend to increase the AC resistance of the coil. A number of methods have been proposed to account for such losses, with many of them starting from plane wave field solutions. For the sake of completeness we briefly reconsider such approaches. As a starting point it would be advantageous to determine the resistance of a circular wire of radius $r_0$ as function of frequency. The wire carries a current $I$.

Now for a good conductor
\[ \nabla \times \mathbf{H} - \mathbf{J} = \sigma \mathbf{E} \quad (A2.1) \]

Faraday's law in phasor form can be written as,
\[ \nabla \times \mathbf{E} = -j \omega \mu \mathbf{H} \quad (A2.2) \]

Using Equations (A2.1) and (A2.2) we get
\[ \nabla \times \mathbf{J} = j \omega \mu \sigma \mathbf{J} \quad (A2.3) \]

For the plain wire using cylindrical co-ordinates; taking current in the $Z$ direction, we have.
\[
\frac{\partial^2 J_z}{\partial r^2} + \frac{1}{r} \frac{\partial J_z}{\partial r} + T^2 J_z = 0 \quad (A2.4)
\]

where \( T^2 = -j\omega\mu\sigma \) \quad (A2.5)

This equation is a Bessel equation whose solution with appropriate boundary conditions (Ramo S., Winney J.R. and Duzer T.V, 1984) is

\[
J_z = \frac{\sigma E_0}{J_0(Tr)} J_0(Tr) \quad (A2.6)
\]

\( J_0 \) is known to be complex. Hence it is convenient to break the function into complex and real parts using the definitions,

\[
B_r (v) = \text{real part of } J_0 \left(j^{v/2}\right) \text{ and } B_i (v) = \text{imaginary part of } J_0 \left(j^{v/2}\right)
\]

Hence Equation (A2.6) can be written as,

\[
J_z = \sigma E_0 \frac{B_r (v) \left(\sqrt{2r/\delta}\right) + j B_i (v) \left(\sqrt{2r/\delta}\right)}{B_r (v) \left(\sqrt{2r_0/\delta}\right) + j B_i (v) \left(\sqrt{2r_0/\delta}\right)} \quad (A2.7)
\]

Here \( \delta \) is the skin depth.

For the cylindrical case,

\[
H_\phi = \frac{dE_z}{j \omega \mu \, dr} \quad (A2.8)
\]

Recalling that \( E_z = J_z/\sigma \)

\[
E_z = E_0 \frac{J_0(Tr)}{J_0(Tr_0)} \quad (A2.9)
\]
Substituting $E_z$ in Equation (A2.8), we get

$$H_z = E_z \frac{\mu_0 J_0'(Tr)}{\mu J_0(Tr_0)} - \frac{-\sigma E_z J_0'(Tr)}{T J_0(Tr_0)}$$  \hspace{1cm} (A2.10)

Now $I = 2 \pi r_0 H_z$

$$\therefore I = \frac{-2 \pi r_0 E_0 J_0(Tr_0)}{T J_0(Tr_0)}$$  \hspace{1cm} (A2.12)

The internal impedance per unit length is

$$Z_1 = \frac{E_z}{I} \frac{T J_0(Tr)}{2 \pi \sigma r_0 J_0'(Tr_0)}$$  \hspace{1cm} (A2.13)

The real part gives the resistance which is

$$R = \frac{R_i}{\sqrt{2} \pi r_0} \left[ \frac{B_{r q} B_{r' q} - B_{i q} B_{r' i q}}{(B_{r' q})^2 + (B_{i q})^2} \right] \Omega / m$$  \hspace{1cm} (A2.14)

This is the expression for the resistance of round wire at any frequency in terms of the parameter 'q' which is $\sqrt{2}$ times the ratio of wire radius to depth of penetration. The curve corresponding to the Equation (A2.14) is shown in Figure 4.8.

Snelling E.C., (1988) has reported an empirical method ascribed to Dowell P.I., (1966) for the calculation of AC resistance of windings. This analysis allows for non-uniform flux density over the thickness of the winding and also takes into account eddy-current screening. The results are expressed in terms of a resistance factor $F_R$. 
For each winding the following parameters are required;

Number of layers \( P \)

Number of turns per layer \( N_l \)

Effective conductor height \( h \)

Copper layer factor \( F_l \)

The layer copper factor is defined by,

\[ F_R = N_l \frac{b}{b_w} \]  
\[ \text{(A2.15)} \]

where 'b_w' is the overall winding breadth and 'b' the conductor breadth.

\[ \phi = h \frac{F_R}{\delta} \]  
\[ \text{(A2.16)} \]

Figure 4.9 shows \( F_R \) as a function of \( \phi \) from which \( R_{ac} = F_R \cdot R_{dc} \) can be computed.