CHAPTER I

INTRODUCTION-BASIC CONCEPTS
A porous medium can be defined as a material consisting of solid matrix with an interconnected void. In recent years, the investigation of flow of fluids through porous media has become an important topic due to the recovery of crude oil from the pores of reservoir rocks. Also, the flow through porous media is of interest in chemical engineering (absorption, filtration), petroleum engineering, hydrology, soil physics, biophysics, and geo physics. With the growing importance of non-Newtonian fluids in modern technology and industries, the thermal instability, thermal solution instability, and Rayleigh–Taylor instability, problems of Walters (model B) fluid and couple stress fluid are desirable.

The definition of the porosity of the porous medium can be given as the ratio of pore volume to the total volume of a given sample of material. A complete graduation exists from large force easily, accessible fluids to very small openings in minerals that are caused by minor lattice imperfection.
According to Kemper et al., (19) the smallest openings contain fluids that are rendered essentially static by the close proximity of atomic force fields at the surface of the solids. To describe various phenomenon’s in the porous medium, many terms relating to porosity are introduced. One such closely related term in the porosity is the specific yield. It is defined as the volume of water drained by the gravity from an initially saturated sample divided by the total volume of the sample.

Permeability of the porous medium is an important property of the solid material and it is independent of the density and viscosity of the fluid. The permeability $K$ can be defined as

$$K = \frac{-Q}{A} \frac{\mu}{\rho g} \left(\frac{\partial h}{\partial s}\right)^{-1}$$  \hspace{1cm} (1)

Where

$Q = \text{Total discharge of the fluid}$

$A = \text{Cross sectional Area of the porous}$

$\mu = \text{Viscosity}$

$\rho = \text{Density}$

$g = \text{Acceleration due to gravity}$

$\frac{\partial h}{\partial s} = \text{The hydraulic gradient in the direction of the flow.}$
The dimensions of the permeability are \( L^2 \). The unit of permeability is Darcy which is used in petroleum industry. The value of one Darcy is \( 0.987 \times 10^{-8} \) cm\(^2\). The hydraulic conductivity of the porous medium is measured in meinzers. If the porous medium has a permeability of one Darcy, then it has the hydraulic conductivity 18.2 meinzers.

Sedimentary materials will respond dynamically as the fluid chemistry changes. Effects of partial or complete hydration which will cause clays, colloids or complete material, to expand and close pore spaces are the most important of the responses to changing fluid chemistry with air and other non polar fluids, permeability are very high partial hydrations by brines will cause values to lower. Full hydration using distilled water as the measuring fluid will cause a reduction in permeability. In the grained sediments, these effects are responsible for different permeability in the sample.

As depth decreases, the porosity and permeability of dense rocks decreases. The decrease of permeability and porosity with depth has been found by many researchers like Frommurze (10) Johnston (17) and so on.

Free surface flows in porous medium, is one of an important branch. The problems concerning free surface porous flows are easy to solve in the steady state. With the aid of conformed mapping for two dimensional cases, these solutions can be obtained. When the flow is transient, the problem becomes more complicated. A perturbation technique is more useful in getting the solution. For transient flows, finite difference method can also be applied to get the solution. Harlow et al., (13) and Harlow and Welch (14) studied the problems of flow under a sluice using finite difference method.
1.1 Darcy Law

This law meaningfully defines the permeability in terms of measurable quantities.

If horizontal clear flow of an incompressible fluid is established through a sample of porous material of length in the direction of the flow and cross sectional area then the permeability of the material is defined as.

\[ k = \frac{q\mu}{A(\Delta P / L)} \]

Here \( q \) is the fluid flow rate in volume per unit time. It is the viscosity of the fluid and \( P \) is the applied pressure difference across the length of the spectrum.

1.2 Brinkman Model

Hubert (1956) derived Darcy’s law by integrating the Navier-stokes equation without considering the geometry of the pore space. Later Pore and Elate (1966) and Whitaker (1966) tried to modify the earlier result of Hubert and Hall. But their results are in terms of aerial averages of velocity and pressure which are not useful for fractional purpose. Further Whitakers’s result is different from traditional from of Darcy’s law. Ahmed and Sunanda (1969) applied volume averaging method in deriving Darcy’s equation. Their results are restricted to isotropic porous media.

Rudraiah et al (1979) obtained the following equation for the creeping flow of a fluid through a porous media.

\[ \nabla(p) = \rho g - \frac{\mu}{k} \overline{\nabla} \]

This equation is valid when the permeability \( k \) is very high. In general the particles of the porous media are loosely packed. So that \( k \) is small. Hence there exists a twin
boundary layer very near to the surface. This is experimentally confirmed by Beavers and Joseph (1967) and Rajasekhara (1974). Saffman (1971) and Rudraiah and Veerabhadraiah (1976, 1977) gave a theoretical justification for the existence of the boundary layer.

The following equation is proposed by Brinkman (1947) for the flow through porous media.

$$\nabla (P) = \rho g - \frac{\mu}{k} \frac{d u}{d t} + \mu \nabla^2 \bar{u}$$

In 1966, Tam supplemented a theoretical proof for this equation.

Katto and Masuoka (1967) experimentally found that Brinkman equation is valid up to the magnitude of $k/h^2$ of order 10 or so. If the porous medium is made up of spherical particles then $k/h^2$ corresponds to considerably high values of $d/h$ where $d$ is the diameter of the fillings and $h$ is the thickness of the porous media. Yamamoto and Youhida (1974) made improvements on Darcy’s law by adding corrective terms. Saffman (1971) gave the equations of motion for the flow through porous medium incorporating viscous stresses. Rudraiah et al. (1979), Yamamoto and Iwamura (1976), Pattabirama Charyulu (1979) and many others solved several flow problems in porous media using non-Darcy laws.

1.3 Non-Darcy Law

In many practical problems, the flow through porous media is curvilinear and the curvature of the path yields the inertia effect, so that the streamlines become more distorted and the drag increase more rapidly. Lapwood (198) was the first person who suggested for the inclusion of convective inertial term $(q \cdot \nabla)q$ in the momentum equation. Subsequently many research articles have been appeared on the non-Darcy
model \((7, 10, 11, 15, 17, \text{ and } 18)\). Now \((1.8)\) with the usual inertia term \(\frac{1}{\delta^2}(q.\nabla)q\) can be written as

\[
\frac{1}{\delta^2}(q.\nabla)q = -\nabla p + \rho g - \frac{\mu_f}{k} q + \mu_c (\nabla^2 q)
\]  

(1.9)

However, equation \((2.9)\) does not take care of possible unsteady nature of velocity. The flow pattern in a certain region may be unsteady and one has to consider the local acceleration term \(\frac{1}{\delta^2} \frac{\partial q}{\partial t}\) also. Adding this term equation \((1.9)\) becomes

\[
\rho \left[ \frac{1}{\delta^2} \frac{\partial q}{\partial t} + \frac{1}{\delta^2}(q.\nabla)q \right] = -\nabla p + \rho g - \frac{\mu_f}{k} q + \mu_c (\nabla^2 q)
\]  

(1.10)

This equation is known as Darcy-Lapwood-Brinkman equation (Rudraiah 1984). For anisotropic porous medium equation \((1.10)\) takes the form

\[
\rho \left[ \frac{1}{\delta^2} \frac{\partial q}{\partial t} + \frac{1}{\delta^2}(q.\nabla)q \right] = -\nabla p + \rho g - \mu_f Q + \mu_c (\nabla^2 q)
\]  

(1.11)

**Forchheimer-Brinkman-Extended-Darcy Equation**

Brinkman’s modification to the Darcy equation of motion has not been considered to be important by the porous media researchers except in the case of sparsely packed beds, primarily because the drag introduced by the boundary is generally small compared to that due to the ensemble of solid particles. However, since the convective heat transfer is mostly a boundary phenomenon, this modification to include the viscous diffusion effects can be of great importance to the energy transport processes. For an isotropic, homogeneous, fluid-saturated porous medium, the governing equations which include both the Forchheimer and Brinkman modifications in vector form are
\[ \nabla \bar{q} = 0 \quad \text{(Equation of continuity)} \quad (1.12) \]

\[
\frac{\rho}{\delta^2} \frac{\partial \bar{q}}{\partial t} + \rho (q \nabla)q = -\nabla p + \rho g - \frac{\mu}{k} \bar{q} - \frac{D^F}{\sqrt{k}} [\bar{q}] + "V^n^2 \bar{q} \quad \text{(Equation of linear momentum)} \quad (1.13)
\]

\[
\rho \tilde{c}_p \left[ \frac{\partial T}{\partial t} + (\bar{q} \nabla)T \right] = \lambda \nabla^2 T \quad \text{(Equation of energy)} \quad (1.14)
\]

\[
\rho - \rho_0 = -\beta g (T - T_0) \quad \text{(Equation of state)} \quad (1.15)
\]

Where \( \bar{q} = (0, 0, u) \) is the velocity, \( T \) is the temperature, \( p \) is the pressure, \( \rho \) is the density of the fluid, \( c_p \) is the specific heat at constant pressure, \( k \) is the permeability of the porous medium, \( \mu \) is the coefficient of viscosity of the fluid, \( \delta \) is the porosity of the medium, \( \beta \) is the co-efficient of thermal expansion, \( \lambda \) is co-efficient of thermal conductivity and \( F \) is a function that depends on the Reynolds number and the microstructure of porous medium.

Here, the thermophysical properties of the solid and fluid have been assumed to be constant except for the density variation in the body force term (Boussinesq approximation), and the solid particles and fluid are considered to be in local thermal equilibrium.

1.3 Governing equations

Based on the experimental research of Darcy inflow through porous medium, Navier-Stoke’s equations are replaced by linear partial differential equations. Suitable approximations are to be made to get the solution, as the governing Equations of porous
media is partial differential equation. In 1856, Henri Darcy formulated the law which governs the flow through a porous medium. Darcy’s law is given by

\[ q = \text{const} \rho \nabla p + \rho g \]  

(4)

Where the \( p \) is the pressure

Equation (4) expresses that Darcy’s velocity \( q \) is proportional to the sum of the pressure gradient and the gravitational force. Moreover, \( q \) is inversely proportional to viscosity. This Darcy law is macroscopic equation of motion for Newtonian fluids in porous media at small Reynolds numbers. Many researchers verified this law, experimentally. The constant in equation (4) is replace by the permeability \( k \) by

Musket (22) Now equation (4) becomes

\[ q = -\frac{k}{\mu} \nabla p \]  

(5)

This law is valid for the flows through isotropic porous media.

By using Darcy’s law various flows through porous media have been investigated by Musket (22), De wiest (9), Bear (2) and many other researchers. The most general form of Darcy’s law is given by

\[ \frac{\rho}{E} \frac{Du_i}{Dt} = -\frac{\partial p}{\partial x_i} + \rho x_i + \rho \nabla x_i - \frac{\mu}{K} u_i \]

where

\( x_i = \) the \( i \)th component of body force per unit mass

\( u_i = \) The \( i \)th component of velocity.

\( E = \) Porosity

\( \frac{D}{Dt} = \) Substantial derivative.
Darcy law is valid when the flow takes place at low speeds. But for high speed flows, Darcy law is not valid. Also Darcy law fails to describe the flows with high speeds or the flow near surfaces which are either permeable or rigid. In such cases, Brinkman (7) equation will be useful. Brinkman obtained the governing equation for the flow through porous media as

\[-\nabla p - \frac{\mu}{K} \vec{V} + \mu \nabla^2 \vec{V} = 0\]  

(6)

Where $\vec{V}$ is the velocity vector.

Rudraiah et al. (30), Chennabasappa and Ranganna (8) Sacheti (34), Vijayakumar Varma and Syam Babu (46) and many others studied numerous flow models past porous media applying Brinkman model.

1.4 Boundary Conditions

i. No-Slip Condition

When the flow takes place over a rigid plate, the velocity Component vanish at the boundaries. This is called no slip condition.

ii. Free Surface Boundary Condition

Vertical component of velocity vanishes at a horizontal free surface. Further if there is no surface tension, the free surface will be free from shear stress.

iii. Beavers and Joseph slip condition

When a fluid flows, an impermeable surface, the no-slip condition is valid on the boundary. But when a fluid flows over a permeable surface, it is necessary to specify some condition on the tangential component of the velocity of the free fluid at the permeable interface. In this case, there will be a migration of fluid tangential to the boundary within the permeable surface, at the permeable interface. In this case, there
will be a migration of fluid tangential to the boundary within the permeable surface. That is, there will be a net tangential drag due to transfer of forward momentum across the permeable interface. The velocity inside the permeable bed will be different from the velocity of the fluid past/over the permeable bed. These two velocities are to be matched at the nominal boundary (surface) of the permeable bed. The nominal boundary of a permeable bed is defined as a smooth geometric surface with the assumption that the outermost perimeters of all surface pores of the permeable material are in this surface. Thus if the surface is filled with solid material to the level of their respective perimeters, a smooth rigid boundary of the assumed shape results.

When Newtonian fluid flows past a permeable bed the no slip condition is not valid there. Firstly, Beavers and Joseph proved that there exists a slip on the velocity at the surface of the porous bed. This slip boundary condition is given by

\[
\frac{du}{dy} = \frac{\alpha}{\sqrt{k}} (u_n - Q)aty = B
\]

Where

- \( u \): Velocity parallel to the nominal surface
- \( \alpha \): Slip parameter
- \( u_n \): Slip Velocity
- \( Q \): Darcy Velocity

And \( y = B \) is the nominal surface.
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HEAT AND MASS TRANSFER

2. HEAT TRANSFER

In nature, so many flows exist which are caused not only by the temperature differences but also by concentration differences. These mass transfer differences do affect the rate of heat transfer. The phenomenon of heat and mass transfer...
frequently exists in chemically processes industries such as, food processing and polymer production. The effect of foreign masses, also known as diffusing species concentration, were studied under different conditions by Somers (18), Mathers et al (9) Eckert and Drake (2) have done pioneering work on heat and mass transfer. Gebhart et al (4) studied the natural convection boundary layer flow due to simultaneous heat and mass transfer with various geometries.

Heat and Mass transfer phenomena is found everywhere in nature and important in all branches of science and technology. The science of heat transfer is concerned with the analysis of the rate of heat transfer taking place in a system. In studying heat transfer, a knowledge of the temperature distribution in a system is essential, as heat temperature distribution in a system is essential. Heat flow takes place wherever there is a temperature gradient in a system. Once the temperature distribution is known, the heat flux, which is the amount of heat transfer permit area, permit time is determined from the law relating the heat flows to the temperature gradient

A detailed knowledge of principles of heat transfer and a through understanding of its back mechanism is necessary to design efficient and economical heat exchangers such as boilers, condensers and combustion engines etc., Design problems in nuclear reactors also demand the knowledge of temperature distribution and heat transfer has become invertible in aeronautics for the durability and proper functioning of space vehicles to avoid structural failures and also for safety considerations construction of dams and other large multistoried buildings need the study of heat transfer. A keen analysis of heat flow is necessary in heating, air conditioning of buildings to estimate the amount of insulation and to reduce the heat
losses or gains. Activities like heating, cooling, evaporation, sublimation in chemical operations in research laboratories involve heat transfer. Designing of chemical processing equipments, fractional analysis. Many metallurgical and industrial mechanisms or operations like extracting metals involve the study of heat transfer. Considerable knowledge of heat transfer is necessary in the controlling or solving or lessening the environmental problems. To develop better variety of seeds, breeders have to deal with heat transfer problems. Food processing needs a lot of knowledge of heat transfer.

Heat transfer can be defined as the transmission of energy from one region to another as a result of temperature difference between them.

There are three different modes of heat transfer,

Conduction
Convection
Radiation.

Heat conduction may be stated as the transfer of internal energy between the molecules. Heat flows from a region of higher temperature to a region of lower temperature by kinetic motion or direct impact of molecules whether the body is at rest or motion.

Heat Conduction is due to the property of matter which allows the passage of heat energy even if a physical body is impermeable to any kind of redaction and it parts is not in motion relative to one another. Heat convection is due to the capacity of moving matter to carry heat energy such as transporting a load from one place to another.

Heat transfer due to convection involves the energy exchange between a solid surface and an adjacent fluid. Convection is a mechanism in which heat flows or
transferred between a fluid and a solid surface because of motion of fluid particles relative to the solid surface when there exists a temperature gradient.

2.2 Equation of Convection

Variations in the properties of the fluid, namely, density and viscosity are caused by the variations in the temperature within a convective flow. Hence, to analyse these effects, we require some approximation methods. One such method is the famous Boussinesq approximation.

In the Boussinesq approximation, variations of all fluid properties other than the density are ignored completely. Variations of the density are also ignored except in so far as they give rise to a gravitational force. Thus, the continuity equation is taken in its constant density form.

\[ \nabla \bar{q} = 0 \quad (1) \]

Similarly, \( \rho \frac{D\bar{q}}{Dt} \) is replaced by \( \rho_0 \frac{D\bar{q}}{Dt} \).

Where \( \rho_0 \) is chosen as constant density at one typical portion. However, a body force term is included to allow for the effect of gravity in the momentum equation. That is in

\[ \rho \frac{D\bar{q}}{Dt} = -\nabla p + \mu \nabla^2 \bar{q} + \bar{F} \quad (2) \]

We take \( \bar{F} = \rho \bar{g} \) \( \quad (3) \)
Here in we consider the density variation and hence write

\[ \rho = \rho_0 + \Delta \rho \quad (4) \]

In addition, the gravitational acceleration is derivable from a potential

\[ \ddot{g} = -\nabla \phi \quad (5) \]

Therefore

\[ \ddot{F} = -(\rho_0 + \nabla \rho) \nabla \phi = -\nabla (\rho_0 \phi) + (\Delta \rho) \ddot{g} \quad (6) \]

Now we introduce

\[ \rho = \rho + \rho_0 \phi \quad (7) \]

As the new pressure

The momentum equation (2) now becomes

\[ \rho_0 \frac{D\dot{q}}{Dt} = -\nabla p + \mu \nabla^2 \dot{q} + (\nabla p) \ddot{g} \quad (8) \]

In addition, Boussinesq approximation linearizes the dependence of \( \rho \) on \( T \) and we can write

\[ \Delta \rho = -\alpha \rho_0 \Delta T \quad (9) \]

\( \alpha \) being the coefficient of expansion of the fluid therefore, the Boussinesq dynamical equation is

\[ \frac{D\dot{q}}{Dt} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \dot{q} - \ddot{g} \alpha \Delta T \quad (10) \]

Where we replace \( \rho_0 \) by \( \rho \) and \( \dot{P} \) by \( \dot{p} \).

Boussinesq approximation requires effective constantly of the physical properties of fluids, viz., \( \mu, k, \alpha \) and \( C_p \).

The temperature and pressure variations must not be so large as to produce large fractional changes in these qualities. By Boussinesq approximation we can suppose that
the fluid has a constant heat capacity per unit volume, \( \rho C_p \). Then \( \rho C_p \frac{D T}{D t} \) is equal to
the of heating per unit volume of a fluid particle. This heating is effected by transfer of heat from neighboring fluid particles by thermal conduction and sometimes also by internal heat generation.

The conductive heat flux

\[ \tilde{\theta} = -K \text{grad}T \] (11)

Where \( K \) is the thermal conductivity of the fluid thus

\[ \rho C_p \frac{D T}{D t} = -\text{div} \tilde{\theta} + \phi \] (12)

Where \( \phi \) is the rate of internal heat generation per unit volume.

Taking \( K \) to be constant, equation (12) may be rewritten as

\[ \frac{\partial T}{\partial t} + (\vec{q} \cdot \nabla) T = K \nabla^2 T + \frac{\phi}{\rho C_p} \] (13)

Where \( K = k / \rho C_p \) (14)

And \( K \) is known as the thermal diffusivity or sometimes as the thermometric conductivity functions (1), (10) and (15) constitute the basic education of convection in the Boussinesq approximation. The additional term in the momentum equation, \( -\vec{g} \alpha \Delta T \), is known as the buoyancy force. Equation (13) requires boundary conditions for the temperature field.

Convection heat transfer may be classified into forced convection heat transfer and Free (or) Natural Convection heat transfer.

2.3 Forced convection and free convection
If the heat transfer between the fluid and the solid surface occurs by fluid motion induced by external agencies (or) forces then the mode of heat transfer is termed as “Forced Convection” in all types of heat. Exchangers, nuclear reactors, air conditioning apparatus are by forced convection.

A free Convection flow field is a self-sustained flow driven by the presence of a temperature gradient. (As opposed to a forced convection flow where external means are used to provide the flow). As a result of the temperature difference, the density field is not uniform also. Buoyancy will induce a flow current due to the gravitational field and the variation in the density field. In general, a free convection heat transfer is usually much smaller compared to a forced convection heat transfer. It is therefore important only when there is no external flow exists.

The density difference is due to the temperature difference and it can be characterized by their volumetric thermal expansion coefficient.

Free Convection results from the buoyancy forces imposed on the fluid. Whether it is gas or a liquid, when its density in the neighborhood of the heat transfer surface is reduced as a result of heating process. Free Convection flow also arises when a heated object is placed in a fluid, otherwise at rest, the density of which varies with temperature. Heat is transferred from the surface of the object to the fluid layers in its neighborhood. The density decreases, when in a normal fluid is convicted to temperature increase, causes these layers to rise and create the free convection flow which now transfer away from the object. Physically such flow is described by stating that it is caused by body
force. The example of a practical device which transfers heat by free convection is a radiator used for heating a room.

Free Convection flow occurs frequently in Nature. It occurs not only due to temperature, but also due to concentration difference or combination of these two.

E.g., in atmospheric flows, there exist differences in the $H_2O$ concentration and hence the flow is affected by such concentration difference. Flows in bodies of water are driven through the comparable effects upon density of temperature, concentration of dissolved materials and suspended particular matter may transport process exist in nature and in industrial applications in which the simultaneous heat and mass transfer occur as a result of combined buoyancy effects of diffusion of chemical species.

The phenomenon or the mode of heat transfer without a media in the form of electromagnetic waves is called radiation. The transfer of heat energy from the sun to the earth is an example for radiation.

2.4 Non-dimensional parameters

a. Grashoff number (Gr)

It plays a significant role in free convection heat and mass transfer. The ratio of the product of the internal force and the buoyant force to the square of viscous force in the convection flow system is interpreted as Grashof number. Grashof number in free convection in analogous to Reynolds number in forced convection.
b. **Prandtl Number (Pr)**

It is an important dimensional parameter dueling with the properties of a fluid. It refers to or relates the relative thickness of velocity boundary layer and thermal boundary layers. It is defined as the ratio of kinematic viscosity ($v$) to thermal diffusivity ($\alpha$) of a fluid. Prandtl number physically means or signifies the relative speed with which the momentum and heat energies are transmitted through a fluid, it thus associates the velocity and temperature fields of a fluid for gasses Prandtl number is of unit order and varied over a wise range in case of liquids.

c. **Rayleigh number (Ra)**

The ratio of the apparent conductivity to the true molecular conductivity is a function, which is the product of Grashof and Prandtl numbers. This function is referred as the Raleigh number.

That is \[ Ra = \frac{g\beta N \theta^3}{v \alpha} \]

Where $\alpha$ is the thermal expansion coefficient of the liquid and $T$ is the temperature difference between the top side and underside of the film.

d. **Schmidt Number (Sc)**
The ratio of molecular diffusivity of momentum to the mass molecular diffusivity is given by Schmidt number. It plays a major role in connective mass transfer.

\[ Sc = \frac{v}{D} \]

e. **Nusselt Number (Nu)**

The ratio of the conductive thermal resistance to the convective thermal resistance of the fluid is called Nusselt number.

This is written as \( \text{Nu} = \frac{hL}{K} \)

f. **Eckert Number (Ec)**

This is defined as two times of the ratio of the dynamic temperature \( t_d \), and the heat transfer temperature difference \( (t_0-t_w) \).

That is

\[ Ec = 2 \frac{t_d}{t_0-t_w} = \frac{U'_w}{g_0 \beta \rho U (t_0-t_w)} \]

2.5 **The Governing Equations**

Dynamical similarity pertains if the ration of each of the velocity components at geometrically similar points of the flow to the velocity scale is the same in both pieces of apparatus, that is if.

\[ U' = \frac{U}{U}, \quad V' = \frac{V}{U}, \quad W' = \frac{W}{U} \]  

(15)
Are the same when

\[ X' = \frac{x}{L}, \quad Y' = \frac{y}{L}, \quad Z' = \frac{z}{L} \] (16)

where \( L \) and \( U \) represent the length and velocity scales.

We can similarly define other physical quantities such as

\[ \theta' = \frac{T - T_0}{T_1 - T_0}, \quad p^1 = \frac{P}{\rho_0 U}, \quad t^1 = \frac{Ut}{L} \] (17)

In equation (17) \( T_0 \) and \( T_1 \) \((T_0 < T_1)\) represents two different temperature fields, for example, \( T_0 \) may represent the ambient temperature and \( T_1 \) the temperature of solid or fluid whose motion is under study.

In view of equations (15), (16) and (17) the dynamical equations governing convective flows (1), (10) and (13) can be written in the non-dimensional form as (draping the primes).

\[ \nabla \cdot \bar{q} = 0 \] (18)

\[ \frac{D\bar{q}}{Dt} = -\nabla p + \frac{1}{Re} \nabla^2 \bar{q} + \frac{Gr}{Re^2} \frac{\phi}{\theta} \] (19)

\[ \frac{D\theta}{Dt} = \frac{1}{Pe'} \nabla^2 \theta + \frac{2E}{Re} \phi \] (20)
Where

\[ \text{Re (Reynolds number)} = \frac{uL}{v} \]

\[ \text{Gr (Grashoff number)} = \frac{g\alpha(T_1 - T_0)L^3}{\nu^2} \]

\[ \text{Pe' (Peclet numbers)} = \frac{UL}{K} \]

\[ \text{Pr (Prandtl number)} = \frac{\nu}{K} \]

\[ \text{Ec (Eckert number)} = \frac{U^2}{C_p(T_1 - T_0)} \quad (21) \]

These are non-dimensional parameters related to the convection flows.

The Grashoff number and the Prandtl number must be the same in two geometrically similar situations for obtaining the same flow patterns. To study a given situation, one obtains a fuel range of the Grashoff number for different Prandtl number fluids. For a given fluid, the Grashoff number indicates expected type of flow. It informs us whether the flow is laminar or turbulent. But it cannot be interpreted as the ration of two dynamical processes. However it gives the relative importance of inertia and viscous forces. The motion can reach a steady state only when other terms balance the buoyancy force. On the other hand, since the buoyancy force is the cause of the motion, these other
terms can’t become large compared to it. If we take the inertial force of the same order
of magnitude as the buoyancy force, we obtain.

\[ U \sim (g\alpha L(T - T_0)^{1/3}) \quad (22) \]

Also in comparing the orders of magnitude of the inertia and viscous forces, we obtain

\[ \frac{|\bar{q} \nabla \bar{q}|}{|\nabla^2 \bar{q}|} \sim Gr^{-1}, \quad (23) \]

When the Grashoff number is large, the above function tells us that the viscous force is
negligible compared with the buoyance and inertia force. This function doesn’t tell us
anything, when the Grashoff number is small.

When Grashoff number is small, we assume that the viscous force is comparable with
the buoyancy force.

This comparison gives us

\[ U \sim \frac{g\alpha(T_i - T_o)L^2}{v} \quad (24) \]

And therefore

\[ \frac{|\bar{q} \nabla \bar{q}|}{|\nabla^2 \bar{q}|} \sim Gr \quad (25) \]

This shows that small Grashoff number implies negligible inertia force.

Thus, the Grashoff number indicates the relative importance of viscous and inertial
effects, because of the different powers to which Grashoff number appears in function
(22) and (24) we aren’t in a position to write a general expression for Gr as a ration of
effects.
In problems relating convection, we need to know not only which dynamical processes are important but also which processes are important in determining the temperature distribution.

\[ \frac{\text{Advection}}{\text{Conduction}} \sim \frac{\bar{q} \nabla T}{K \nabla^2 T} \sim \frac{UL}{K} = P' = P_r \quad (26) \]

Since

\[ \frac{\bar{q} \nabla T}{K \nabla^2 T} \sim |\text{Gr}^{1/2} \cdot \text{Pr. when Gr is large} \]

\[ \text{Gr} \cdot \text{Pr. when Gr is small} \quad (27) \]

We have

The non-dimensional parameter

\[ R_a = \text{Gr} \cdot \text{Pr} = \frac{\alpha(T_1 - T_0)L^3}{\nu K} \quad (28) \]

In equation (27) is called the Rayleigh number and plays an important role in studies of convection in horizontal layers. Considering gases and liquids, for which Prandtl Number is nearly equal to one, we see that the advection dominate over conduction as well as inertial forces dominates over viscous forces when Pr is large or small, this phenomena doesn’t occur

We have to consider another non-dimensional parameter, viz., the Nusselt number, indicating the heat transfer. It can be defined as

\[ Nu = \frac{d\theta}{dy} = f(\text{Gr}, \text{Pr}) \quad (29) \]

When the flow is produced by maintaining the temperature differences between impermeable boundaries, thermal conduction is responsible for the introduction of temperature differences in to the fluid. Omitting the action of conductivity would
remove the convection problems altogether. The correct inference when $Gr^{1/2}$ and $Gr^{1/2}\Pr$ are large is that the flow will have a boundary layer character.

2.6 Mass transfer

Mass transfer is defined as the transfer of matter by virtue of species concentration difference in a system. The difference in concentration provides a driving force for the transfer of mass. Mass transfer always occurs in the direction of redacting concentration gradient.

The phenomena of mass transfer are very common in the theory of stellar structure and observable effects are detectable at least on the solar surface. The effect of mass transfer on the Stokes problems (1851) was first studied by Soundalgekar (1979).

The involvement and application of mass transfer process goes to greater lengths in numerous fields of science, engineering and technology. Mass transfer operations quite often occur in the fields of electric engineering, civil engineering, aeronautics, metallurgy, environmental engineering, refrigeration, air conditioning, biological and industrial processes. The study of geophysics, astronomy, meteorology, agricultural oceanography and food processing demands the knowledge of heat and mass transfer. Mass transfer flows are highly significant for their varied practical importance many examples of mass transfer applications can be cited from the environment.

Mass transfer broadly occurs in biological chemical physical and engineering fields. It involves in biological functions or process like respiratory mechanisms, oxygenation (or) purification of blood, kidney functions, osmosis and assimilation of food and drugs.
Evaporation of clouds, smoke formation, dispersion of fog, distribution of temperature and moisture over agricultural fields and grooves of fruit trees, damages of crops due to freezing and pollution of the nature. Mass transfer finds its place in ablative coding transpiration and film cooling of rocket and jet engines. Mass transfer applications are widely found in chemical engineering process like distillation, absorption of gases, interaction of solids and liquids from their mixtures, crystallization ads option (solid taking up vapor on it surface) and chromatography processes like air humidification, cooling of water, ion exchange involve mass transfer.

Mass transfer occurs by two mechanisms.

1. Diffusion mass transfer
2. Convective mass transfer.

2.7 Diffusion Mass transfer

In diffusion mass transfer the transfer of matter occurs by the movement of molecules or species or particles of one component to another. Diffusion mass transfer may occur either due to concentration gradient or temperature gradient or pressure gradient (pressure diffusion).

2.8 Convective Mass transfer

Convective mass transfer is mechanism in which mass is transferred between the fluid and the solid surface as a result of movement of matter from the fluid to the solid surface or fluid.

Convection mass transfer is again classified into
a) Natural or free convective mass transfer

b) Forced convection mass transfer

In natural convection mass transfer, the transfer of mass occurs by the motion of species due to the density differences resulting from temperature or concentration differences or mixture of varying composition.

2.9 SORRET EFFECT

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. Mass fluxes can be created by temperature gradients and this is the Soret effect or the thermo-diffusion effect. The Soret effect dramatically lowers the thermal convection threshold, since concentration gradients release much more slowly than temperature gradients due to the disparate values of the mass diffusion coefficient and of the thermal diffusivity.

The name ‘Soret effect’ is usually attributed to mass separation induced by temperature gradients. The effect was discovered in 1879 by the swiss scientist Charles Soret who noticed that a salt solution contained in a tube with two ends at different temperatures did not remain uniform in composition. The salt was more concentrated near the cold end than near the hot end of the tube. Charles Soret concluded that a flux of salt was generated by a temperature gradient resulting, in steady state conditions, in a concentration gradient. Although the German C.Ludwig described the same phenomenon several years before in 1856 in a short communication, the phenomenon bears his name because Soret studied the effect rather in detail and formulated the fundamental equations describing the phenomenon.
The Soret effect plays an important role in the operation of solar ponds, biological systems and the microstructure of the world oceans. In biological systems, mass transport across biological membranes induced by small thermal gradients in living matter is an important factor. One of the challenges in optimizing exploitation of oil reservoirs is a good knowledge of the fluid physics in crude oil reservoirs. Today, the modelling methods are based on pressure – temperature equilibrium diagrams and on gravity segregation of the different components of crude oil. However, improved models which more accurately predict the concentration of the different components are necessary. The concentration distribution of the different components in hydrocarbon mixtures is mainly driven by phase separation and diffusion and the Soret effect plays an important role.

2.10 RADIATION:

All substances (solids, liquids and gases) at normal and especially at elevated temperatures emit energy in the form of radiation and are also capable of absorbing such energy. This shows that all heat transfer processes are accompanied by a heat exchange by radiation. However, in some cases heat exchange by radiation may be very small fraction of the total quantity of heat exchanged, as such it may be neglected. In case significant amount of heat transfer occurs by radiation, then use may be made of the various laws of radiation.

The relative importance of the various modes of heat transfer differs considerably with the temperature. Heat transfer by conduction and convection depends basically on the temperature difference and is little affected by the temperature level. For example, other factors remaining constant, heat transfer by conduction or convection from a body at 1000\(^\circ\)C to a body at 200\(^\circ\)C remains the same as that from same body at
900°c to a body at 100°c. In case of radiation this, however, does not hold good. There may be about 35% more heat transfer at higher temperature even for the same temperature difference assuming all other factors as constant. Another difference between the radiation and the other modes of heat transfer lies in the fact that radiation heat transfer does not require any intermediate medium where as in case of conduction, and convection, medium for heat transfer is essentially required. Moreover, in case of conduction, heat flows from a body at high temperature to a body at low temperature if a third body, colder than either of the two, is interposed at any point between the two bodies, then heat will flow from both bodies to colder body. In case of radiation, however, this does not necessarily hold good. Radiative heat transfer may occur from a hot body, through a cold non-absorbing medium leaving it unaffected, and then reach a 'warmer' body.

Several theories have been proposed to explain the transport of energy by radiation. Whichever theory is used, radiant energy is the same type of wave motion as radio waves, X-rays and light waves except for the wave length. In fact, there is a whole spectrum of electromagnetic radiation in which the various arbitrary divisions are referred to by names reflecting the methods of origin or some characteristic quality. All forms have the same velocity of propagation but different wave lengths and sources of origin. All forms produce heat when absorbed. Never-the less, it is only the electromagnetic radiation produced by virtue of the temperature of the emitter that we call thermal radiation. Table 11-A gives the approximate ranges of wave length of some forms of radiation.

The amount of thermal radiation emitted by a body depends on its temperature and surface condition. Radiant energy emitted by a hot body is not confined to the visible range of wave length.
### Table 1

**Characteristic wave lengths of Radiation**

<table>
<thead>
<tr>
<th>Name</th>
<th>Wave length Range in Micons *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosmic rays</td>
<td>upto $10^{-6}$</td>
</tr>
<tr>
<td>Gamma rays</td>
<td>$1(10^{-6})$ to $140(10^{-6})$</td>
</tr>
<tr>
<td>X-rays</td>
<td>$6(10^{-6})$ to $100,000(10^{-6})$</td>
</tr>
<tr>
<td>Ultraviolet rays</td>
<td>0.014 to 0.4</td>
</tr>
<tr>
<td>Visible or light rays</td>
<td>0.4 to 0.8</td>
</tr>
<tr>
<td>Infrared rays</td>
<td>0.8 to 400</td>
</tr>
<tr>
<td>Radio</td>
<td>$10(10^6)$ to $30,000(10^6)$</td>
</tr>
</tbody>
</table>

*1 micron = $10^{-6}$ metre.

But extends itself on both sides somewhat beyond this region. A thermometer placed in the dark or invisible region beyond the red end of a solar spectrum will detect a temperature rise.

**LAWS OF THERMAL RADIATION:**

(a) **Planck's Law:**

In 1900, Max Planck developed the quantum theory of electromagnetic waves and with the help of this, he has suggested the following formula for the monochromatic emissive power ($E_{h\nu}$) of a black-body which is based on his theoretical analysis and it is given as
where the values of $c_1$ and $c_2$ are given as follows:

$$c_1 = 3.21 \times 10^{-16} \text{ kcal-m}^2/\text{hr}$$

$$= 3.21 \times 10^{-8} \frac{\text{kcal}}{\text{m}^2 \cdot \text{hr}} \times \text{cm}^4$$

As $1 \mu = 10^{-3} \text{ cm}$

$$\therefore \lambda cm = \frac{\mu}{10^{-4}}$$

$$\therefore \lambda^4 = \frac{\mu^4}{10^{-16}}$$

Substituting this in the above equation

$$c_1 = \frac{3.21 \times 10^{-8} \frac{\text{kcal}}{\text{m}^2 \cdot \text{hr}} \mu^4}{10^{-16}}$$

$$= 3.21 \times 10^8 \frac{\text{kcal}}{\text{m}^2 \cdot \text{hr}}$$

and

$$c_2 = 1.438 \text{ cm-k}$$

$$= 14380 \mu \text{-k}$$

(b) Rayleigh-Jean's Law:

Planck's law has two limiting cases one of which is that when the product $\lambda T$ is large compared with the constant $c_2$. With this provision we can confine ourselves to only two terms of the exponential function (1) expanded into a series with the $\frac{c_2}{\lambda T}$ exponent:
The equation (1) becomes

$$E_{0\lambda} = \frac{2\pi c T^4}{c^2 \lambda^3}$$

This relationship expresses Rayleigh-Jean’s law.

**Wien’s Law of Deviation:**

The second extreme case corresponds to a small value of the product $\lambda T$ as compared to constant $c_2$. Then, the unity present in the denominator of equation (1) can be neglected, and the relationship becomes Wien’s law (1893).

$$E_{0\lambda} = \frac{2\pi c T^4}{c^2 \lambda^3}$$

Coordinates of the maximum values of the emissive power can be obtained from the extreme value of (1). For this purpose the derivative of the function is found for the wave length. Equating the derivative to zero, we obtain the following transcendental equation:

$$e^{-c_2} + \frac{c_2}{5\lambda_{max} T} - 1 = 0$$

whose solution is

$$\frac{c_2}{\lambda_{max} T} = 4.965$$

from which

$$\lambda_{max} T = 2.8978 \times 10^{-3}$$


where $\lambda_{\text{max}}$ is the wavelength corresponding to a maximum intensity of radiation; the product $\lambda_{\text{max}} T$ is measured in units of m.k.

(d) **Planck’s Law in Dimensionless Form:**

Wien’s law of deviation, \[ (E_{0,\lambda})_{\text{max}} = c_3 T^5 \] (5)

permits Planck’s law (1) to be expressed in dimensionless form:

\[
\frac{E_{0,\lambda}}{(E_{0,\lambda})_{\text{max}}} = \frac{2\pi c_1}{c_3} \left( \frac{1}{\frac{c_2}{e^{\frac{c_2}{\lambda T}} - 1}} \right) = \phi(\lambda T)
\] (6)

If we substitute the value of $T$ from (4) in (6), then the latter acquires the form of:

\[
\frac{E_{0,\lambda}}{(E_{0,\lambda})_{\text{max}}} = f\left(\frac{\varphi}{\varphi_{\text{max}}}\right)
\] (7)

Planck’s law is represented here graphically not by a family of isotherms, as illustrated. But by a single curve that holds for any wavelength and temperature of the body. The maximum of this relationship corresponds to the values

\[
\frac{E_{0,\lambda}}{(E_{0,\lambda})_{\text{max}}} = 1 \quad \text{ and } \quad \left(\frac{\lambda}{\lambda_{\text{max}}}\right) = 1
\]

(e) **Stefan-Boltzmann’s Law:**

The Stefan-Boltzmann law relates the hemispherical total emissive power and temperature. This relationship was first established by Stefan in 1879, measuring the radiation emitted from a model of a black body, well before the appearance of Planck’s quantum theory. Later, in 1884, the relationship was found theoretically (departing from the laws of thermodynamics) by Boltzmann and that is why the law is called Stefan-Boltzmann’s law. Stefan-Boltzmann’s law can also be obtained from Planck’s law. For
the emissive power $E_0$, W/m², Stefan-Boltzmann’s law may be presented in the following form:

$$E_0 = \int_{0}^{\infty} E_0 d\lambda = \sigma_0 T^4$$  \hspace{1cm} (8)

where $\sigma_0$ is Stefan-Boltzmann’s constant.

To facilitate practical calculations, equation (8) is usually presented in the following form:

$$E_0 = c_0 \left( \frac{T}{100} \right)^4$$  \hspace{1cm} (9)

Where $c_0 = 5.6687 = 5.67$ is the radiation constant of a black body, measured in W/m²·K⁴.

The Stefan-Boltzmann law also applies to grey bodies. It is them assumed that, as with black bodies, the inherent radiation of a grey body is proportional to the fourth power of the absolute temperature, but the emissive power is smaller than that of black bodies at the same temperature, as illustrated. The law thus acquires the following form for grey bodies:

$$E = \varepsilon\sigma_0 \left( \frac{T}{100} \right)^4 = c \left( \frac{T}{100} \right)^4$$  \hspace{1cm} (10)

where $\varepsilon = E / E_0 = c / c_0 = $ integral emissivity of a grey body;

$$c =$$ its radiation constant (or factor), W/m²·K⁴.

Thus, by total emissivity is meant the ratio of the total radiation density of a surface to that of a black body at the same temperature.

Stefan-Boltzmann’s law is strictly valid for a grey body to the same extent as the assumption that emissivity remains strictly constant, independent of temperature.

(f) **Kirchhoff’s Law of Radiation**
Kirchhoff’s law (1882) establishes the quantitative relation between the emissivity and absorptivity of grey and black bodies. It can be derived from the heat balance of an emitting system consisting of a relatively large enclosure with heat-insulated walls, with two bodies inside it. Under conditions of thermodynamics equilibrium the radiant energy of each of the three bodies is equal to the absorbed energy

\[ E_1 = E_{ab1} = A_1 E_0 = A_1 E_0 \]

\[ E_2 = E_{ab2} = A_2 E_0 = A_2 E_0 \]

whence we get

\[ E_1 / A_1 = E_2 / A_2 = - - - = E_0 = f(T) \]

Relationship (11) expresses Kirchhoff’s law according to which the ratio of radiant energy to absorbed energy is independent of the nature of bodies and is equal to the radiation of a black body at the same temperature.

REFERENCE:


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3. MAGNEOHYDRODYNAMICS

We can describe scientifically the interaction of electromagnetic fields and fluids by the proper application of the principles of the special theory of relativity. The practical application of these principles, in physical Engineering, Astrophysics, Geophysics etc., have become an important in recent years. The study of three applications to continuum is known as Magneto hydrodynamics or Magneto fluid dynamics.

The study of magneto hydrodynamics (MHD) plays an important role in agriculture, engineering and petroleum industries. MHD has won practical applications, for instance, it may be used to deal with problems such as cooling of nuclear reactors by liquid
sodium and induction flow water which depends on the potential difference in the fluid
direction perpendicular to the motion and goes to the magnetic filed.

The study of magneto-hydro-dynamics (MHD) of viscous conducting fluids is playing
a significant role, owing to its practical interest and abundant applications, in astro-
physical and geo-physical phenomenon. Astro-Physicists and geo-physicists realized the
importance of MHD in stellar and planetary processes. The main impetus to the
engineering approach to the electro magnetic fluid interaction studies has come from the
concept of the magneto hydro dynamics, direct conversion generator, ion propulsion
study of flow problems of electrically conducting fluid, particularly of ionized gases is
currently receiving considerable interest. Such studies have made for years in
convection with astro-physical and geo-physical problems such as Sun spot theory,
motion of the interstellar gas etc., Recently, some engineering problems need the studies
of the flow of an electrically conducting fluid, in ionized gas is called plasma. Many
names have been used in referring to the study of plasma phenomena. Hartman (20)
called it mercury dynamics, as he worked with mercury. Astro-physics called it comical
electro dynamics, and some called it magneto-hydro-dynamics. Physics and electrical
engines commonly use the term plasma dynamics. The aerodynamicist has spoken of
magneto-hydro-dynamics. The more general name magneto fluid mechanics was
suggested by T.Van Karman (43).

3.1 Basic MHD Equations

When an electro magnetic body force acts on the fluid, the motion of the fluid may
generate in induced electro magnetic field in the presence of the electromagnetic field
and alters the field. Due to the interaction of the fluid forces and the external
electromagnetic force, the velocity field in the fluid and electric and magnetic fields
inside the fluid get disturbed. Now to discuss the magneto fluid dynamics, we have to combine the basic concepts of fluid dynamics and electromagnetism. The equations governing motion will be the Navier-Stokes equation of fluid and Maxwell’s equations which are usually coupled and must be solved simultaneously. The electro-magnetic body force is usually non-conservative (rotational) and not derivable from a scalar potential function. Only under rare circumstances it may be approximately conservative and derivable from a scalar potential function. In such cases the electromagnetic body force alter simply the pressure, provided the boundary conditions on velocity and pressure remain the same. However this happens in exceptional cases and the force is rotational in general and does alter the flow.

The Maxwell’s equations in RMKS units are

\[ \nabla \cdot \vec{D} = \rho \quad \text{(Coulomb's law)} \]  

(1)

\[ \nabla \cdot \vec{B} = 0 \quad \text{(Absence of free magnetic poles)} \]  

(2)

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{(Faraday's law)} \]  

(3)

\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad \text{(Amper's law)} \]  

(4)

And Ohm’s law

\[ \vec{J} = \sigma \left( \vec{E} + \vec{q} \times \vec{B} \right) + \vec{\lambda} \vec{q} \]  

(5)

The current conservation equation

\[ \nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0 \]  

(6)
The continuity equation is

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0 \]  \hspace{1cm} (7)

The equation of motion (Navier-Stoke’s equation) is

\[ \rho \frac{D\vec{q}}{Dt} = -\nabla P + \mu \nabla^2 \vec{q} + \vec{F} + \vec{fe} \]  \hspace{1cm} (8)

These equations hold in any frame of reference.

3.2 MHD Approximations

In addition to the non-relativistic approximation, certain additional simplifications can be made for flow which is quasi-steady (Steady of low frequency oscillatory), and in which the electric field is of the order of magnitude of the induced quantity \( \vec{q} \times \vec{B} \).

Elsasser (13) has pointed out that the ratio of the displacement current to the conduction current in large class of phenomena is so small that displacement current can be neglected entirely. Thus both space charges and the electrostatic potential are omitted.

The following assumptions are made under MHD approximation.

1. \( \vec{q}^2 \ll c^2 \) This allows the quantity \( \sqrt{1 - \frac{\vec{q}^2}{c^2}} \) to be taken as unity and use the non-relativistic Newtonian form of equations of motion, where \( c \) is the velocity of the light.

2. The electric field \( \vec{E} \) is of the same order of magnitude as the induced electric field \( \vec{q} \times \vec{B} \). This assumption allows us to assume that the induced magnetic field is much smaller than the externally applied magnetic field.
3. The problems of very high frequency are not considered, so that the displacement current \( \frac{\partial D}{\partial t} \) is neglected compared to \( \tilde{J} \), the conduction current. Thus the equations of electromagnetic fields (6) and (7) become.

\[
\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \tag{9}
\]

\[
\nabla \times \bar{H} = \bar{J} \tag{10}
\]

Equations (9) and (10) are called pre-Maxwell's equations for electromagnetic fields.

4. The electric energy is negligible compared to the magnetic energy. Since the displacement current and electric field energy are neglected, the main interaction is between the magnetic field and the fluid.

5. The current density vector \( \tilde{J} \) is approximately the same in any inertial frame. This means, using Ohm's law

\[
\tilde{J} = \tilde{J}' = \sigma (\bar{E} + \bar{q} \times \bar{B}) \tag{11}
\]

6. The magnetic force density is represented by

\[
\vec{f}_c = \ell \bar{E} + \tilde{J} \times \bar{B} \tag{12}
\]

In comparison with magnetic term, the electric term is negligible. For this reason \( \ell \bar{E} \) is often neglected in MHD.
7. In using \( \nabla . D = \ell \), great care must be taken. If the equation \( \nabla . J + \frac{\partial \ell}{\partial t} = 0 \) is used in metals (Even in alternating current problem under the MHD approximation), the \( \frac{\partial \rho_e}{\partial t} \) term may be dropped and \( \rho_e \) need not be mentioned although it is identically zero.

Also \( \nabla . J = 0 \) is valid even for high frequency phenomena as long as the fluid velocity \( \vec{q} \) is very small compared to \( c \).

### 3.3 Equations under the MHD Approximation

The basic equation under the MHD approximation takes the following form

\[
\nabla \times \vec{H} = \vec{J} \tag{13}
\]

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{14}
\]

\[
\nabla . \vec{H} = 0 \tag{15}
\]

\[
\nabla . \vec{J} = 0 \tag{16}
\]

and the Ohm's law is

\[
\vec{J} = \sigma (\vec{E} + \vec{q} \times \vec{B}) \tag{17}
\]

The equation of conservation of mass is

\[
\frac{\partial \rho}{\partial t} + \nabla . (\rho \vec{q}) = 0 \tag{18}
\]

The equation of motion is

\[
\rho \frac{D\vec{q}}{Dt} = -\nabla p + \mu \nabla ^2 \vec{q} + \vec{J} \times \vec{B} - \rho \nabla \psi \tag{19}
\]

Where \( \Psi \) is the Gravitational Potential. Magnetic diffusion equation is

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\[
\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times (\vec{q} \times \vec{B}) \tag{20}
\]

Where \( \eta = \frac{1}{\sigma \mu_c} \) is magnetic diffusivity (or) magnetic viscosity.

### 3.4 Non-Dimensionalisation

We introduce the following dimensionless parameters.

\[
x^* = \frac{X}{L}, \quad \nabla^* = \nabla L
\]

\[
t^* = \frac{1}{t_0} = \frac{tU}{L}, \quad \vec{q}^* = \frac{\vec{q}}{U}, \quad \psi^* = \frac{\psi}{gL}
\]

\[
\vec{E}^* = \frac{\vec{E}}{UB_0}, \quad \vec{H}^* = \frac{\vec{H}}{H_0}, \quad \vec{J}^* = \frac{\vec{J}}{\sigma \mu_c U H_0},
\]

\[
P^* = \frac{P}{P_0 u^*}, \quad \rho^* = \frac{\rho}{\rho_0}, \quad \mu^* = \frac{\mu}{\mu_0}
\]

By using the above stated dimensionless parameters the equations (13) to (20) after dropping the asterisks are

\[
\nabla \times \vec{H} = R_c \vec{J} \tag{21}
\]

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{22}
\]

\[
\nabla \cdot \vec{H} = 0 \tag{23}
\]

\[
\nabla \cdot \vec{J} = 0 \tag{24}
\]

\[
\vec{J} = \sigma (\vec{E} + \vec{q} \times \vec{B}) \tag{25}
\]

\[
\frac{\partial P}{\partial t} + \nabla (\rho \vec{q}) = 0 \tag{26}
\]

\[
\frac{D\vec{q}}{Dt} = -\nabla P + \frac{1}{R_e} \nabla^2 \vec{q} + \frac{M^2}{R_e} \vec{J} \times \vec{B} - \frac{\rho}{F_r} \nabla \psi \tag{27}
\]

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\[
\frac{\partial B}{\partial t} = -\frac{1}{Re} \nabla \times (\nabla \times B) + \nabla \times (\eta \nabla \times B)
\]  

(28)

The dimensionless parameters appearing in the above equations are

1. \( Re = \) Reynolds number \( \frac{UL}{v} \) which is a measure of the ratio of inertial to viscous.

2. \( R_a = \) Magnetic Reynolds number \( \frac{UL}{\eta} - \sigma \mu \), which is a measure of the ratio of magnetic convection to magnetic diffusion. A small \( R_a (R<<1) \) indicates that the induced magnetic field is small compared to the total or the applied magnetic field.

3. \( F_r = \) Froud number \( \frac{q_h^2}{gL} \), which is a measure of the ratio of inertial force to the gravitational force.

4. \( M = \) Hartmann number \( \sqrt{\frac{E_0^2 L^2}{\mu}} = \sqrt{\frac{\sigma_0 B_0^2 L^2}{\mu \rho_0}} \) of the ratio of pondernotive force to the viscous forces. Here \( \mu_f \) is the fluid viscosity.

5. \( P_m = \) Magnetic Prandtl number \( \frac{\nu}{\eta} = \frac{R_a}{Re} \), which is a measure of the ratio of vorticity diffusion to magnetic diffusion. This number is usually very small, so that the magnetic field diffuses much more easily than the vorticity.

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