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1.1 INTRODUCTION

An Experiment is a scientific procedure undertaken to make a discovery, test a hypothesis, or demonstrate a known fact. John Stuart Mill was probably the first to give a clear notion on how to carry out an experiment. He classified experiments into spontaneous experiments and artificial experiments. His classification, in course of time, has developed into two branches namely observational studies and controlled experiments. Mill and his co-workers found that controlled experiments are better if the subject matter allowed it.

1.1.1 Design of Experiments

Experimental design is the branch of statistics that deals with the design and analysis of experiments. This method of study is widely used in the fields of agriculture, medicine, biology, marketing research, and industrial production. The study of Statistics as a scientific discipline and its applications, has been introduced in India by Prof. Mahalanobis (1944, 46). He gathered a group of young and talented scientists from diverse fields and encouraged them to do research
work in statistics. A large amount of research work was carried out in the design of experiments, which formed a fascinating branch of statistics for Indian research scientists. Bose (1939) started research work in the mathematical construction of designs. He has influenced a large number of research workers in India and in U.S.A. The renowned Indian scientists who made great contributions in this area are Bose (1939), Bhattacharya (1945), Das (1960), Dey (1975), Rao (1947), Raghavarao (1960), Shrikhande (1950), Saha (1973), Nair (1943) and Bose et. al. (1953, 54, 55).

In an experimental study, one or more of the variables of interest, referred to as the factors of the study, are controlled so that data may be obtained about how the factors influence another variable referred to as the response variable. For example, consider an experiment designed to determine the effect of three different exercise programs on the cholesterol level of patients with elevated cholesterol. Each patient is referred to as an experimental unit, the response variable is the cholesterol level of the patient at the completion of the program, and the exercise program is the factor whose effect on cholesterol level is being investigated. Each of the three exercise programs is referred to as a treatment.

Three of the most widely used experimental designs are the completely randomized design, the randomized block design, and the factorial design. In a completely randomized experimental design, the treatments are randomly assigned to the experimental units. For instance, applying this design method to the cholesterol-level study, the three types of exercise program (treatment) would be randomly assigned to the experimental units (patients). The use of a completely randomized design will yield less precise results when factors not accounted for by the experimenter affect the response variable.

Consider, for example, an experiment designed to study the effect of two dif-
different gasoline additives on the fuel efficiency, measured in miles per gallon (mpg), of full-size automobiles produced by three manufacturers. Suppose that 30 automobiles, 10 from each manufacturer, were available for the experiment. In a completely randomized design the two gasoline additives (treatments) would be randomly assigned to the 30 automobiles, with each additive being assigned to 15 different cars. Suppose that manufacturer 1 has developed an engine that gives its full-size cars a higher fuel efficiency than those produced by manufacturers 2 and 3. A completely randomized design could, by chance, assign gasoline additive 1 to a larger proportion of cars from manufacturer 1. In such a case, gasoline additive 1 might be judged to be more fuel efficient when in fact the difference observed is actually due to the better engine design of automobiles produced by manufacturer 1.

To prevent this from occurring, a statistician could design an experiment in which both gasoline additives are tested using five cars produced by each manufacturer. In this way, any effects due to the manufacture would not affect the test for significant differences due to gasoline additive. In this revised experiment, each of the manufacturers is referred to as a block, and the experiment is called a randomized block design. In general, blocking is used in order to enable comparisons among the treatments to be made within blocks of homogeneous experimental units.

Factorial experiments are designed to draw conclusions about more than one factor, or variable. The term factorial is used to indicate that all possible combinations of the factors are considered. For instance, if there are two factors with \( a \) levels for factor 1 and \( b \) levels for factor 2, the experiment will involve collecting data on \( ab \) treatment combinations. The factorial design can be extended to experiments involving more than two factors and experiments involving partial factorial designs.
1.1.2 Optimality of Block Designs

As such, the study of experimental research has its origin in agricultural research. The modern concepts of experimental designs are primarily due to Sir R.A. Fisher. He, and his followers, developed the basic ideas of statistical designing during the period 1919 - 1930 at the Rothamsted Experimental Station in England. A number of useful concepts such as balance, orthogonality, blocking etc were introduced by them. Prof. R.C. Bose and his colleagues gave general structure to classes of these designs, and invented general methods for constructing them.

At present, a large variety of experimental designs are available in the literature. Since high speed computers can be used for analysis, mere ease of analysis is not the sole criterion for selecting a design for a particular problem, but the statistical properties of the design have to be taken care of. This led to the study of optimality of experimental designs. It is desirable to maximise the amount of information gained on the treatments being studied by using optimal designs. The modern theory of optimum designs has its root in the decision theory founded by Abraham Wald. Together with Wald, Jacob Wolfowitz and Jack Kiefer made substantial contribution to the development of the theory of optimal designs.

In an experimental design, the values chosen for the explanatory variables lead to certain desirable properties. In the multiple regression model the matrix of variances and covariances is proportional to $X'X$, where $X$ is the design matrix. A $D$- optimal design is one that maximizes the determinant of $X'X$. This is equivalent to minimizing the volume of the corresponding confidence ellipsoid. An $A$- optimal design is one that minimizes the trace of $(X'X)^{-1}$. That is, it minimizes the average variance of the parameter estimates.
1.2 SUMMARY OF THE PRESENT STUDY

This Thesis consists of seven chapters.

In Chapter 1, the basic concepts in Optimal Design of Experiments are explained with examples, a chapter-wise summary of the thesis is given and applications of the present work are presented.

In chapter 2, the basic concepts and notations required in the later chapters are defined.

In Chapter 3, three different methods for the construction of regular graph (RG) designs are given. The first method of construction of RG designs, by augmenting a new treatment in each of the $b$ blocks of the BIB design, and augmenting $(r - \lambda + 1)$ blocks which contain all the $v$ treatments of the BIB design only, is given in section 3.4. The parameters of the constructed RG designs with unequal block sizes are given in Table 3.5.2. Further, it is verified that these RG designs belong to a particular class of PEB designs with two efficiency classes. The second method of construction of RG designs, by adding two RG designs having the same number of treatments and same size of blocks, is provided in Section 3.6. The third method of construction of RG designs from a BIB design having parameters $v > 4$, $k > 2$ and $\lambda = 2$, by deleting a particular treatment and those blocks in which that particular treatment is present, is provided in Section 3.7. The parameters of the constructed RG designs are given in Table 3.7.3. Further, the adjacency graph of RG designs is given in Section 3.8, and thereby some new RG designs are obtained.

In Chapter 4, two different methods for the construction of semi-regular graph
(SRG) designs are given. The construction is carried out by augmenting a BIB design whose parameters satisfy certain conditions on the availability of SRG designs. In the first method, the SRG design is constructed by augmenting a new treatment in each of the \( b \) blocks of the BIB design, and augmenting one more block which contains all the \((v + 1)\) treatments. In the second method, the SRG design is constructed by augmenting a new treatment in each of the \( b \) blocks of the BIB design, and augmenting \((r - \lambda + 1)\) blocks which contain all the \( v \) treatments of the BIB design only. Further, it is verified that these SRG designs belong to a particular class of PEB designs with two efficiency classes. The parameters of the constructed SRG designs are given in Table 4.5.2 and Table 4.5.4. The dual designs of the constructed SRG designs, are also obtained and are given in section 4.6. It is found that they are RG designs, and belong to a particular class of PEB designs with three efficiency classes. The parameters of the dual RG designs are given in Table 4.6.1.

In Chapter 5, sections 5.3 and 5.4 deal with the construction of two series of Group Divisible (GD) designs from \( v \equiv 3 \pmod{6} \) BIB designs (i) by deleting any one set of \((t/3)\) disjoint blocks, and (ii) by adding any one set of \((t/3)\) disjoint blocks respectively. Section 5.5 contains another method of obtaining a new Semi-Regular Group Divisible (SRGD) design by adding two SRGD designs. The parameters of the constructed GD designs are given in Table 5.3.3 and Table 5.4.3.

In Chapter 6, the designs for comparing test treatments with a control are classified into three categories namely (i) \( r_o = bt \) (R-type) (ii) \( r_o > bt \) (S-type) and (iii) \( r_o < bt \) (S-type). In Section 6.4, it is shown that step (S-) type \( r_o < bt \) balanced treatment incomplete block (BTIB) designs and step (S-) type \( r_o > bt \) BTIB designs for comparing test treatments with a control are partially efficiency balanced (PEB) designs. It is proved by using the \( M_{r_o} \)-matrix of the design. Further, it is established in Section 6.5, that the binary (R-) type BTIB designs and
the binary (R-) type GDT designs are simple PEB designs. It is also pointed out in Section 6.6 that, the binary \((r_o \leq b)\) BTIB designs with \(\lambda_o = \lambda_1\) are proper efficiency balanced designs.

In Chapter 7, the \(E-\)optimality of the constructed RG, SRG and GD designs are considered. Section 7.4 deals with \(E-\)optimality of RG designs. The \(E-\)optimality of SRG designs is established in section 7.5. That is, the \(E-\)optimality of unequi-replicated designs with unequal block sizes is given in Section 7.5.1 and \(E-\)optimality of unequi-replicated proper designs is given in Section 7.5.2. Again, the \(E-\)optimality of GD designs is established in section 7.6.

1.3 APPLICATIONS

Block designs have immense applications in almost all areas of scientific investigation. They are commonly used in agricultural, industrial and biological experiments to eliminate heterogeneity in one direction. Block designs have applications in sampling theory also.

Yates (1936a, 1936b) introduced BIB designs in statistical design of experiments for varietal trials in the context of biological experiments. Bose (1939) developed the construction of BIB designs and derived some important combinatorial results. It is well known that BIB designs are optimal for a number of criteria under the usual homoscedastic linear additive model for observations. BIB designs could not satisfy the requirements of various agricultural experiments. Bose and Nair (1939) developed another class of designs called partially balanced incomplete block (PBIB) designs, which are available for all number of treatments having small number of replications.

Traditional work on the construction of block designs has been concentrated
on the equally replicated case, i.e. the total number of experimental units is a multiple of the total number of varieties. For a given number of varieties to be compared and a given block size, the assumption of equal replication imposes a severe constraint on the number of blocks which can be used. From practical viewpoint, it is desirable to have a method of constructing efficient block designs for the unequally replicated case. In this investigation, a new class of $E$-optimal equi-replicated RG designs with unequal block sizes is constructed in chapter 3. Also, new classes of $E$-optimal unequi-replicated SRG designs with unequal block sizes and $E$-optimal unequi-replicated proper SRG designs constructed using BIB designs are given in Chapter 4. Further, a new method of construction of $E$-optimal GD designs is presented in Chapter 5.

Construction of repeated block designs is also discussed in Chapter 5. From the point of view of application, there is no reason to exclude the possibility that a BIB design would contain repeated blocks. Indeed, the statistical optimality of a BIB design is unaffected by the presence of repeated blocks. Such designs are useful in various experimental designs, controlled sampling etc.

Comparing test treatments with a control is a very common problem to be solved. Various examples of test treatments include different makes of some apparatus, different brands of a drug, different types of fertilizers, different varieties of agricultural products etc. Statistical methods of experimental study are required to retain or replace the control treatment, which is used at present. The earliest work on this problem was carried out by Dunnett (1955, 1964). He also posed the problem of optimally allocating experimental units to control and test treatments, but didn’t solve it. This optimal allocation problem was solved by Bechhofer and his co-workers (1969, 1970, 1971). In this investigation, BTIB and GDT designs are discussed in Chapter 6.

In some other situations the purpose of the trial may be to compare some new
treatments not with each other, but with an established one, to find out if any are worth further study. For example, the Strawberry Weedkiller trial described by Pearce (1953) where there were Four blocks each of seven plots, and there were Four treatments A, B, C and D besides an untreated control (O). Similar designs were described by Tocher (1952). The purpose of the trial was to find out whether any of the Weedkillers A, B, C or D, all of which were apparently suitable for controlling weeds in Strawberry fields, would harm the fruiting plants. The data represent the total spread in inches of 12 plants per plot approximately 2 months after the application of the Weedkillers. The comparisons of most interest are those of A, B, C and D with the control, O.
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