Chapter II

FACTOR PROPORTIONS THEORY

The factor proportions theory has reigned the world for the last fifty years. The credit for replacing the classical theory of International trade by a modern theory goes to two Swedish authors, Eli Heckscher and Bertril Ohlin. Heckscher's remarkable paper published in Sweden in 1919\(^1\) made the first departure from the classical analysis. Following him, Bertril Ohlin claimed in his bulky volume on International Trade to have sharply broken with the classical theory of comparative advantage by putting International trade theory in terms of a multiple market theory.\(^2\) The general equilibrium approach to International trade theory employed by both Heckscher and Ohlin set in motion many other scholarly works in this field. The whole doctrine has been re-examined from different angles and the discussion has centred round a closer scrutiny of two positions of the theory. The first is that the cause of comparative cost differences between countries lies in


differences in their relative factor endowment, the second that free International trade in commodities tends to equalise factor prices between countries, thus serving to some extent as a substitute for mobility of factors. An attempt has been made in this chapter to examine the theoretical validity of these two propositions.

**Factor endowments and pattern of trade:**

The central theme of the Heckscher-Ohlin model is the demonstration of the minimum requirement for International trade. Countries trade with each other because of differences in their relative cost of production of commodities. In the absence of trade, each country has a set of commodities which are relatively cheaper than others. One might order these commodities as per their relative cost of production. The problem is which are the factors that determine this order. Both Heckscher and Ohlin attribute it to differences in the factor endowments of countries. Differences in factor endowments cause differences in factor prices and thereby commodity prices and the interplay of economic factors makes it incumbent

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3 In the classical model, the "production conditions" alone determined the International trade. But the main weakness of the model is that they did not specify what exactly contained in the production condition. Only there are loose references to differing climates, skill, ingenuity of the population etc. The classical economists were more concerned with the welfare aspects of trade and so left the question of what determines trade hanging in the air.
upon different countries to export those goods which require relatively a higher proportion of their abundant factor. This proposition has come to be called as "the Heckscher-Ohlin theorem".

Before we attempt for a proof of the theorem, let us note the main assumptions of the model.

(a) There are two countries, two commodities and two factors of production.

(b) The production functions are identical for same commodity in both countries and are different for different commodities.

(c) Both commodities use both factors. Their individual production functions are homogeneous, convex and with constant returns to scale.

(d) The relative factor intensities of both the goods are same at all factor prices so that the labour intensive good remains the labour intensive in both the countries and vice versa.

4 Ohlin's analysis of factor requirements in commodities is rather loose. There are only statements like "each region is best equipped to produce the goods which require large proportions of the factor relatively abundant there" (See Ohlin, Op.Cit., p.12.) Both Heckscher and Ohlin did not say what is the precise meaning of their assertion that a given commodity requires a high proportion of a certain factor. Later Samuelson's "strong factor-intensity" assumption permitted for an unambiguous statement with regard to the direction of trade.

(e) There is perfect competition in both factor market and product market and full employment of resources.

(f) There exists complete free trade between the two countries and transport costs are zero.

(g) Both countries have different factor endowments, but a fixed supply of both the factors.

(h) Consumer preferences are identical in both countries at each relevant commodity price ratio.

(i) Labour and capital are of identical quality in both the countries.  

Of these assumption, conditions (a) and (i) are merely for convenience. Condition (e) implies that factor payments are determined entirely by their marginal product and under equilibrium conditions the payments to identical factors are equal. The conditions (b), (e) and (d) are the main cornerstones of the model and need further elaboration. The

Ohlin did not share this and is more realistic when he says, "yet it cannot be overlooked that the various groups of labour perform different tasks and receive unequal wages and that the flow of individuals from one group to another is not free and easy. Should not such groups of labour be regarded as different factors of production? Has not the fact that some of them receive relatively much higher pay in one country than in another anything to do with the International division of labour? Undoubtedly. Countries with a large supply of labour with high technical skill will be able to produce manufactured goods more cheaply than countries with a scanty supply of this labour quality ...." (See Ohlin. Op.Cit., p.69) Ohlin divided labour into three groups; unskilled, skilled and technical labour. Similarly he also makes distinction between long and short term capital. But over the time, most economists have come to label trade models with a two or three factor breakdown as a simplification to the Heckscher-Ohlin theory.
condition (b) implies that the isoquant map for a given commodity is same in both countries. This may be demonstrated with the help of Figure 1. Here we have the isoquant drawn for one unit of output for a commodity, say, food in country 1 and 2. MM is the factor price ratio line which is tangential to the food isoquant at point P. Reading along vertically from this point we get the capital inputs and horizontally labour inputs per unit of food output. If both countries have identical factor price ratios, P will indicate the optimum combination of capital and labour inputs for the production of one unit of good output. If the second country is favourably positioned with respect to labour compared to the first country, NN is the factor price ratio line which is tangential to the isoquant at Q. The same applies to another commodity, say textiles.

Condition (c) states that production functions are subject to constant returns to scale. This implies that we can derive the whole production surface for these two commodities, food and textiles, if we are given their respective unit isoquant. This is demonstrated in Figure 2. Isoquants for food are drawn for one unit and two units of

To be more specific, it would show the ratio, independent of the scale of output, of quantities of labour and capital employed in the production of a commodity at a given price of the two factors.
output. If we suppose that the factor price ratio is depicted by the slope of MM, then to produce one unit of food it requires two units of capital and one unit of labour implying a labour capital ratio of 1/2. If we apply the same MM line to the isoquant representing two units of food, it will be seen then that it requires two units of labour and four units of capital implying a labour capital ratio of 1/2.

With constant returns to scale and perfect competition, Euler's identity holds. Therefore each factor of production must be paid the value of its marginal physical product. This means that the actual price of a commodity under equilibrium, is equal to the cost of production per unit of output, where the cost itself must be equal to the factor inputs per unit of output valued at their market prices.

Condition (d) implies that the isoquants for food and textiles are not identical. They need different capital-labour combination to produce a unit of output at a given factor-price ratio. When we compare food with textiles, the former is said to be labour-intensive, if at all relevant price ratios of the two factors, the ratio between the quantity of labour and capital is higher in the production of food than in textiles. This relative factor intensity remains fixed for all feasible capital labour ratios. This is demonstrated in Figure 3 where we have drawn two isoproduct curves, one for
Figure 3.
food and another for textiles. MM is the factor price ratio line at which textiles are shown to have a high capital-labour ratio as compared to food. (M'M' is parallel to MM); i.e.

$$\frac{NQ}{OQ} > \frac{PZ}{OZ}$$

does not intersect more than once. If they intersect more than once, factor intensity cannot hold good.

9. This is because of the strong factor-intensity assumption.

10. Condition (h) implies not only identical tasks but also that the aggregate preference map of the two countries have homothetic isoquants (implying unitary income elasticities).
intensive. Assuming country 1 is capital rich and country 2 labour rich, we can define factor abundance in its physical connotation as:

\[ \frac{c^1}{L^1} > \frac{c^2}{L^2} \]

Let us indicate the outputs of textiles and food in countries 1 and 2 by \( X^1_t, X^1_f, X^2_t, X^2_f \)

Through algebraic manipulation, we can then prove,

\[ \frac{X^1_t}{X^1_f} > \frac{X^2_t}{X^2_f} \]

i.e. country 1 produces relatively more of textiles and country 2 more of food. It follows then, from the assumptions of similar tastes in both countries and perfect competition, that textiles must be cheaper than food in country 1.

Alternatively we can define factor abundance under its price connotation. Then country 1 is relatively capital abundant if,

\[ (PC/PL)^1 < (PC/PL)^2 \]

before trade is initiated. Through the algebraic manipulation, we can then derive,

\[ \frac{P^1_t}{P^1_f} < \frac{P^2_t}{P^2_f} \]
implying thereby textiles relatively cheaper in country 1.  

These results are demonstrated in Figure 4. $T_1$ and $T_2$ are the transformation curves of country 1 and 2 respectively. $U_1$ and $U_2$ are the community indifference curves of country 1 and 2. The pre-trade price ratios for the two commodities in the two countries are obtained at the point of tangency between transformation curve and indifference curve. Points 'g' and 'i' denote these positions in the figure. At 'g' the marginal opportunity cost for producing one more unit of textiles is less in country 1 compared to country 2. Therefore the relative price ratio of textiles to food is less in country 1 compared to country 2. Similarly the marginal opportunity cost of food in terms of textile is less at 'i'.


12 The transformation curve shows all possible output combinations of textiles (measured alongOX) and food (measured along OY) when the system is in equilibrium. The slope of the transformation curve at any point indicates the ratio of the marginal opportunity costs of an extra unit of one commodity in terms of the sacrifice of the output of the other. The negative slope of the curve shows that as more of one commodity is produced, the less of the other is the result. The convexity shows the condition of increasing marginal cost of production of one commodity in terms of the other.

13 This is also the point of domestic equilibrium. It will be noted from the figure that the points of domestic equilibrium are different for the two countries.
Figure 4.
than at 'g' and the relative price ratio of food to textiles is less. Thus with trade, country 1 concentrates in the production of textiles and country 2 in the production of food.

So far the Heckscher-Ohlin theorem was examined under rigid conditions of two country, two commodities and two factors. Let us relax these conditions one by one. If there are more than two countries or commodities, while the factors remaining two, it does not alter the analytical structure of the model. One may have as many countries as possible and each one will have different transformation curves so long as their factor endowments differ. Similarly if there are more than two commodities, they can be ordered as per their relative factor intensities. But when there are more than two factors, say capital, labour and land, one cannot have the same rigorous proof as in the case of two factors. We can no more qualify now a specific commodity traded in International market as necessarily capital intensive or labour intensive. A clear demonstration of the proof depends upon two preconditions: first, it is possible to apply the concept of factor intensity meaningfully to the various products; second, countries can be identified as abundant in a particular factor. A basic attempt in this regard has been

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that of Vanek\textsuperscript{15} who got over these problems by assuming a complementarity relation between capital and natural resources. His study of American trade structure substantiated this belief. But the universal applicability of this assumption is open to doubt.

We might sum up here stating that in a generalized case of many commodities, many countries, and many factors, the identification of factor abundance and factor intensity becomes more and more difficult. As observed by Professor Harrod,\textsuperscript{16} and more conclusively by Pearce,\textsuperscript{17} the probability of factor ratio curves intersecting each other increases, a priori, with the increase in number of factors.

**Factor Price Equalization:**

So far we were concerned with the pre-trade price ratios. What is the effect of free International trade on factor prices? Both Heckscher and Ohlin held the view that free commodity trade acts as a substitute for factor mobility in the equalization of factor prices between the trading countries.

\textsuperscript{15}Vanek Jaroslav. \textit{Op.Cit.} pp. 33-34


Will the tendency towards equalization of factor prices be complete? To cite Ohlin:

A complete local adaptation of production through inter-regional factor movements and the resulting complete price equalization would make the prices just the same as if there were only one region and no geographical distribution of the industrial agents. These would be used and combined just as it is explained in the one-market theory. Space would be of no consequence. In such a state prices would be different from what they are, when we have a number of isolated regions. Clearly, the state of prices caused by inter-regional trade, under the assumption in Part I lies somewhere between these two extremes. The tendency is to push prices from complete independence state to complete equalization state, but it is not carried through. The price differences as regards the productive factors are reduced, but they do not disappear.†

Professor Samuelson, in two of his wellknown papers,‡ gave the proof for a complete equalization of factor prices, subject to certain assumptions. Samuelson was followed by Meade, Timmergen, Mekensiz, Kahn, Nikado and Gale. This constitutes the second major proposition of the factor proportions theory dubbed as "Factor price equalization theorem."

Verbally stated the proof of the theorem is as follows:

In the pre-trade position, country 1 has to produce more of

(\text{Post} - \text{Trade})


textiles to meet foreign consumption in addition to domestic consumption. The increased production of textiles in country 1 will cause an increased demand for capital while a decline in the production of food will be accompanied with a low demand for both factors but especially labour. The increased demand for capital raises the marginal productivity of this factor and increases the price of capital. On the other hand, the decline in demand for labour lowers down its marginal productivity and the price of labour must fall. Thus after trade is established between two countries, the price of capital, in the capital abundant country, i.e. country 1 must increase while the price of scarce factor, labour, must fall. This ultimately cause a change in relative price of two factors in country 1. Under a similar argument, it follows that the price of labour must increase and the price of capital must fall in country 2. Since the ratio in which the commodities are produced would continuously vary with their relative prices, there prevails a unique relationship between relative prices of the two commodities, their production levels and the marginal productivities and wages of the factors. Under free trade, when there are no transport costs, the price of textile and food must be equal in both countries. This means the relative

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20 Since the production functions are assumed to be homogeneous and subject to constant returns to scale, the Euler's theorem holds and there is no residual. So there is no doubt on the assumption of unique relationship.
price ratios of the two traded commodities are same. From the unique relationship, it follows that relative commodity prices must be equal to relative factor prices in each country. Since the production functions are same in both countries, this relationship must be the same for both countries. Therefore, when there is a single price ratio in two countries, it means a single factor price ratio and the linear homogeneity of the production function ensures the equality of both the marginal productivities and factor prices of comparable factors in the two countries.

In order to demonstrate it, let us use the box diagram. In figure 5 ACBD and A'C'B'D' represent such a pair of diagrams for country 1 and country 2 respectively where AC (A'C') = BD (B'D'), AD (A'D') = BC (B'C'). Each box represents an economy with a fixed supply of capital and labour. AC, AC' represents endowments of labour in country 1 and country 2. In the figure the boxes are drawn such that AD > A'D', AC < A'C' so that country 1 has relatively more capital while country 2 has relatively more labour. Both countries produce food and textiles. The points inside the boxes indicate allocations of capital and labour between the two industries. Points A, A' represent the origins of isoquants for textile while B, B' are the origins for food isoquants and their slope measures the ratio of marginal productivities of two factors. AB, A'B' are the contract curves (production efficiency curve) of the two.
Figure C

Country 1

Country 2

Figure 5.
countries and all the points along this curve represent optimum production condition. Both the contract curves lie to the South-East diagonal indicating textiles as relatively capital-intensive at all relative prices of factors. Draw a ray such as $AX$ which cuts all the isoquants at the same angle so that

$$\frac{MPL_A}{MPC_A}$$

is constant along $AX$. Draw another ray from $B$ so that it meets the contract curve at $X$. $A'F'$ and $B'G'$ are drawn such that they are parallel to $AX$ and $BX$ respectively. Both $A'F'$ and $B'G'$ intersects at $X'$. Then $X$ and $X'$ are called "corresponding points." Then the following relationship holds good at $X$ and $X'$.

At $X$ we have:

$$\frac{MPL_t^1}{MPC_t^1} = \frac{MPL_t^2}{MPC_t^2} \ldots \ldots (1)$$

at $X'$ we have:

$$\frac{MPL_{t'}^1}{MPC_{t'}^1} = \frac{MPL_{t'}^2}{MPC_{t'}^2} \ldots \ldots (2)$$

Where $MPL$ stands for marginal physical productivity of labour and $MPC$ for marginal physical productivity of capital.

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21 This is due to homogeneity property.
Manipulation of (1) and (2) yields:

\[
\frac{\text{MPL}_1}{\text{MPL}_2} = \frac{\text{MPL}_t^1}{\text{MPL}_t^2} \quad \ldots \quad \ldots \quad \ldots \quad (3)
\]

\[
\frac{\text{MPC}_1}{\text{MPC}_2} = \frac{\text{MPC}_t^1}{\text{MPC}_t^2} \quad \ldots \quad \ldots \quad \ldots \quad (4)
\]

From (3) and (4) it follows:

\[
\frac{t_p^1}{f_p^1} = \frac{t_p^2}{f_p^2}
\]

i.e. the price ratio is same at both \(X\) and \(X'\).

For full equilibrium conditions at \(X\) and \(X'\), the factor price ratios must equal the ratios of marginal physical product i.e.

at \(X\)

\[
\frac{W^1}{R^1} = \frac{\text{MPL}_t^1}{\text{MPC}_t^1} = \frac{\text{MPL}_t^1}{\text{MPC}_t^1} \quad \ldots \quad \ldots \quad (5)
\]

\[
\frac{W^2}{R^2} = \frac{\text{MPL}_t^2}{\text{MPC}_t^2} = \frac{\text{MPL}_t^2}{\text{MPC}_t^2} \quad \ldots \quad \ldots \quad (6)
\]

Where \(W\) stands for wage and \(R\) stands for rent to capital.
Since the textile isoquant is tangential to food isoquant at X and X', it follows:

\[
\frac{\text{MPL}_t^1}{\text{MPC}_t^1} = \frac{\text{MPL}_t^2}{\text{MPC}_t^2} \quad \ldots \quad \frac{\text{MPL}_f^1}{\text{MPC}_f^1} = \frac{\text{MPL}_f^2}{\text{MPC}_f^2}
\]

Then it follows:

\[
\frac{W^1}{R^1} = \frac{W^2}{R^2}
\]

i.e. relative factor prices are equal in both countries.

It follows further that the absolute factor prices must also be equalised in both countries. Under the assumption of constant returns to scale and production functions homogeneous of the first degree, Euler's theorem hold and so all along any ray from the origin, the output of a given commodity must equal the marginal physical productivity of capital multiplied by the quantity of capital employed to produce plus and marginal physical productivity of labour multiplied by the quantity of labour. This may be expressed as:

\[
t = (L_t)(\text{MPL}_t) + (C_t)(\text{MPC}_t) \quad \ldots \quad \ldots (7)
\]

\[
f = (L_f)(\text{MPL}_f) + (C_f)(\text{MPC}_f) \quad \ldots \quad \ldots (8)
\]
From expression (7)

$$\frac{t}{L_t} = MFC_t \left( \frac{C_t}{L_t} + \frac{MPL_t}{MPC_t} \right) \ldots \ldots (9)$$

In expression (9) $\frac{t}{L_t}$ is the ratio of output of textiles to the input of labour. Along the ray this ratio is fixed. The ratios $\frac{MPL_t}{L_t}$ and $\frac{C_t}{L_t}$ are also constant along this ray. Then it follows that $MPC_t$ is too constant. This also implies the constancy of $MPL_t$ along the ray. Similar arguments hold good for expression (8). Therefore we have, at points X and X'.

$$MPL_1 = MPL_2, \quad MPC_1 = MPC_2 \ldots \ldots (10)$$

and $$MPL_f = MPL_f, \quad MPC_f = MPC_f \ldots \ldots (11)$$

Given the assumption of perfect competition, factor remunerations must be equal to the marginal value productivity in each country. Then we have:

$$W^1 = P_t^1 (MPL_1) = P_f^1 (MPL_f)$$

$$W^2 = P_t^2 (MPL_2) = P_f^2 (MPL_f)$$

$$R^1 = P_t^1 (MPL_1) = P_t^1 (MPC_f)$$

$$R^2 = P_t^2 (MPC_2) = P_t^2 (MPC_f)$$
Under free commodity trade and transport cost being zero, not only the commodity price ratio is same in both countries, but also the price of commodities in the two countries i.e. $P_t^1 = P_t^2$, $P_f^1 = P_f^2$. From (10) and (11) we know the marginal physical productivities of both factors are equal in both countries. If the price of food and textiles is the same in both countries and if marginal physical productivities of capital and labour is same in both countries, then factor prices also must be the same in both countries i.e. $W^1 = W^2$, $R^1 = R^2$. Absolute factor prices are also equal in both countries.

**Empirical tests of the Heckscher-Ohlin theory:**

There has been little controversy about the chain leading to the determination of factor prices, but the first proposition of the theory has been empirically searched in the recent years. MacDougall\(^2\) was the first economist to test the

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Hecksher-Ohlin hypothesis. He, in his attempt to verify the theory, examined the relative share of export of the United States in the world market for capital-intensive goods because he postulated that the United States having larger capital per worker than the United Kingdom should have a larger share of it in the world market. Using horse power as an index of capital, he found no such systematic relationship.

Kravis, in his effort to verify the Hecksher-Ohlin hypothesis, found that the United States export industries are high wage industries as compared to import-competing industries. But, to the question whether these high wages stemmed from relatively more capital per unit of output in these industries, he found no such relationship.

In yet another test, Tarshis, instead of studying exports in the light of factor endowments, examined relative internal commodity prices within nations. He found that price ratios of capital-intensive goods relative to labour-intensive goods were lower in the United States than in less capital abundant countries.


The most extensive test of Heckscher-Ohlin theory has been that of Professor Leontief who, using his input-output analysis, attempted to find out capital and labour content of a representative bundle of exports and competitive imports. His results were in contrary to the expectations and have come to be known as Leontief paradox.

**Conclusion:**

To sum up we might say that both Heckscher and Ohlin attempted for a general equilibrium approach to International trade theory, hitherto unattempted by classical economists. However, the theory is not without its lapses. The assumptions of the model are mere abstractions. Professor Harrod seems to think that the "strong factor intensity" assumption on which the theory stands is "unlikely" and hence any conclusion derived from it is a "serious curiosism in International trade theory and should be presented as such, rather than a fundamental principle or as an analysis of any probable development." The real world does not provide as simple and


27 The details regarding Leontief's methodology and his findings are given in the next chapter.

ordered a situation as envisaged by Ohlin and his followers. Countries are not only endowed with different stocks of economic factors but also with a diversity in the quality of such factors which do introduce disparities in the production functions of various countries. Then there are institutional and cultural disparities which also directly or indirectly influence the productivity of the countries concerned. Ohlin was well aware of these and while presenting what he regarded as a simplified version of his model, divided labour into three skill groups and capital into long-term and short-term capital. However the impossibility of quantifying in a mathematically functional way the influence of the diverse and intangible factors on the productivity of countries concerned made him to oversimplify the theory and to focus attention, for the first time, on certain mutually interdependent aspects that determine the source and course of trade.