CHAPTER 5
SHAPELET BASED 3D RECONSTRUCTION ALGORITHM FOR
MRI/CT IMAGES
3D image reconstruction has seen extensive research and some algorithms have been developed as solutions for obtaining a 3D model of an object with multiple 2D views. 3D reconstruction techniques can be widely classified as Active and Passive. Active reconstruction technique includes shape from optical triangulation, where 2D images of the object is taken, by turning it 360 degrees. From each 2D image thus obtained, sample points are taken, which are then used to form triangles to finally arrive at its shape. The passive techniques include techniques like, shape from silhouette, shape from shading, voronoi based reconstruction, reconstruction from point clouds and reconstruction from unorganized points.

All the 3D reconstruction techniques existing till date aims to decrease the processing time, reduce the storage space and to increase the accuracy of the reconstructed view. An approximate 3D reconstruction was conventionally achieved in these techniques by selecting different ways to develop an efficient mesh structure of the object, rather than manipulating the depth information from the dataset. Most of the existing 3D reconstruction techniques for MRI/CT are voxel based techniques, which tries to reconstruct the 3D view based on the intensity value stored at each voxel location. The voxel based techniques available for 3D reconstruction of MRI/CT images are Marching cubes algorithm [112], Isosurface reconstruction [138] and surface to volume reconstruction. These techniques do not make use of the shape or depth information available in the 2D image stack.

This research work has developed a 3D reconstruction algorithm, by making use of the depth information hidden in each of the MRI or CT image slices. Any 3D reconstruction technique requires the shape of the object to be obtained, in order to reconstruct it. After exhaustive study, a variant of the wavelet family, Shapelets [139, 140] has been identified for an exact representation of the shape of the object in the 2D image. Wavelets decompose an image on several scales, but fail to describe the real shape of the object. Shapelets do not decompose an image on different scales, but on different elementary shapes. This enables shapelets to capture the shape information in the image. Hence, it can be used to extract the shape information in an image, which in turn is hidden in the depth information. The proposed algorithm gets
the depth information from each slice in the 2D stack of a MRI or CT image and generates the shape for each image slice. Experimental results show that the proposed algorithm provides a more accurate 3D representation of the organ, than the voxel based reconstruction techniques.

5.1 Shapelets

Wavelet decomposition decomposes an image at several scales. But they fail to describe the real shape of the object. Shapelets decompose an image based on different elementary shapes. Shapelet decomposition is a linear decomposition of an image into a series of localized basis functions with different shapes. Shapelets are of two types namely, Cartesian shapelets and Polar shapelets.

5.1.1 Cartesian shapelets

They are weighted Hermite polynomials [141], which correspond to perturbations about a circular Gaussian. The dimensionless basis function is defined as:

\[
\phi_n(x) = \left[\frac{2^n \sqrt{\pi n!}}{\sqrt{\pi}}\right]^{1/2} H_n(x) e^{-x/2}
\]

\(H_n\) is an \(n^{th}\) order Hermite polynomial. They are orthogonal. i.e.

\[
\int_{-\infty}^{\infty} \phi_n(x) \phi_m(x) dx = \delta_{mn}
\]

For practical applications, the dimensional basis function is used:

\[
B_n(x; \beta) = \beta^{-\frac{n}{2}} \varphi_n\left(\frac{x}{\beta}\right)
\]

\(\beta\) is a character size, close to that of the object which is to be decomposed. The basis functions \(B_n\) are also orthogonal. The object to be decomposed is thus represented as:

\[
f(x) = \sum_{n=0}^{\infty} f_n B_n(x; \beta)
\]

The shapelet coefficient is given by:

\[
f_n = \int_{-\infty}^{\infty} f(x) B_n(x; \beta) dx
\]

If the object is adequately localized, the series converges quickly; so that the first few basis functions are sufficient to approximate the shape information of the object of interest. Then its decomposition into shapelets can be truncated to some maximum order of decomposition.
The object can thus be represented as:

\[ f(x) = \sum_{n=0}^{N} \alpha_n B_n(x, \beta) \]

Figure 5.1 shows 1D and 2D Cartesian shapelets.

**5.1.2 Polar Shapelets**

They are similar to Cartesian shapelets, with a Gaussian weighting function of scale size \( \beta \). Since they are polar, they are separable in ‘\( \theta \)’ and ‘\( r \)’ and hence easier to understand. Also, the operations on them are more intuitive.

A function \( f(r, \theta) \) in polar coordinates can be decomposed as a weighted sum of the basis functions \( \chi_{n,m}(r, \theta; \beta) \):

\[ f(r, \theta) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} f_{n,m} \chi_{n,m}(r, \theta; \beta) \]

\[ f_{n,m} = \int_{R} f(r, \theta) \chi_{n,m}(r, \theta; \beta) r dr d\theta \]

\( f_{n,m} (n \in N, -n \leq m \leq n) \) are the polar shapelet coefficients of order \((n, m)\). The real and imaginary parts of the first few polar shapelets [142] are given in Figure 5.2.
5.2 Constructing the surface normals

Traditionally, surfaces were reconstructed from surface normals via integration. However, to do this, the surface gradients should be integrable. This approach is also sensitive to noise.

Since differentiation is linear, a correlation performed between the signal gradient and the gradients of the basis function provide information which is proportional to the direct correlation between the signal and the basis function. Hence, if the surface gradient information is correlated with the gradients of a bank of shapelet basis functions, the surface shape can be reconstructed by summing the results of correlation. The summing of the correlated values of the basis, automatically imposes a continuity constraint and performs an implicit integration of the surface from its gradients [140, 143].

Correlation between the gradients of the surface and each shapelet is defined in terms of slant and tilt values separately and then combined. The correlation of the surface and the shapelet slants is done in terms of the gradient magnitude, which is given by the ‘tan’ of the slant.

\[ |\nabla| = \tan(\sigma) \]

The gradient correlation is then formed as:

\[ C_v = \sum_x \sum_y |\nabla_x| |\nabla_s| \]

where, ‘f’ – denotes the surface and ‘s’ denotes the shapelet.

Tilt values are used to make a distinction between positive and negative shapes in the image. If the surface and shapelet gradient magnitudes match at some point and the tilt directions also match, then the component of the shapelet at this point must be
positive. If the tilts of the surface and the shapelet are in opposite directions, then the shapelet component must be negative. If the tilts are orthogonal, then there is no correlation between the surface and the shapelet. Thus, the gradient correlation must be multiplied by a tilt correlation measure, which varies between +1 and -1, when the tilts are in opposite directions. A measure that satisfies this requirement is the cosine of the tilt angle difference.

\[
C_r = \sum_x \sum_y \cos(\tau_f - \tau_s)
\]

\[
= \sum_x \sum_y \cos(\tau_f) \cos(\tau_s) + \sum_x \sum_y \sin(\tau_f) \sin(\tau_s)
\]

The overall correlation measure between the surface and the shapelet is obtained by the product of the gradient and the tilt correlations:

\[
C = C_v C_r
\]

\[
= \sum_x \sum_y |\nabla_f| |\nabla_s| \left[ \sum_x \sum_y \cos(\tau_f) \cos(\tau_s) + \sum_x \sum_y \sin(\tau_f) \sin(\tau_s) \right]
\]

\[
= \sum_x \sum_y |\nabla_f| |\nabla_f| |\nabla_s| |\nabla_s| \cos(\tau_f) \cos(\tau_s) + \sum_x \sum_y |\nabla_f| |\nabla_f| |\nabla_s| |\nabla_s| \sin(\tau_f) \sin(\tau_s)
\]

This is performed over multiple scales of the shapelet and the result is summed to form the final reconstruction.

\[
R = \sum_i C_i
\]

5.3 Choosing the Shapelet Basis

A large number of basis functions are available that can be used as shapelets. Correlating the gradient of a shapelet filter with the gradient of the signal corresponds to extracting a band of frequencies from the signal gradient. The need to reconstruct a surface from correlations between surface normals and shapelet gradients imposes the following constraints on the shapelet function:

- Minimal ambiguity of shape with respect to its gradient.
- Preservation of phase information in the signal.
- Uniform coverage of the signal spectrum, so that it is faithfully reconstructed.

The commonly used shapelet for image decomposition is the 2D Cosine Gabor wavelet. Figure 5.3a shows its smoothly varying normals. But the gradient magnitudes shown in Figure 5.3b, have discontinuities of slope.
Figure 5.3: a) Cosine Gabor function, b) its gradient magnitude

Figure 5.4a shows a shapelet based on low-pass Butterworth filter.

Figure 5.4: a) Butterworth shapelet with transition parameter $n=2$, along with its cross-section, b) its gradient magnitude

In order to select a shapelet with minimal ambiguity of shape with respect to its gradient, it must be simple and ideally take the form of a single peak, so that the gradient function will have a single positive and a single negative peak. A function which satisfies this criterion is the Gaussian filter. Hence, for the experiments presented in this work, the shapelet used is the Gaussian shapelet. Figure 5.5a shows the Gaussian shapelet.

Figure 5.5: a) A shapelet bank of 5 Gaussians, with height proportional to scale, b) The corresponding transfer functions of their gradients. The sum of the transfer functions is shown by the dashed line.
5.4 Proposed 3D Reconstruction Algorithm – Shape-3DRecon

3D reconstruction aims at generating a 3D model of the organ of interest, with minimal ambiguity. At the same time, it should preserve all the information in each slice of the 2D image stack. The reconstruction algorithm proposed in this research work ensures that the above requirements are met. The highlight of this algorithm is that, it makes use of the shape/depth information which is hidden in the 2D stack and manipulates it to generate the 3D view. This ensures a greater accuracy in the final generated 3D view.

The input to the method is a 2D stack of ‘N’ MRI/CT images. Each image in the stack is called a slice. The following are the steps carried out on each of the slices $S_i$, $i = 1, 2, \ldots, N$, in order to generate the final 3D view, in the proposed Shape-3DRecon reconstruction algorithm.

5.4.1 Generating the gradient values in the two directions

Each slice $S_i$ is taken and the gradients in the $x$ and $y$ directions are obtained respectively as $S_{dx}$ and $S_{dy}$.

5.4.2 Obtaining the Slant and Tilt values for each slice

The gradients thus obtained are used to get the slant and tilt values.

5.4.3 Finding the gradient correlation

The gradient correlation measure is obtained by multiplying and summing the gradient magnitudes of the surface $|\nabla_f|$ and that of the shapelet $|\nabla_s|$.

$$C_v = \sum_x \sum_y |\nabla_f| |\nabla_s|$$

5.4.4 Getting the Tilt correlation

The tilt correlation measure is generated using the expression:

$$C_r = \sum_x \sum_y \cos(\tau_f - \tau_s)$$

5.4.5 Obtaining the overall correlation between the surface and the shapelet

The gradient and tilt correlation measures are multiplied to obtain the overall correlation measure.

$$C = C_v C_r$$

5.4.6 Getting the final reconstruction

This process is carried out on different scales of the shapelet and the correlation measures thus obtained for each case is then summed up to get the final reconstruction.
\[ R = \sum_i C_i \]

### 5.4.7 Generating the final 3D view

The above steps are repeated on each of the image slice in the 2D stack and the summed up correlations for each slice is finally stored together to build a volume.

The volume thus obtained after applying the Shape-3DRecon algorithm on the MRI or CT dataset, is then rendered using isosurface rendering and surface to volume rendering.

### 5.5 Results and Discussion

The proposed 3D reconstruction algorithm, Shape-3DRecon, has been evaluated on sets of 2D stack of MRI and CT images. The results pertaining to three datasets are discussed in this thesis. Dataset 1 is a stack of CT image of the head, with 40 2D grayscale images. Dataset 2 is a stack of CT image of the head with 99 2D grayscale images. Dataset 3 is a stack of MRI image of the head, with 99 2D grayscale images.

Each 2D image in the stack, for all the three datasets, is having a size of \(256 \times 256\).

To compare the accuracy of the generated 3D model, the 3D reconstruction of the same datasets has also been carried out using voxel based isosurface rendering and voxel based surface to volume rendering.

All the existing and proposed 3D reconstruction algorithms were written and run on MATLAB, version 7.11 (MatlabR2010b).
Figure 5.6: SET 1 - a) Voxel based isosurface rendering, b) Shapelet based isosurface rendering, Voxel based surface to volume rendering c) Front view e) Back view, Shapelet based surface to volume rendering d) Front view f) Back view
Figure 5.7: SET 2 - a) Voxel based isosurface rendering, b) Shapelet based isosurface rendering, c) Voxel based surface to volume rendering, d) Shapelet based surface to volume rendering
5.5.1 Discussion and Inference for the Shape-3DRecon Algorithm

The results obtained after rendering the output of the Shape-3DRecon algorithm using isosurface rendering and surface to volume rendering, for the three datasets, is shown in Figures 5.6, 5.7 and 5.8. Also is shown the 3D views obtained for the same datasets using voxel based isosurface rendering and voxel based surface to volume rendering.

For the first dataset, in Figure 5.6, the results for surface to volume rendering for both voxel based rendering and the proposed algorithm is shown in two views. The two views clearly show the comparison between the existing and the proposed algorithms. The isosurface rendering technique gives a view of the structure of the organ, while the surface to volume rendering creates a smooth surface of the organ. The former is used by doctors to get a detailed knowledge of the organ under study, while the latter assists doctors in plastic surgery, cosmetic surgeries like maxillofacial surgery,
craniofacial surgery and micro-vascular reconstruction to study head and neck cancer etc.

From the results, it is clearly evident that the proposed shapelet based 3D reconstruction algorithm, Shape-3DRecon, gives a more accurate 3D view of the concerned organ, than its voxel based 3D view. It can be seen that even the minute details about the organ is clearly visible in the 3D view, when the proposed algorithm is used. These minute details are not evident in the conventional voxel based rendering techniques. Also, the surfaces obtained by the proposed algorithm is smoother and better resembles the organ than the conventional techniques. This is because; the proposed method extracts the complete shape information from the input stack and then constructs a 3D model.

5.6 Conclusion and contribution

A 3D reconstruction algorithm, Shape-3DRecon, has been developed by manipulating the shape information in a MRI or CT dataset. Its performance was evaluated on different MRI and CT datasets. The 3D reconstructed results thus obtained were visually compared with conventional voxel based isosurface reconstruction and voxel based surface to volume reconstruction.

The results show a visibly efficient 3D view of the MRI or CT image stack, for the proposed Shape-3DRecon algorithm, when rendered using both isosurface rendering and surface to volume rendering. The proposed algorithm outperforms conventional voxel based isosurface rendering and voxel based surface to volume rendering methods.

The work carried out in this research is summarized in the following chapter and its importance has been illustrated. The contributions made by this research work to the existing literature are discussed and conclusions drawn. The scope for future extension of the work is also presented.