Abstract

The thesis starts with a few words on Dirichlet himself. The properties of standard real type-1 and type-2 Dirichlet distributions are then discussed. Matrix-variate analogues of type-1 and type-2 Dirichlet models are also presented. A survey of some of the important areas of applications and extensions of Dirichlet models is attempted. All these are done in the first chapter.

In the second chapter we introduce a new extension of type-1 Dirichlet model both in scalar variables case and in the matrix-variate case. The generalization of type-1 Dirichlet model in the scalar variables case is derived by using a property which we will call as a ‘short memory property’. Several properties of this new model, which are applicable in many fields, are presented. Here the matrix-variate analogue of an extension of a type-1 Dirichlet model is developed and its properties are also discussed.

In certain studies, successive sums of variables also enter into the picture. Hence an extension of the type-2 Dirichlet model with successive sums incorporated into it, is introduced in the third chapter. Many types of properties of this new model are studied which enhance the possibility of application in different directions. Further, the matrix-variate analogue of this new model is given and its properties are also examined.

The fourth chapter explores the application of a generalized real type-1 Dirichlet model in multivariate statistical analysis. It is shown that the exact null distribution of likelihood ratio criteria for testing a number of hypotheses on the parameters of one or more multivariate Gaussian populations can be obtained as a marginal distribution of this generalized Dirichlet model having a specific set of parameters. The exact distribution of the likelihood ratio criterion so obtained has a very simple and general format for every \( p \). Various types of properties and relations involving hypergeometric series are also established. A novel idea is introduced here through which the complicated exact null distribution of the likelihood ratio criterion in multivariate statistical analysis is converted into an easily tractable marginal density in a generalized Dirichlet model. It
provides a direct and easy method of computation of $p$-values. Numerical tables are also provided to illustrate the method of computation of $p$-values.

An application of the generalized type-1 Dirichlet model in geometrical probability problems is considered in the last chapter. The volumes of random paralleloptopes when the linearly independent random points in Euclidean $n$-space have very general real rectangular matrix-variate type-1 beta density is discussed. Structural decompositions are given and the random volumes are connected to generalized Dirichlet models as well as to likelihood ratio criteria in testing statistical hypotheses. This enables one to compute percentage points of random volumes through the $p$-values in the generalized Dirichlet marginal. Several special cases are explicitly evaluated in terms of elementary functions. Numerical tables are also provided to illustrate the method.

**Keywords:** generalized Dirichlet model; beta density; short memory property; neutrality principle; matrix-variate distribution; Jacobians of matrix transformations; Meijer’s G-function; likelihood ratio criterion; exact distribution; random volume; geometrical probability.