CHAPTER VI

NUMERICAL STUDY OF CONVECTIVE HEAT AND MASS TRANSFER FLOW OF A VISCOUS, ELECTRICALLY CONDUCTING FLUID IN A VERTICAL WAVY CHANNEL WITH CHEMICAL REACTION AND SORET EFFECT
Coupled heat and mass transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair (2), Lai and Kulacki (9) and Murthy and Singh (11). Coupled heat and mass transfer by mixed convection in Darcian fluid-saturated porous media has been analysed by Lai (8). The free convection heat and mass transfer in a porous enclosure has been studied recently by Angirasa et al. (1). The combined effects of thermal and mass diffusion in channel flows has been studied in
recent times by a few authors, notably Nelson and Wood (14, 15), Lee et al. (10) and others (23, 25).

In recent years, energy and material saving considerations have prompted an expansion of the efforts at producing efficient heat exchanger equipment through augmentation of heat transfer. It has been established (5a) that channels with diverging – converging geometries augment the transportation of heat transfer and momentum. As the fluid flows through a tortuous path viz., the dilated – constricted geometry, there will be more intimate contact between them. The flow takes place both axially (primary) and transversely (secondary) with the secondary velocity being towards the axis in the fluid bulk rather than confining within a thin layer as in straight channels. Hence it is advantageous to go for converging – diverging geometries for improving the design of heat transfer equipment. Vajravelu and Nayfeh (23a) have investigated the influence of the wall waviness on friction and pressure drop of the generated coquette flow. Vajravelu and Sastry (24) have analysed the free convection heat transfer in a viscous, incompressible fluid confined between long vertical wavy walls in the presence of constant heat source. Later Vajravelu and Debnath (25) have extended this study to convective flow in a vertical wavy channel in four different geometrical configurations. This problem has been extended to the case of wavy walls by McMichael and Deutsch (109), Deshikachar et al. (49), Rao et al. (17a) and Sree Ramachandra Murthy (20a). Hyan Goo Kwon et al. (7a) have analyzed that the Flow and heat/mass transfer in a wavy duct with various corrugation angles in two dimensional flow regimes. Mahdy et al. (79) have studied the mixed convection heat and mass transfer on a vertical wavy plate embedded in a saturated porous media (PST/PSE). Comini et al. (3a) have analyzed the Convective
heat and mass transfer in wavy finned-tube exchangers. Jer-Huan Jang et al. (58) have analyzed that the mixed convection heat and mass transfer along a vertical wavy surface.

In many chemical engineering processes, there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz., polymer production, manufacturing of ceramics or glassware and food processing. Das et al. (5) have studied the effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthukumaraswamy (13) has studied the effects of reaction on a long surface with suction. Recently Gnaneswar (6) has studied radiation and mass transfer on an unsteady two-dimensional laminar convective boundary layer flow of a viscous incompressible chemically reacting fluid along a semi-infinite vertical plate with suction by taking into account the effects of viscous dissipation.

The present trend in the field of chemical reaction analysis is to give a mathematical model for the system to predict the reactor performance. A large amount of research work has been reported in this field. In particular the study of heat and mass transfer with chemical reaction is of considerable importance in chemical and hydrometallurgical industries. Chemical reaction can be codified as either heterogeneous or homogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. Frequently the transformations proceed in a moving fluid, a situation encountered in a number of technological fields. A common area of interest in the field of aerodynamics is the analysis of thermal boundary layer problems for two dimensional steady and incompressible laminar flow passing a wedge. Simultaneous heat and mass transfer from different geometries embedded in a porous media has many
engineering and geophysical application such as geothermal reservoirs, drying of porous solids thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes. The effort has arisen largely due to the need to optimize industrial system such as furnaces, ovens and boilers and the interest in our environment and in no conventional energy sources, such as the use of salt-gradient solar ponds for energy collection and storage. In particular, natural convection induced by the simultaneous action of buoyancy forces resulting from thermal and mass diffusion is of considerable interest in nature and in many industrial application such as geophysics, oceanography, drying process, solidification of binary alloy and chemical engineering. Kandaswamy et al(9a) have discussed the Effects of chemical reaction, heat and mass transfer on boundary layer flow over a porous wedge with heat radiation in the presence of suction or injection.

In this chapter we deal with the two-dimensional laminar simultaneous heat and mass transfer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid through a porous medium confined in a vertical wavy channel. The equations of continuity, linear momentum, energy and diffusion which govern the flow fields are solved by employing a regular perturbation technique with slope $\delta$ as the perturbation parameter. The behaviour of the velocity, temperature and concentration, skin friction, Nusselt number and Sherwood Number has been discussed for variations in the governing parameters.
2. FORMULATION OF THE PROBLEM

We consider the coupled heat and mass transfer flow of a viscous electrically conducting fluid through a porous medium confined in a vertical channel bounded by wavy walls in the presence of constant heat sources, transverse magnetic field effects and a first order chemical reaction. The flow is assumed to be steady, laminar and two-dimensional and the surface is maintained at constant temperature and concentration. It is also assumed that the applied magnetic field is uniform and that magnetic Reynolds number is small so that the induced magnetic field is neglected. In addition, there is no applied electric field and all of the Hall effect, viscous dissipation and Joule heating are neglected. All thermophysical properties are constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq approximation. We consider rectangular Cartesian coordinate system $O(x,y,z)$ with walls at $y = \pm Lf\left(\frac{\delta x}{L}\right)$ with slope $\delta$. Under these assumptions, the equations describing the physical situation are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + \beta g(T - T_e) + \beta' g(C - C_e) -
\]

\[-\frac{\sigma B_0^2 u}{\rho} - \left(\frac{v}{k}\right)u \tag{2.2}\]

\[
u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \left(\frac{v}{k}\right)v \tag{2.3}\]
\[ \rho_b C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \lambda \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q \]  \hspace{1cm} (2.4)

\[ u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - \gamma C + k_1 \frac{\partial^2 T}{\partial y^2} \]  \hspace{1cm} (2.4a)

where \( y \) is the horizontal or transverse co-ordinate, \( u \) is the axial velocity, \( v \) is the transverse velocity, \( T \) is the fluid temperature, \( C \) is the concentration, \( T_e \) is the ambient temperature, \( C_e \) is the ambient concentration and \( \rho, g, \beta, \beta^*, \mu, \sigma, B_o, Q, D \) are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation/absorption coefficient, mass diffusion coefficient and chemical reaction parameter respectively. The physical boundary conditions for the problem are

\[ u(-f) = 0, v(-f) = 0, T(-f) = T_1, C(-f) = C_1 \]

\[ u(+f) = 0, v(+f) = 0, T(+f) = T_2, C(+f) = C_2 \]  \hspace{1cm} (2.5)

where \( T_1, T_2 \) and \( C_1, C_2 \) are the surface temperature and concentrations on \( y = \pm L \) respectively.

The flow is maintained by a constant volume flux for which a characteristic velocity is defined by

\[ q = \frac{1}{L} \int_{-f}^{+f} u \, dy \]  \hspace{1cm} (2.6)

In view of the equation of continuity we define the stream function \( \psi \) as

\[ u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x} \]  \hspace{1cm} (2.7)
the equation governing the flow in terms of stream function $\psi$ are

$$\frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x^2} = \mu \frac{\partial^4 \psi}{\partial x^2 \partial y^2} + \frac{\sigma \mu H_0^2}{\rho_0} \frac{\partial^2 \psi}{\partial y^2} + \beta g \frac{\partial}{\partial y} (T - T_e) +$$

$$+ \beta^* g \frac{\partial}{\partial y} (C - C_c) \quad (2.8)$$

$$\rho_0 C_r \left( - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \lambda \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + Q \quad (2.9)$$

$$\left( - \frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = D \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) - \gamma C + k_{11} \left( \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial x^2} \right) \quad (2.10)$$

and

$$\psi(+f) - \psi(-f) = -1 \quad \frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial x} = 0$$

$$T(+f) = T_1 \quad T(-f) = T_2$$

$$C(+f) = C_1 \quad C(-f) = C_2$$

In order to write the governing equations and boundary conditions in the dimensionless form the following non-dimensional quantities are introduced

$$xy = \frac{x}{L}, \quad y' = \frac{y}{L}, \quad \psi' = \frac{\psi}{(\nu)}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad \psi' = \frac{C - C_1}{C_2 - C_1} \quad (2.11)$$

the equations after dropping the dashes are

$$\left( \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial x} \right) = \nabla^4 \psi + D^{\frac{1}{2}} \frac{\nu^2}{k} \nabla^2 \psi +$$

$$+ M \frac{\partial^2 \psi}{\partial y^2} - C \left( \frac{\partial \theta}{\partial y} + N \frac{\partial C}{\partial y} \right) \quad (2.12)$$

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\[
\frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) + \alpha
\]  
(2.13)

\[
Sc\left( -\frac{\partial \psi}{\partial y} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} \right) = \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - K C + \frac{Sc\alpha}{N} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right)
\]  
(2.14)

and

\[
\psi(+f) - \psi(-f) = -1 \quad \frac{\partial \psi}{\partial y} = 0 \quad \frac{\partial \psi}{\partial x} = 0
\]

\[
\theta(+f) = 1, \quad \theta(-f) = 0
\]  
(2.15)

\[
C(+f) = 1 \quad C(-f) = 0
\]

where

\[
G = \frac{\beta \kappa (T_2 - T_1) L^4}{\nu^2} \quad \text{(Grashof Number)}
\]

\[
M^3 = \frac{\sigma \mu \kappa H^2 L^3}{\nu^2} \quad \text{(Hartman Number)}
\]

\[
D^{-1} = \frac{L^3}{k} \quad \text{(Darcy parameter)}
\]

\[
P = \frac{\mu C_p}{\lambda} \quad \text{(Prandtl Number)}
\]

\[
Sc = \frac{\nu}{D} \quad \text{(Schmidt Number)}
\]

\[
K = \frac{\gamma L^2}{\nu} \quad \text{(Chemical reaction parameter)}
\]

\[
N = \frac{\beta^* \Delta C}{\beta \Delta T} \quad \text{(Buoyancy ratio)}
\]
\[ \alpha = \frac{QL^2}{\lambda} \quad \text{(Heat source parameter)} \]

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \]

3. ANALYSIS OF THE FLOW

The main aim of the analysis is to discuss the perturbations created over a combined free and forced convective flow due to the non-uniform slowly varying boundaries. We introduce the transformation

\[ \bar{x} = \delta x \quad \text{and} \quad \eta = \frac{y}{f(\bar{x})} \quad (3.1) \]

On using the transformation (3.1) the equations (2.13)-(2.15) reduce to

\[ \delta \bar{f} \left( \frac{\partial \psi}{\partial x} \frac{\partial (F^2 \psi)}{\partial \eta} - \frac{\partial \psi}{\partial y} \frac{\partial (F^2 \psi)}{\partial \bar{x}} \right) = F^4 \psi + (D^{+1} f^2) F^2 \psi + \\
+ (M^2 f^2) \frac{\partial^2 \psi}{\partial y^2} - (Gf^3) \left( \frac{\partial \theta}{\partial \eta} + N \frac{\partial \phi}{\partial \eta} \right) \quad (3.2) \]

\[ \delta \bar{p} \left( - \frac{\partial \psi}{\partial \eta} \frac{\partial \theta}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \eta} \right) = F^2 \theta + \alpha f \quad (3.3) \]

\[ \delta \bar{c} \left( - \frac{\partial \psi}{\partial \eta} \frac{\partial T}{\partial x} + \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial \eta} \right) = F^2 C - Kf^2 C + \frac{ScS_{\phi}}{N} \quad (3.4) \]

where

\[ F^2 = \delta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \eta^2} \]

The governing equations of the flow, temperature and concentration are coupled non-linear differential equations. Assuming the slope of the wavy wall \( \delta \) to be small, we write
\[ \psi(x, y) = \psi_0(x, y) + \delta \psi_1(x, y) + \delta^2 \psi_2(x, y) + \ldots \quad (3.5a) \]

\[ \theta(x, y) = \theta_0(x, y) + \delta \theta_1(x, y) + \delta^2 \theta_2(x, y) + \ldots \quad (3.5b) \]

\[ C(x, y) = C_0(x, y) + \delta C_1(x, y) + \delta^2 C_2(x, y) + \ldots \quad (3.5c) \]

Substituting the above expansions (3.5a)-(3.5c) in the equations (3.2)-(3.4) and equating the like powers of \( \delta \), we obtain equations to the zeroth order as

\[ \frac{\partial^4 \psi_0}{\partial \eta^4} - M_1^2 \frac{\partial^3 \psi_0}{\partial \eta^3} = G_1(\theta_{0,y} + N C_{0,y}) \quad (3.6) \]

\[ \frac{\partial^2 \theta_0}{\partial \eta^2} = -\alpha f^2 \quad (3.7) \]

\[ \frac{\partial^3 C_0}{\partial \eta^3} - \beta_1^2 C_0 = -\frac{ScSo}{N} \frac{\partial^2 \theta_0}{\partial \eta^2} \quad (3.8) \]

The first order equations are

\[ \frac{\partial^4 \psi_1}{\partial \eta^4} - M_1^2 \frac{\partial^3 \psi_1}{\partial \eta^3} = G_1(\theta_{1,y} + N C_{1,y}) + f(\psi_{0,\eta} \psi_{0,y} \psi_{0,\eta,y} - \psi_{0,y} \psi_{0,\eta}) \quad (3.9) \]

\[ \frac{\partial^2 \theta_1}{\partial \eta^2} = Pf(\theta_{0,\eta} \psi_{0,x} - \theta_{0,x} \psi_{0,\eta}) \quad (3.10) \]

\[ \frac{\partial^3 C_1}{\partial \eta^3} - \beta_1^2 C_1 = -\frac{ScSo}{N} \frac{\partial^2 \theta_1}{\partial \eta^2} \quad (3.11) \]
The second order equations are

\[
\frac{\partial^4 \psi_x}{\partial \eta^4} - M_1^2 \frac{\partial^3 \psi_x}{\partial \eta^3} = G_1 (\theta_{1,\eta} + NC_{2,\eta}) + f(\psi_{0,\eta} \psi_{1,\eta} + \psi_{1,\eta} \psi_{0,\eta}) - \psi_{1,\eta} \psi_{0,\eta} \psi_{1,\eta} - \psi_{0,\eta} \psi_{1,\eta}
\]  

(3.12)

\[
\frac{\partial^3 \psi_x}{\partial \eta^3} = Pf (\theta_{0,\eta} \psi_{1,\eta} + \theta_{1,\eta} \psi_{0,\eta} - \theta_{\eta,\eta} \psi_{0,\eta} - \theta_{\eta,\eta} \psi_{1,\eta})
\]  

(3.13)

\[
\frac{\partial^2 \psi_x}{\partial \eta^2} - \beta^2 \psi_x = -\frac{ScSo \theta_{\eta,\eta}}{N} \frac{\partial^2 \psi_x}{\partial \eta^2}
\]  

(3.14)

The corresponding boundary conditions are

\[
\psi_0 (+1) - \psi_0 (-1) = -1 \quad \frac{\partial \psi_0}{\partial \eta} = 0 \quad \frac{\partial \psi_0}{\partial x} = 0
\]

\[
\theta_0 (+1) = 1 \quad \theta_0 (-1) = 0
\]  

(3.15a,b,c)

\[
C_0 (+1) = 1 \quad C_0 (-1) = 0
\]

and

\[
\psi_i (+1) - \psi_i (-1) = 0 \quad \frac{\partial \psi_i}{\partial \eta} = 0 \quad \frac{\partial \psi_i}{\partial x} = 0
\]

\[
\theta_i (+1) = 0 \quad \theta_i (-1) = 0
\]  

(3.16a,b,c)

\[
C_i (+1) = 0 \quad C_i (-1) = 0 \quad (i \geq 1)
\]

4. **SOLUTION OF THE PROBLEM**

Solving the equations (3.6)-(3.8) & (3.9)-(3.11) subject to the boundary conditions (3.15a,b,c) we obtain
\[\psi_0(\eta) = a_5 \text{Ch}(\beta_2 \eta) + a_{10} \text{Sh}(\beta_2 \eta) + a_{11} \eta + a_{12} + \phi_1(\eta)\]

\[\phi_1(\eta) = a_5 \text{Ch}(\beta_1 \eta) + a_6 \text{Sh}(\beta_1 \eta) - a_7 \eta^2 - a_8 \eta^3\]

\[\theta_n(\eta) = 0.5 \alpha f^2 (\eta^2 - 1) + 0.5 (\eta + 1)\]

\[C_n(\eta) = \frac{a_{17}}{\beta_1^2} \left( \frac{\text{Ch}(\beta_1 \eta)}{\text{Ch}(\beta_1)} - 1 \right) + 0.5 \left( \frac{\text{Sh}(\beta_1 \eta)}{\text{Sh}(\beta_1)} + 1 \right)\]

\[\theta_1(\eta) = a_{65} (\eta^3 - 1) + a_{66} (\eta^3 - \eta) + a_{67} (\eta^4 - 1) + a_{68} (\eta^4 - \eta) + \]

\[+ (a_{70} + \eta a_{74})(\text{Ch}(\beta_1 \eta) - \text{Ch}(\beta_1)) + a_{71} (\text{Sh}(\beta_2 \eta) - \eta \text{Sh}(\beta_2)) + \]

\[+ (a_{72} + \eta a_{76})(\text{Ch}(\beta_1 \eta) - \text{Ch}(\beta_1)) + a_{73}((\text{Sh}(\beta_1 \eta) - \eta \text{Sh}(\beta_1)) + \]

\[+ (a_{75} + \eta a_{79})(\eta \text{Sh}(\beta_2 \eta) - \text{Sh}(\beta_1)) + (a_{77} + \eta a_{81})(\eta \text{Sh}(\beta_1 \eta) - \text{Sh}(\beta_1)) + \]

\[+ a_{78} (\eta^3 \text{Ch}(\beta_2 \eta) - \text{Ch}(\beta_1)) + a_{80} (\eta^3 \text{Ch}(\beta_2 \eta) - \text{Ch}(\beta_1))\]
\[ C_1(\eta) = b_{47}(1 - \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) + b_{64}(\eta^2 - \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) + b_{68}(\eta^4 - \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) + \\
+ b_{68}(\text{Ch}(\beta_2, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) \text{Ch}(\beta_2) + b_{70}(\eta^2 \text{Ch}(\beta_2, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) \text{Ch}(\beta_2) + \\
+ b_{71}(\eta \text{Sh}(\beta_2, \eta) - \text{Ch}(\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{72}(\eta^3 \text{Sh}(\beta_2, \eta) - \text{Ch}(\beta_1, \eta) - \\
- \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) \text{Sh}(\beta_2)) + b_{77}(\eta^2 \text{Sh}(\beta_2, \eta) - \text{Ch}(\beta_1, \eta) - \\
- \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) \text{Sh}(\beta_2)) + b_{78}(\eta^3 \text{Sh}(\beta_2, \eta) - \text{Ch}(\beta_1, \eta) - \\
- \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) \text{Sh}(\beta_2)) + b_{79}(\eta^4 \text{Sh}(\beta_2, \eta) - \text{Ch}(\beta_1, \eta) - \\
- \frac{\text{Ch}(\beta_1, \eta)}{\text{Ch}(\beta_1)}) \text{Sh}(\beta_2)) + b_{80}(\eta \text{Sh}(\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_1)) + \\
+ b_{81}(\eta^2 \text{Sh}(\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Ch}(\beta_1) + b_{82}(\eta^3 \text{Sh}(\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Ch}(\beta_1) + \\
+ b_{83}(\eta^4 \text{Sh}(\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Ch}(\beta_1) + b_{84}(\eta \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2)) + \\
+ b_{85}(\eta \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{86}(\eta^2 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + \\
+ b_{87}(\eta^3 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{88}(\eta^4 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2)) + \\
+ b_{89}(\eta \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{90}(\eta^2 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + \\
+ b_{91}(\eta^3 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{92}(\eta^4 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + \\
+ b_{93}(\eta \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{94}(\eta^2 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + \\
+ b_{95}(\eta^3 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{96}(\eta^4 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + \\
+ b_{97}(\eta \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{98}(\eta^2 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + \\
+ b_{99}(\eta^3 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2) + b_{100}(\eta^4 \text{Sh}(2\beta_1, \eta) - \frac{\text{Ch}(\beta_1, \eta)}{\text{Sh}(\beta_1)}) \text{Sh}(\beta_2)) \]
\( \psi_1(\eta) = e_1 \text{Ch}(M_1 \eta) + e_2 \text{Sh}(M_1 \eta) + e_3 \eta + e_4 + \phi_2(\eta) \)

\( \phi_2(\eta) = -h_{166} \eta^2 - b_{170} \eta^4 + b_{169} \eta^3 - h_{170} \eta^6 - h_{171} \eta^7 + (h_{172} + \eta^2 h_{189} + + \eta^4 h_{191}) \text{Ch}(\beta, \eta) + (b_{173} + \eta^2 b_{191} + \eta^4 b_{197} + \eta^6 b_{198}) \text{Ch}(\beta, \eta) + (b_{174} + \eta \beta_{202}) 
+ \eta^2 b_{203} \text{Ch}(\beta, \eta) + (b_{175} + \eta \beta_{205} + \eta^5 b_{209} \text{Ch}(\beta, \eta) + (b_{176} + \eta \beta_{210}) \text{Ch}(2 \beta, \eta) + + (b_{177} + h_{212} + \eta^2 b_{214}) \text{Ch}(2 \beta, \eta) + (b_{178} + \eta^2 b_{188} + \eta^4 b_{186} + \eta^6 b_{187}) \text{Sh}(\beta, \eta) + + (b_{180} + \eta^2 b_{193} + \eta^4 b_{194}) \text{Sh}(\beta, \eta) + (b_{185} + \eta \beta_{202} + \eta^5 b_{201}) \text{Sh}(\beta, \eta) + (b_{185} + \eta \beta_{205} 
+ \eta^2 b_{209}) \text{Sh}(\beta, \eta) + (b_{176} + \eta \beta_{210}) \text{Ch}(2 \beta, \eta) + (b_{179} + b_{212} + \eta^2 b_{211}) \text{Sh}(2 \beta, \eta) 

In the case of \( \beta = 0 \) and \( S_0=0 \) the results are in good agreement with that of Sudha(22a)

5. STRESS, NUSSELT NUMBER AND SHERHWOOD NUMBER

The shear stress on the boundaries \( y = \pm 1 \) are given by

\[ \tau^* = (\frac{\mu}{\delta} \frac{du}{dy})_{y=\pm 1} \]

which in the non-dimensional form reduces to

\[ \tau = \frac{\tau^*}{(v^2/L^2)} = (\frac{du}{dy})_{y=\pm 1} \]

\[ = (\frac{du_0}{dy} + \delta \frac{du_1}{dy} + \delta^2 \frac{du_2}{dy})_{y=\pm 1} \]

and the corresponding expressions are

\[ \tau_{y=1} = b_3 + \delta b_3 + \delta^2 b_3 + \ldots \]

\[ \tau_{y=-1} = b_3 + \delta b_3 + \delta^2 b_3 + \ldots \]

The rate of heat transfer(Nusselt Number) on the boundaries \( y = \pm 1 \) are given by

169
The rate of mass transfer (Sherwood Number) on the boundaries \( y = \pm 1 \) are given by

\[
Nu_{y = \pm 1} = \left( \frac{d\theta_0}{dy} + \delta \frac{d\theta_1}{dy} + \delta^2 \frac{d\theta_2}{dy} \right)_{y = \pm 1}
\]

and the corresponding expressions are

\[
Nu_{y = 1} = b_7 \\
Nu_{y = -1} = b_8
\]

The rate of mass transfer (Sherwood Number) on the boundaries \( y = \pm 1 \) are given by

\[
Sh_{y = \pm 1} = \left( \frac{dC_0}{dy} + \delta \frac{dC_1}{dy} + \delta^2 \frac{dC_2}{dy} \right)_{y = \pm 1}
\]

and the corresponding expressions are

\[
Sh_{y = 1} = b_9 \\
Sh_{y = -1} = b_{10}
\]

Where \( b_1, b_2 \ldots \ldots \ b_{10} \) are constants given in appendix.
6. DISCUSSION OF NUMERICAL RESULTS

In this analysis we investigate the convective heat and mass transfer of a viscous, electrically conducting, chemically reacting fluid in a vertical wavy channel. The walls are taken in the form $\eta=1+\beta \exp(-x^2)$. The channel is converging or diverging according as $\beta>0$ or $\beta<0$. We confine our attention for $\beta>0$. The velocity, temperature and concentration are shown in figs 1-25 for different values of $G$, $M$, $D^1$, $\alpha$, $Sc$, $So$, $N$ and $x$. It is found that from fig 1 that the actual axial flow is negative and hence $u>0$ represents the reversal flow. The velocity exhibits reversal flow for $G<0$ and no such flow exists for $G>0$. The region of reversal flow enlarges with increase in $|G|(<0)$. Also $|u|$ enhances with increase in $|G|(<0)$. The variation of $u$ with $D^1$ and $M$ shows that lesser the permeability of the porous medium higher the Lorentz force smaller $|u|$ in the entire flow region (fig-2). Fig-3 represents the variation of $u$ with heat source parameter $\alpha(\geq 0)$ fixing the heat parameter. We find that the reversal flow for $\alpha<0$ and the region of reversal flow enlarges with increase in $|\alpha|(<0)$. Also $|u|$ experiences an enhancement with increase in the strength of the heat source/sink. The effect of waviness of the channel walls on $u$ is shown in fig-4. It is found that higher the dilation of the channel walls larger $|u|$ everywhere in the region (fig-4). The variation of $u$ with $Sc$ & $So$ shows that lesser the molecular diffusivity larger $|u|$ in the region. An increase in $|So|(<0)$ results in an enhancement in $|u|$. $|u|$ experiences a depreciation with increase in $k\leq 1$ and enhances with $k\geq 2$. Also we notice reversal flow in the entire flow region for $k=2$. Moving along the axial direction the axial velocity depreciates in the flow region (fig-6).

The secondary velocity ($v$) which is due to the waviness of the boundaries shown in figs 7-12 for different variations. It is found that the secondary velocity is towards the mid region for all $|G|$, except for $|G|\geq 2 \times 10^2$, at which it is towards the
Fig-1: Variation of $u$ with $G$
$M=0.2 \alpha=2, K=0.5, N=1$

<table>
<thead>
<tr>
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<th>I</th>
<th>II</th>
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<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>$10^3$</td>
<td>$3\times10^3$</td>
<td>$5\times10^3$</td>
<td>$-10^3$</td>
<td>$-3\times10^3$</td>
<td>$-5\times10^3$</td>
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Fig-2: Variation of $u$ with $M$ & $D^{-1}$
$\alpha=2, K=0.5, N=1$

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$M$</td>
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<td>0.2</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$D^{-1}$</td>
<td>$10^3$</td>
<td>$10^2$</td>
<td>$10^2$</td>
<td>$3\times10^2$</td>
<td>$5\times10^2$</td>
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</tbody>
</table>
Fig-3: Variation of $u$ with $\alpha$
$M=0.2, K=0.5, N=1$

<table>
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<tr>
<th>$\alpha$</th>
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<tr>
<td>II</td>
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<td>III</td>
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<td>V</td>
<td></td>
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Fig-4: Variation of $u$ with $\beta$
$\alpha=2, K=0.5, N=1$

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.9</th>
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<tr>
<td>IV</td>
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</table>
Fig-5: Variation of $u$ with Sc & So
$a=2, K=0.5, N=1$

<table>
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<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
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<tbody>
<tr>
<td>Sc</td>
<td>2.01</td>
<td>1.3</td>
<td>0.6</td>
<td>0.24</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
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<tr>
<td>So</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>-0.5</td>
<td>-1</td>
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</tbody>
</table>

Fig-6: Variation of $u$ with K & x
$M=0.2, \alpha=2, N=1$

<table>
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<th>VII</th>
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</tr>
<tr>
<td>x</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\pi}{2}$</td>
<td>$\pi$</td>
<td>$2\pi$</td>
<td></td>
</tr>
</tbody>
</table>
Fig-7: Variation of $v$ with $G$
\[ \alpha = 2, \ K = 0.5, \ N = 1 \]
\[ \begin{array}{cccccc}
G & 10^3 & 3 \times 10^3 & 5 \times 10^3 & -10^3 & -3 \times 10^3 & -5 \times 10^3 \\
I & & IV & V & VI & \\
\end{array} \]

Fig-8: Variation of $v$ with $M$ & $D'$
\[ \alpha = 2, \ K = 0.5, \ N = 1 \]
\[ \begin{array}{cccc}
M & 2 & 4 & 6 & 2 & 2 \\
D' & 10^2 & 10^2 & 10^2 & 3 \times 10^2 & 5 \times 10^2 \\
\end{array} \]
Fig-9: Variation of $v$ with $\alpha$
M=0.2, K=0.5, N=1

$\alpha$  2  4  6  -4  -6

Fig-10: Variation of $v$ with $\beta$
$\alpha$=2, K=0.5, N=1

$\beta$  0.3  0.5  0.7  0.9
Fig-11. Variation of $v$ with $S_c$&$S_o$

$\alpha=2$, $K=0.5$, $N=1$

<table>
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<th>III</th>
<th>IV</th>
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<th>VII</th>
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<tr>
<td>$S_c$</td>
<td>2.01</td>
<td>1.3</td>
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<td>1.3</td>
<td>1.3</td>
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<tr>
<td>$S_o$</td>
<td>0.5</td>
<td>0.5</td>
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<td>1</td>
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Fig12. Variation of $v$ with $K$ & $x$

$M=0.2$, $\alpha=2$, $N=1$

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<td>$\frac{\pi}{2}$</td>
<td>$\pi$</td>
<td>$2\pi$</td>
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Fig-15: Variation of $\theta$ with $\alpha$
$M=0.2$, $K=0.5$, $N=1$

<table>
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<td>6</td>
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<td>-4</td>
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<tr>
<td>-6</td>
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Fig-16: Variation of $\theta$ with $\beta$
$\alpha=2$, $K=0.5$, $N=1$

<table>
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<th>$\beta$</th>
<th>I</th>
<th>II</th>
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<tbody>
<tr>
<td>0.3</td>
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<tr>
<td>0.5</td>
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<tr>
<td>0.7</td>
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<tr>
<td>0.9</td>
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Fig-17: Variation of $\theta$ with $Sc\&So$

$\alpha=2, K=0.5, N=1$

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<tr>
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<th>III</th>
<th>IV</th>
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<tr>
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<td>2.01</td>
<td>1.3</td>
<td>0.6</td>
<td>0.24</td>
<td>1.3</td>
<td>1.3</td>
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<tr>
<td>$So$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>-0.5</td>
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Fig-18: Variation of $\theta$ with $K\&x$

$M=0.2, \alpha=2, N=1$

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<th>VII</th>
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<tbody>
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<td>$K$</td>
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<td>0.5</td>
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<tr>
<td>$x$</td>
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<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>$2\pi$</td>
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Fig-19: Variation of $c$ with $G$
$\alpha=2, K=0.5, N=1$

<table>
<thead>
<tr>
<th>$G$</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>-0.2</th>
<th>-0.3</th>
<th>-0.4</th>
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Fig-20: Variation of $c$ with $M$ & $D^{-1}$
$\alpha=2, K=0.5, N=1$

<table>
<thead>
<tr>
<th>$M$</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{-1}$</td>
<td>$10^2$, $10^4$, $10^6$, $3\times10^2$, $5\times10^2$</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>
boundary. $|v|$ enhances with increase in $G>0$ while it depreciates with $|G|<3\times10^2$ and enhances with higher $|G|>5\times10^3$ (fig 7). Lesser the permeability of the porous medium / higher the Lorentz force smaller $|v|$ in the flow region (fig 8). Fig 9 shows that for $a>0$, $v$ is towards the mid region and is towards the boundary for $a<0$ and its magnitude enhances with $|a|$ ($>0$). Also higher the dilation of the channel walls larger $|v|$ in the flow region (Fig 10). Lesser the molecular diffusivity larger $|v|$ and also $|v|$ enhances marginally with $|S_o|(<0)$ (fig 11). An increase in the chemical parameter $K$ leads to a depreciation in $|v|$. Moving along the axial direction the secondary velocity reduces marginally in the flow region (fig 12).

The non-dimensional temperature distribution ($\theta$) is shown in figs. 13-18 for different values of $G$, $M$, $D^{-1}$, $\alpha$, $\beta$, $S_c$, $S_o$, $K$ & $X$. We follow the convention that the temperature is positive or negative according as the actual temperature is greater/lesser than the equilibrium temperature. It is found from fig 13 that for $G>0$ $\theta$ is positive except in the mid- region and this region enlarges with increase in $G>0$ while for $G<0$ $\theta$ is positive in the entire flow region. The actual temperature depreciates with increase in $G>0$ and enhances with $|G|$ in the entire flow region. Fig 14 represents the variation of $\theta$ with different $M$ and $D^{-1}$. The region of transition from positive to negative spreads towards the boundary with increase in $M$ & $D^{-1}$. Also lesser the permeability of the porous medium larger the actual temperature and higher the Lorentz force smaller the actual temperature in the flow region. The influence of heat source parameter on $\theta$ is exhibited in fig 15. With increase in the strength of heat sources/sinks the region of transition enlarges towards the boundaries. Also the actual temperature decreases with increase in the strength of heat source/sinks. The depreciation in the actual temperature is more
Fig-22: Variation of $\theta$ with $P$
$a=2$, $K=0.5$, $N=1$

I  II  III  IV
$p$  0.3  0.5  0.7  0.9

Fig-21: Variation of $\theta$ with $a$
$M=0.2$, $K=0.5$, $N=1$

I  II  III  IV  V
$a$  2  4  6  -4  -6

$\beta$  0.3  0.5  0.7  0.9
Fig-23: Variation of $c$ with $Sc$ & $So$

$\alpha=2$, $K=0.5$, $N=1$

<table>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
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<td>1.3</td>
<td>0.6</td>
<td>0.24</td>
<td>1.3</td>
<td>1.3</td>
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<td>0.5</td>
<td>1.0</td>
<td>-0.5</td>
<td>-1.0</td>
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Fig-24: Variation of $c$ with $N$

$M=0.2$, $\alpha=2$, $K=0.5$

<table>
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<tbody>
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<td>$N$</td>
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<td>-0.5</td>
<td>-0.8</td>
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</tbody>
</table>
Fig-25: Variation of c with K & x
M=0.2, α=2, G=N=1
I II III IV V VI
K 0.5 1 2 0.5 0.5 0.5
X -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8
y 0
predominant with enhancement in $\alpha < 0$ than in the case of heat sources. The influence of wall waviness on $\theta$ is shown in fig-16. It is found that higher the dilation of channel walls smaller the actual temperature everywhere in the region. From fig-17, we find that lesser the molecular diffusivity, smaller the actual temperature in the flow region. An increase in the soret parameter $S_0 > 0$ enlarges the transition region while no such phenomenon is observed with $S_0 < 0$. An increase in $S_0 > 0$ results in a depreciation of actual temperature while it enhances with $|S_0|(<0)$. The variation of $\theta$ with chemical reaction parameter $k$ and axial distance $x$ is shown in fig-18. The actual temperature experiences an enhancement with increase in chemical reaction parameter $k$. Moving along the axial direction, the actual temperature enhances except near the boundaries $\eta= \pm 1$ with $x \leq \pi/2$ while it depreciates with higher $x \geq \pi$.

The non-dimension concentration distribution ($C$) is exhibited in fig19-25 for different $G$, $M$, $D^1$, $\alpha$, $\beta$, $S_c$, $S_0$, $N$, $k$ & $x$. The concentration is positive or negative according as the actual concentration is greater or lesser than the equilibrium concentration. From fig-19, we notice that the concentration is negative for $G>0$ and positive for $G<0$. The actual concentration experiences an enhancement with increase in $|G|(<0)$. Lesser the permeability of the porous medium / higher the Lorentz force smaller the actual concentration in the entire flow region. The depreciation in the actual concentration with $D^1$ is more predominant than that with $M$(fig-20). The influence of heat sources on $C$ is shown in fig-21. We notice that for all values of $\alpha(<0)$ the actual concentration is always lesser than the equilibrium concentration. Also the actual concentration reduces with increase in the strength of either heat sources or sinks except that $C$ is positive in the vicinity of $\alpha=2$ and $\alpha=-4$. From fig-22, we find that higher the
### Table 1
Nusselt Number at $\eta=1$

<table>
<thead>
<tr>
<th>$G$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>-5.7299</td>
<td>-19.2842</td>
<td>-3.0542</td>
<td>-5.4967</td>
<td>0.7615</td>
</tr>
<tr>
<td>3x10^3</td>
<td>-0.6766</td>
<td>-1.1819</td>
<td>-1.0669</td>
<td>-1.16796</td>
<td>-0.6814</td>
</tr>
<tr>
<td>5x10^3</td>
<td>-0.2486</td>
<td>-0.6835</td>
<td>-0.5126</td>
<td>-0.1223</td>
<td>-0.6524</td>
</tr>
<tr>
<td>-10^3</td>
<td>0.5493</td>
<td>0.1179</td>
<td>-0.2800</td>
<td>-0.1862</td>
<td>-0.5237</td>
</tr>
<tr>
<td>-3x10^3</td>
<td>0.4889</td>
<td>0.06242</td>
<td>-0.3233</td>
<td>-0.2339</td>
<td>-0.54101</td>
</tr>
<tr>
<td>-5x10^3</td>
<td>0.4493</td>
<td>0.0255</td>
<td>-0.3514</td>
<td>-0.2653</td>
<td>-0.55124</td>
</tr>
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### Table 2
Nusselt Number at $\eta=1$

<table>
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<tr>
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<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>-5.7299</td>
<td>0.2541</td>
<td>0.4797</td>
<td>1.2026</td>
<td>1.0088</td>
<td>0.9344</td>
</tr>
<tr>
<td>3x10^3</td>
<td>-0.6766</td>
<td>0.3764</td>
<td>0.5303</td>
<td>1.1847</td>
<td>0.9929</td>
<td>0.9208</td>
</tr>
<tr>
<td>5x10^3</td>
<td>-0.2486</td>
<td>0.4232</td>
<td>0.5521</td>
<td>1.1739</td>
<td>0.9839</td>
<td>0.9134</td>
</tr>
<tr>
<td>-10^3</td>
<td>0.5493</td>
<td>0.6507</td>
<td>0.6852</td>
<td>0.9213</td>
<td>0.8505</td>
<td>0.8199</td>
</tr>
<tr>
<td>-3x10^3</td>
<td>0.4889</td>
<td>0.6181</td>
<td>0.6628</td>
<td>1.0255</td>
<td>0.8921</td>
<td>0.8462</td>
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<tr>
<td>-5x10^3</td>
<td>0.4493</td>
<td>0.5992</td>
<td>0.6504</td>
<td>1.0606</td>
<td>0.9095</td>
<td>0.8579</td>
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</table>

### Table 3
Nusselt Number at $\eta=1$

<table>
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<tr>
<th>$G$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^3</td>
<td>-0.3171</td>
<td>-0.7299</td>
<td>1.5672</td>
<td>1.1572</td>
<td>0.173</td>
<td>0.7386</td>
<td>0.6496</td>
</tr>
<tr>
<td>3x10^3</td>
<td>0.0333</td>
<td>-0.6766</td>
<td>2.5033</td>
<td>1.3170</td>
<td>0.1839</td>
<td>0.7179</td>
<td>0.6279</td>
</tr>
<tr>
<td>5x10^3</td>
<td>0.1409</td>
<td>-0.2486</td>
<td>5.8319</td>
<td>1.4891</td>
<td>0.2449</td>
<td>0.7048</td>
<td>0.6154</td>
</tr>
<tr>
<td>-10^3</td>
<td>0.5318</td>
<td>0.5493</td>
<td>0.5832</td>
<td>0.6177</td>
<td>0.5228</td>
<td>0.1408</td>
<td>0.3922</td>
</tr>
<tr>
<td>-3x10^3</td>
<td>0.4861</td>
<td>0.4889</td>
<td>0.4953</td>
<td>0.5032</td>
<td>0.4847</td>
<td>0.4558</td>
<td>0.4701</td>
</tr>
<tr>
<td>-5x10^3</td>
<td>0.4583</td>
<td>0.4493</td>
<td>0.4274</td>
<td>0.3954</td>
<td>0.4624</td>
<td>0.5293</td>
<td>0.5003</td>
</tr>
</tbody>
</table>

| $Sc$  | 2.01      | 1.30      | 0.6       | 0.24       | 1.30      | 1.30       | 1.30       |
| $So$  | 0.5       | 0.5       | 0.5       | 0.5        | 1.0       | -0.5       | -1.0       |
### Table 4
**Nusselt Number at η=1**

\[ \text{M}=2, \, \alpha=2, \, N=1, \, S_{0}=0.5 \]

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^3)</td>
<td>-13.7253</td>
<td>-5.7299</td>
<td>-3.7266</td>
<td>0.8043</td>
<td>0.7972</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>-1.0695</td>
<td>-0.6766</td>
<td>-0.3997</td>
<td>0.7479</td>
<td>0.7480</td>
</tr>
<tr>
<td>(5\times10^3)</td>
<td>-0.4781</td>
<td>-0.2486</td>
<td>-0.0908</td>
<td>0.6615</td>
<td>0.6941</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>0.5203</td>
<td>0.5493</td>
<td>0.4974</td>
<td>0.9138</td>
<td>0.9563</td>
</tr>
<tr>
<td>(-3\times10^3)</td>
<td>0.4490</td>
<td>0.4889</td>
<td>0.4523</td>
<td>0.9284</td>
<td>0.9888</td>
</tr>
<tr>
<td>(-5\times10^3)</td>
<td>0.4019</td>
<td>0.4493</td>
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<td>0.9406</td>
<td>1.0189</td>
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<table>
<thead>
<tr>
<th>β</th>
<th>0.3</th>
<th>0.5</th>
<th>0.7</th>
<th>0.5</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

### Table 5
**Nusselt Number at η=1**

\[ \text{M}=2, \, \alpha=2, \, K=0.5, \, N=1, \, \beta=0.5 \]

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^3)</td>
<td>-5.7299</td>
<td>4.2352</td>
<td>0.9815</td>
<td>0.3789</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>-0.6766</td>
<td>-1.7793</td>
<td>3.1847</td>
<td>3.1731</td>
</tr>
<tr>
<td>(5\times10^3)</td>
<td>-0.2486</td>
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<td>1.8699</td>
<td>2.4701</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>0.5493</td>
<td>0.4238</td>
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<td>-2.0117</td>
</tr>
<tr>
<td>(-3\times10^3)</td>
<td>0.4889</td>
<td>0.3395</td>
<td>-2.1976</td>
<td>-2.2034</td>
</tr>
<tr>
<td>(-5\times10^3)</td>
<td>0.4493</td>
<td>0.2831</td>
<td>-2.3429</td>
<td>-2.3492</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>(\pi/4)</th>
<th>(\pi/2)</th>
<th>(\pi)</th>
<th>(2\pi)</th>
</tr>
</thead>
</table>

### Table 6
**Nusselt Number (Nu) at η=-1**

\[ \alpha=2, \, K=0.5, \, N=1, \, \beta=0.5 \]

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>(10^3)</td>
<td>4.3000</td>
<td>5.3665</td>
<td>2.8016</td>
<td>3.6038</td>
<td>1.0491</td>
</tr>
<tr>
<td>(3\times10^3)</td>
<td>1.2089</td>
<td>1.3746</td>
<td>1.4714</td>
<td>1.0997</td>
<td>0.9064</td>
</tr>
<tr>
<td>(5\times10^3)</td>
<td>0.6657</td>
<td>1.1036</td>
<td>1.1575</td>
<td>0.4952</td>
<td>0.3496</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>-0.6352</td>
<td>-0.1842</td>
<td>0.2564</td>
<td>0.1494</td>
<td>0.5551</td>
</tr>
<tr>
<td>(-3\times10^3)</td>
<td>-0.52002</td>
<td>-0.2746</td>
<td>0.3462</td>
<td>0.2466</td>
<td>0.5993</td>
</tr>
<tr>
<td>(-5\times10^3)</td>
<td>-0.4464</td>
<td>-0.0639</td>
<td>0.4025</td>
<td>0.3084</td>
<td>0.6247</td>
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</tbody>
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<table>
<thead>
<tr>
<th>M</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>(10^2)</td>
<td>(10^2)</td>
<td>(10^2)</td>
<td>(3\times10^2)</td>
<td>(5\times10^2)</td>
</tr>
</tbody>
</table>
### Table 7

Nusselt Number (Nu) at $\eta=-1$

<table>
<thead>
<tr>
<th>$G$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>4.3000</td>
<td>-0.1160</td>
<td>-0.3505</td>
<td>-1.1993</td>
<td>-1.0417</td>
<td>-0.9649</td>
</tr>
<tr>
<td>$3\times 10^3$</td>
<td>1.2089</td>
<td>-0.2058</td>
<td>-0.4321</td>
<td>-1.2036</td>
<td>-1.0584</td>
<td>-0.9500</td>
</tr>
<tr>
<td>$5\times 10^3$</td>
<td>0.6657</td>
<td>-0.2819</td>
<td>-0.4676</td>
<td>-1.2062</td>
<td>-1.0209</td>
<td>-0.9419</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>-0.6352</td>
<td>-0.6686</td>
<td>-0.6879</td>
<td>-1.2819</td>
<td>-0.9044</td>
<td>-0.8371</td>
</tr>
<tr>
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<td>-0.5202</td>
<td>-0.6114</td>
<td>-0.6503</td>
<td>-1.2470</td>
<td>-0.9418</td>
<td>-0.8669</td>
</tr>
<tr>
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<td>-0.4464</td>
<td>-0.5785</td>
<td>-0.6296</td>
<td>-1.2365</td>
<td>-0.9571</td>
<td>-0.8801</td>
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</table>

### Table 8

Nusselt Number (Nu) at $\eta=-1$

<table>
<thead>
<tr>
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<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>0.7608</td>
<td>4.3001</td>
<td>-2.9181</td>
<td>-1.8287</td>
<td>0.3016</td>
<td>-0.9211</td>
<td>-0.7495</td>
</tr>
<tr>
<td>$3\times 10^3$</td>
<td>0.2707</td>
<td>1.2089</td>
<td>-5.8371</td>
<td>-2.1252</td>
<td>0.0429</td>
<td>-0.8662</td>
<td>-0.7008</td>
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<tr>
<td>$5\times 10^3$</td>
<td>0.1036</td>
<td>0.6657</td>
<td>-4.568</td>
<td>-2.4487</td>
<td>-0.0575</td>
<td>-0.8324</td>
<td>-0.6730</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>-0.5838</td>
<td>-0.6352</td>
<td>-0.7400</td>
<td>-0.8537</td>
<td>-0.5579</td>
<td>0.2417</td>
<td>-0.2273</td>
</tr>
<tr>
<td>$-3\times 10^3$</td>
<td>-0.4962</td>
<td>-0.5202</td>
<td>-0.5765</td>
<td>-0.6519</td>
<td>-0.4848</td>
<td>-0.2778</td>
<td>-0.3734</td>
</tr>
<tr>
<td>$-5\times 10^3$</td>
<td>-0.4438</td>
<td>-0.4464</td>
<td>-0.4531</td>
<td>-0.4636</td>
<td>-0.4427</td>
<td>-0.4255</td>
<td>-0.4325</td>
</tr>
</tbody>
</table>

| Sc  | 2.01 | 1.30 | 0.6  | 0.24  | 1.30  | 1.30  |
| So  | 0.5  | 0.5  | 0.5  | 0.5   | 1.0   | -0.5  | -1.0  |

### Table 9

Nusselt Number (Nu) at $\eta=-1$

<table>
<thead>
<tr>
<th>$G$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>4.8688</td>
<td>4.3001</td>
<td>3.9185</td>
<td>-1.6024</td>
<td>-1.4076</td>
</tr>
<tr>
<td>$3\times 10^3$</td>
<td>1.4555</td>
<td>1.2089</td>
<td>1.0457</td>
<td>-1.8764</td>
<td>-1.4774</td>
</tr>
<tr>
<td>$5\times 10^3$</td>
<td>0.8391</td>
<td>0.6657</td>
<td>0.5679</td>
<td>-2.3529</td>
<td>-1.5555</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>-0.6581</td>
<td>-0.6352</td>
<td>-0.5459</td>
<td>-1.1358</td>
<td>-1.1889</td>
</tr>
<tr>
<td>$-3\times 10^3$</td>
<td>-0.5245</td>
<td>-0.5202</td>
<td>-0.4491</td>
<td>-1.0792</td>
<td>-1.1456</td>
</tr>
<tr>
<td>$-5\times 10^3$</td>
<td>-0.4389</td>
<td>-0.4164</td>
<td>-0.3869</td>
<td>-1.0332</td>
<td>-1.1058</td>
</tr>
</tbody>
</table>

| $\beta$ | 0.3  | 0.5  | 0.7  | 0.5   | 0.5   |
| $K$     | 0.5  | 0.5  | 0.5  | 1.0   | 2.0   |
dilation of channel walls smaller the actual concentration in the entire flow field. Thus the wall waviness of the boundary effects the concentration remarkably. Fig-23 represents the variation of concentration with Schmidt number \( S_c \) and soret parameter \( S_0 \).

For smaller values of \( S_c \leq 0.6 \) the concentration is positive in the flow region and \( S_c > 1.3 \) the concentration is negative in entire flow region. Also for \( S_0 > 0 \) the concentration is negative in entire flow region and for \( S_0 < 0 \) the region of transition which appears in vicinity of \( \eta = -1 \) extends towards mid-region with increase in \( |S_0| \). Lesser the molecular diffusivity smaller the actual concentration in the entire flow field. An increase in the soret parameter \( |S_0| \) leads to a depreciation in the actual concentration everywhere in the flow region. Fig.24 exhibits the variation of \( C \) with buoyancy ratio \( N \). When molecular buoyancy force dominates over the thermal buoyancy force, the actual concentration enhances when the buoyancy forces act in the same direction while the forces acting in opposite directions it experiences a depreciation in the flow region. The variation of \( C \) with chemical reaction parameter \( k \) and axial distance ‘\( x \)’ exhibits an increasing tendency in the entire flow region with increase chemical reaction parameter \( k \). Moving along the axial direction the actual concentration experiences an enhancement in the flow region (fig-25).

The Nusselt number (\( Nu \)) at \( \eta = \pm 1 \) are evaluated for different values of \( G \), \( D^{-1} \), \( M \), \( \alpha \), \( S_c \), \( S_0 \), \( \beta \), \( k \) & \( x \). The variation of \( Nu \) with Grashoff number \( G \) shows that \( |Nu| \) experiences a depreciation with increase in \( |G| (\geq 0) \). Lesser the permeability of the porous medium smaller the magnitude of the rate of heat transfer at both the walls in the heating case while in the cooling case smaller \( |Nu| \) and for further lowering of the permeability larger \( |Nu| \) at \( \eta = \pm 1 \). Also higher the Lorentz force larger \( |Nu| \) at \( \eta = \pm 1 \) and for further
enhancement in the force, smaller $|\text{Nu}|$ in the heating case and a reversed effect is observed in the cooling case. At $\eta = -1$ higher the Lorentz force larger $|\text{Nu}|$ for $G > 0$ and smaller $|\text{Nu}|$ for $G < 0$ (tables 1&6). Tables 2 & 7 represent the variation of $\text{Nu}$ with heat source parameter $\alpha$. It is found that $|\text{Nu}|$ experience a depreciation with increase in the strength of the heat source parameter in $\alpha \leq 4$ and enhancement with higher $\alpha \geq 6$ at both the walls in the heating case while it enhances with $\alpha$ in the cooling case. An increase in the strength of the heat sink depreciates the rate of heat transfer at both the walls for $G < 0$. With reference to variation of $\text{Nu}$ with Schmidt number $\text{Sc}$, we find that lesser the molecular diffusivity smaller $|\text{Nu}|$ at $\eta = \pm 1$ and for further lowering of the diffusivity smaller $|\text{Nu}|$ in the heating case while a reversed effect is observed in the case of cooling of the channel walls. At $\eta = -1$ smaller the diffusivity larger $|\text{Nu}|$ and smaller $|\text{Nu}|$ for still lowering of diffusivity while it reduces with increase in $\text{Sc}$ in the case of cooling of the channel walls. An increase in the soret parameter $S_0(0)$ enhances $|\text{Nu}|$ at $\eta = \pm 1$ and reduces it at $\eta = -1$ for $G > 0$ while it reduces at both the walls for $G < 0$. An increase in $|S_0|(<0)$ reduces $|\text{Nu}|$ at $\eta = \pm 1$ in the heating case while it reduces in the cooling case (tables 3&8). The influence of the surface geometry on $\text{Nu}$ is exhibited in tables 4& 9. It is found that higher dilation smaller the rate of heat transfer at $\eta = \pm 1$ in the heating case while in the cooling case larger $|\text{Nu}|$ and for higher dilation smaller $|\text{Nu}|$ at $\eta = \pm 1$. At $\eta = -1$, $|\text{Nu}|$ depreciates with $k$ at $G = 10^3$ and at higher $G \geq 3 \times 10^3$ it enhances with $k \leq 1.0$ and depreciates with higher $k \geq 2.0$ and $G > 0$ and it experiences an enhancement in the cooling case. Moving along the axial direction the rate of heat transfer depreciates with $x$ at $G = 10^3$ and enhances at higher $G \geq 3 \times 10^3$ in the heating case and reduces with $x \leq \pi/2$ and
### Table 10

**Nusselt Number (Nu) at \( \eta = -1 \)**  
\( M = 2, \ a = 2, \ K = 0.5, \ N = 1, \ \beta = 0.5 \)

<table>
<thead>
<tr>
<th>( G )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>4.3001</td>
<td>5.3659</td>
<td>6.1642</td>
<td>10.4195</td>
</tr>
<tr>
<td>( 3 \times 10^3 )</td>
<td>1.2089</td>
<td>1.8219</td>
<td>7.0096</td>
<td>7.0341</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>0.6657</td>
<td>1.1277</td>
<td>5.3046</td>
<td>5.3212</td>
</tr>
<tr>
<td>( -10^3 )</td>
<td>-0.6352</td>
<td>-0.6358</td>
<td>2.6539</td>
<td>2.6939</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>-0.5202</td>
<td>-0.6738</td>
<td>2.8614</td>
<td>2.8713</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>-0.4464</td>
<td>-0.6706</td>
<td>2.9792</td>
<td>2.9893</td>
</tr>
<tr>
<td>( x )</td>
<td>( \pi/4 )</td>
<td>( \pi/2 )</td>
<td>( \pi )</td>
<td>( 2\pi )</td>
</tr>
</tbody>
</table>

### Table 11

**Sherwood Number (Sh) at \( \eta = 1 \)**  
\( a = 2, \ K = 0.5, \ N = 1, \ \beta = 0.5 \)

<table>
<thead>
<tr>
<th>( G )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>0.7701</td>
<td>0.3624</td>
<td>0.1684</td>
<td>0.1972</td>
<td>0.1667</td>
</tr>
<tr>
<td>( 3 \times 10^3 )</td>
<td>0.6158</td>
<td>0.3549</td>
<td>0.2113</td>
<td>0.2350</td>
<td>0.1932</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>0.5499</td>
<td>0.3515</td>
<td>0.2325</td>
<td>0.2536</td>
<td>0.2064</td>
</tr>
<tr>
<td>( -10^3 )</td>
<td>0.1530</td>
<td>0.3236</td>
<td>0.4152</td>
<td>0.4086</td>
<td>0.3249</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>0.2185</td>
<td>0.3287</td>
<td>0.3754</td>
<td>0.3754</td>
<td>0.2985</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>0.2549</td>
<td>0.3312</td>
<td>0.3552</td>
<td>0.3584</td>
<td>0.2853</td>
</tr>
<tr>
<td>( M )</td>
<td>2</td>
<td>5</td>
<td>10</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( D^1 )</td>
<td>( 10^2 )</td>
<td>( 10^2 )</td>
<td>( 10^2 )</td>
<td>( 3 \times 10^2 )</td>
<td>( 5 \times 10^2 )</td>
</tr>
</tbody>
</table>

### Table 12

**Sherwood Number (Sh) at \( \eta = 1 \)**  
\( M = 2, \ K = 0.5, \ N = 1, \ \beta = 0.5 \)

<table>
<thead>
<tr>
<th>( G )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^3 )</td>
<td>0.7701</td>
<td>0.5569</td>
<td>0.4989</td>
<td>0.0232</td>
<td>0.2148</td>
<td>0.2729</td>
</tr>
<tr>
<td>( 3 \times 10^3 )</td>
<td>0.6158</td>
<td>0.4990</td>
<td>0.4625</td>
<td>0.1361</td>
<td>0.2672</td>
<td>0.3073</td>
</tr>
<tr>
<td>( 5 \times 10^3 )</td>
<td>0.5499</td>
<td>0.4715</td>
<td>0.4449</td>
<td>0.1921</td>
<td>0.2935</td>
<td>0.3243</td>
</tr>
<tr>
<td>( -10^3 )</td>
<td>0.1530</td>
<td>0.2479</td>
<td>0.2899</td>
<td>0.6856</td>
<td>0.5402</td>
<td>0.4893</td>
</tr>
<tr>
<td>( -3 \times 10^3 )</td>
<td>0.2185</td>
<td>0.2924</td>
<td>0.3214</td>
<td>0.5781</td>
<td>0.4851</td>
<td>0.4531</td>
</tr>
<tr>
<td>( -5 \times 10^3 )</td>
<td>0.2549</td>
<td>0.3153</td>
<td>0.3372</td>
<td>0.5240</td>
<td>0.4579</td>
<td>0.4354</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>-2</td>
<td>-4</td>
<td>-6</td>
</tr>
</tbody>
</table>
**Table 13**

Sherwood Number (Sh) at $\eta = 1$

$M=2, \alpha=2, K=0.5, N=1, \beta=0.5$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>0.7644</td>
<td>0.7701</td>
<td>1.0248</td>
<td>0.8943</td>
<td>0.5093</td>
<td>0.0148</td>
<td>0.1779</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>0.6300</td>
<td>0.6158</td>
<td>0.5247</td>
<td>0.8897</td>
<td>0.4559</td>
<td>0.1121</td>
<td>0.2236</td>
</tr>
<tr>
<td>$5\times10^3$</td>
<td>0.5714</td>
<td>0.5499</td>
<td>0.4417</td>
<td>0.8854</td>
<td>0.4306</td>
<td>0.1603</td>
<td>0.2465</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.1955</td>
<td>0.1530</td>
<td>0.2479</td>
<td>0.9158</td>
<td>0.2237</td>
<td>0.5851</td>
<td>0.4616</td>
</tr>
<tr>
<td>$-3\times10^3$</td>
<td>0.2593</td>
<td>0.2185</td>
<td>0.2647</td>
<td>0.9222</td>
<td>0.2649</td>
<td>0.4927</td>
<td>0.4138</td>
</tr>
<tr>
<td>$-5\times10^3$</td>
<td>0.2942</td>
<td>0.2549</td>
<td>0.2754</td>
<td>0.9290</td>
<td>0.3861</td>
<td>0.4462</td>
<td>0.3902</td>
</tr>
</tbody>
</table>

| Sc   | 2.01 | 1.30 | 0.6 | 0.24 | 1.30 | 1.30 | 1.30 |
| So   | 0.5  | 0.5  | 0.5 | 0.5  | 1.0  | -0.5 | -1.0 |

**Table 14**

Sherwood Number (Sh) at $\eta = 1$

$M=2, \alpha=2, K=0.5, \beta=0.5$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>0.7701</td>
<td>0.7015</td>
<td>0.8362</td>
<td>0.8455</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>0.6158</td>
<td>0.5609</td>
<td>0.6792</td>
<td>0.6891</td>
</tr>
<tr>
<td>$5\times10^3$</td>
<td>0.5499</td>
<td>0.5080</td>
<td>0.6025</td>
<td>0.6110</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.1530</td>
<td>0.2533</td>
<td>-0.0444</td>
<td>-0.0239</td>
</tr>
<tr>
<td>$-3\times10^3$</td>
<td>0.2185</td>
<td>0.2889</td>
<td>0.0919</td>
<td>0.0620</td>
</tr>
<tr>
<td>$-5\times10^3$</td>
<td>0.2549</td>
<td>0.3096</td>
<td>0.1614</td>
<td>0.1399</td>
</tr>
</tbody>
</table>

| N     | 1     | 2     | -0.5  | -0.8  |

**Table 15**

Sherwood Number (Sh) at $\eta = 1$

$M=2, \alpha=2, K=0.5, N=1$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>0.7957</td>
<td>0.8101</td>
<td>0.8437</td>
<td>-9.0158</td>
<td>-6.4460</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>0.5828</td>
<td>0.6158</td>
<td>0.6950</td>
<td>-9.9050</td>
<td>-6.4768</td>
</tr>
<tr>
<td>$5\times10^3$</td>
<td>0.5030</td>
<td>0.5499</td>
<td>0.6276</td>
<td>-10.9587</td>
<td>-6.5077</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.1246</td>
<td>0.1530</td>
<td>0.1617</td>
<td>-6.5162</td>
<td>-6.3237</td>
</tr>
<tr>
<td>$-3\times10^3$</td>
<td>0.1773</td>
<td>0.2185</td>
<td>0.2467</td>
<td>-6.0667</td>
<td>-6.2933</td>
</tr>
<tr>
<td>$-5\times10^3$</td>
<td>0.2079</td>
<td>0.2549</td>
<td>0.2923</td>
<td>-8.6649</td>
<td>-6.2630</td>
</tr>
</tbody>
</table>

<p>| $\beta$ | 0.3 | 0.5 | 0.7 | 0.5 | 0.5 |
| $K$     | 0.5 | 0.5 | 0.5 | 1.0 | 2.0 |</p>
<table>
<thead>
<tr>
<th>$G$</th>
<th>$I$</th>
<th>$II$</th>
<th>$III$</th>
<th>$IV$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>0.7701</td>
<td>1.6083</td>
<td>-0.1899</td>
<td>-0.1895</td>
<td></td>
</tr>
<tr>
<td>$3 \times 10^3$</td>
<td>0.6158</td>
<td>0.5525</td>
<td>-0.2102</td>
<td>-0.2094</td>
<td></td>
</tr>
<tr>
<td>$5 \times 10^3$</td>
<td>0.5499</td>
<td>0.4442</td>
<td>-0.2341</td>
<td>-0.2328</td>
<td></td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.1530</td>
<td>0.1298</td>
<td>-0.1309</td>
<td>-0.1328</td>
<td></td>
</tr>
<tr>
<td>$-3 \times 10^3$</td>
<td>0.2185</td>
<td>0.1609</td>
<td>-0.1219</td>
<td>-0.1206</td>
<td></td>
</tr>
<tr>
<td>$-5 \times 10^3$</td>
<td>0.2549</td>
<td>0.1804</td>
<td>-0.1126</td>
<td>-0.1104</td>
<td></td>
</tr>
</tbody>
</table>

| $x$ | $\pi/4$ | $\pi/2$ | $\pi$ | $2\pi$ |

<table>
<thead>
<tr>
<th>$G$</th>
<th>$I$</th>
<th>$II$</th>
<th>$III$</th>
<th>$IV$</th>
<th>$V$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-1.9078</td>
<td>-1.0726</td>
<td>-0.8317</td>
<td>-0.8574</td>
<td>-0.8797</td>
</tr>
<tr>
<td>$3 \times 10^3$</td>
<td>-1.1012</td>
<td>-0.7988</td>
<td>-0.7309</td>
<td>-0.7309</td>
<td>-0.8511</td>
</tr>
<tr>
<td>$5 \times 10^3$</td>
<td>-0.8305</td>
<td>-0.6996</td>
<td>-0.6894</td>
<td>-0.6704</td>
<td>-0.8368</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.2581</td>
<td>0.2859</td>
<td>-0.3046</td>
<td>-0.3976</td>
<td>-0.7087</td>
</tr>
<tr>
<td>$-3 \times 10^3$</td>
<td>0.1209</td>
<td>-0.2698</td>
<td>-0.3855</td>
<td>-0.2932</td>
<td>-0.7371</td>
</tr>
<tr>
<td>$-5 \times 10^3$</td>
<td>0.0386</td>
<td>-0.1393</td>
<td>-0.4270</td>
<td>-0.3430</td>
<td>-0.7514</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M$</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^{-1}$</td>
<td>$10^2$</td>
<td>$10^2$</td>
<td>$10^2$</td>
<td>$3 \times 10^2$</td>
<td>$5 \times 10^2$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$G$</th>
<th>$I$</th>
<th>$II$</th>
<th>$III$</th>
<th>$IV$</th>
<th>$V$</th>
<th>$VI$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-1.9078</td>
<td>-0.5629</td>
<td>-0.3946</td>
<td>0.2287</td>
<td>0.0476</td>
<td>-0.0673</td>
</tr>
<tr>
<td>$3 \times 10^3$</td>
<td>-1.1012</td>
<td>-0.4389</td>
<td>-0.3279</td>
<td>0.1147</td>
<td>-0.0103</td>
<td>-0.0582</td>
</tr>
<tr>
<td>$5 \times 10^3$</td>
<td>-0.8308</td>
<td>-0.3810</td>
<td>-0.2955</td>
<td>0.0625</td>
<td>-0.0389</td>
<td>-0.0788</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.2581</td>
<td>0.0345</td>
<td>-0.0412</td>
<td>-0.2981</td>
<td>-0.2820</td>
<td>-0.2565</td>
</tr>
<tr>
<td>$-3 \times 10^3$</td>
<td>0.1209</td>
<td>-0.0458</td>
<td>-0.0948</td>
<td>-0.2311</td>
<td>-0.2289</td>
<td>-0.2160</td>
</tr>
<tr>
<td>$-5 \times 10^3$</td>
<td>0.0386</td>
<td>-0.0886</td>
<td>-0.1228</td>
<td>-0.1951</td>
<td>-0.2018</td>
<td>-0.1955</td>
</tr>
</tbody>
</table>

| $\alpha$ | 2 | 4 | 6 | -2 | -4 | -6 |
Table. 19
Sherwood Number (Sh) at $\eta = -1$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-1.0472</td>
<td>-1.9078</td>
<td>1.8619</td>
<td>0.5994</td>
<td>-0.6043</td>
<td>-0.0232</td>
<td>-0.1066</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>-0.7381</td>
<td>-1.1012</td>
<td>2.9545</td>
<td>0.5868</td>
<td>-0.4827</td>
<td>-0.1061</td>
<td>-0.1565</td>
</tr>
<tr>
<td>$5\times10^3$</td>
<td>-0.6119</td>
<td>-0.8305</td>
<td>6.4261</td>
<td>0.5744</td>
<td>-0.4258</td>
<td>-0.1441</td>
<td>-0.1813</td>
</tr>
<tr>
<td>$-10^4$</td>
<td>0.0660</td>
<td>0.2581</td>
<td>0.6497</td>
<td>0.6526</td>
<td>-0.0463</td>
<td>-0.4081</td>
<td>-0.3910</td>
</tr>
<tr>
<td>$-3\times10^4$</td>
<td>-0.0401</td>
<td>0.1209</td>
<td>0.5379</td>
<td>0.6666</td>
<td>-0.0956</td>
<td>-0.3588</td>
<td>-0.3451</td>
</tr>
<tr>
<td>$-5\times10^4$</td>
<td>-0.0106</td>
<td>0.0386</td>
<td>0.4513</td>
<td>0.6810</td>
<td>-0.1378</td>
<td>-0.3323</td>
<td>-0.3217</td>
</tr>
<tr>
<td>Sc</td>
<td>2.01</td>
<td>1.30</td>
<td>0.6</td>
<td>0.24</td>
<td>1.30</td>
<td>1.30</td>
<td>1.30</td>
</tr>
<tr>
<td>So</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>-0.5</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

Table. 20
Sherwood Number (Sh) at $\eta = -1$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-1.9078</td>
<td>-9.4536</td>
<td>-0.6886</td>
<td>-0.5858</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>-1.1012</td>
<td>-2.0371</td>
<td>-0.5595</td>
<td>-0.4952</td>
</tr>
<tr>
<td>$5\times10^3$</td>
<td>-0.8305</td>
<td>-1.2563</td>
<td>-0.4924</td>
<td>-0.4458</td>
</tr>
<tr>
<td>$-10^4$</td>
<td>0.2581</td>
<td>0.2789</td>
<td>0.1935</td>
<td>0.1704</td>
</tr>
<tr>
<td>$-3\times10^4$</td>
<td>0.1209</td>
<td>0.1585</td>
<td>0.0258</td>
<td>-0.021</td>
</tr>
<tr>
<td>$-5\times10^4$</td>
<td>0.0386</td>
<td>0.0798</td>
<td>-0.0548</td>
<td>-0.0297</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
<td>2</td>
<td>-0.5</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

Table. 21
Sherwood Number (Sh) at $\eta = -1$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-3.1934</td>
<td>-1.9078</td>
<td>-1.8090</td>
<td>1.9446</td>
<td>2.0523</td>
</tr>
<tr>
<td>$3\times10^3$</td>
<td>-1.2629</td>
<td>-1.1012</td>
<td>-1.1928</td>
<td>1.8867</td>
<td>2.0599</td>
</tr>
<tr>
<td>$5\times10^3$</td>
<td>-0.8356</td>
<td>-0.8305</td>
<td>-0.9570</td>
<td>1.9611</td>
<td>2.0677</td>
</tr>
<tr>
<td>$-10^4$</td>
<td>0.3341</td>
<td>0.2581</td>
<td>0.1868</td>
<td>1.5467</td>
<td>2.0214</td>
</tr>
<tr>
<td>$-3\times10^4$</td>
<td>0.2208</td>
<td>0.1209</td>
<td>0.0222</td>
<td>1.4844</td>
<td>2.0138</td>
</tr>
<tr>
<td>$-5\times10^4$</td>
<td>0.1491</td>
<td>0.0386</td>
<td>-0.0133</td>
<td>1.4238</td>
<td>2.0061</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$K$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>
Table 22
Sherwood Number (Sh) at $\eta = -1$

$M=2$, $a=2$, $K=0.5$, $N=1$, $\beta=0.5$

<table>
<thead>
<tr>
<th>G</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3$</td>
<td>-1.9078</td>
<td>7.5672</td>
<td>0.7380</td>
<td>0.7375</td>
</tr>
<tr>
<td>$3 \times 10^3$</td>
<td>-1.1012</td>
<td>-2.5705</td>
<td>0.7764</td>
<td>0.7754</td>
</tr>
<tr>
<td>$5 \times 10^3$</td>
<td>-0.8305</td>
<td>-1.1392</td>
<td>0.8184</td>
<td>0.8170</td>
</tr>
<tr>
<td>$-10^3$</td>
<td>0.2581</td>
<td>0.3811</td>
<td>0.6117</td>
<td>0.6130</td>
</tr>
<tr>
<td>$-3 \times 10^3$</td>
<td>0.1209</td>
<td>0.2962</td>
<td>0.5869</td>
<td>0.5873</td>
</tr>
<tr>
<td>$-5 \times 10^3$</td>
<td>0.0386</td>
<td>0.2384</td>
<td>0.5628</td>
<td>0.5634</td>
</tr>
<tr>
<td>$\times$</td>
<td>$\pi/4$</td>
<td>$\pi/2$</td>
<td>$\pi$</td>
<td>$2\pi$</td>
</tr>
</tbody>
</table>
enhances with $x > \pi$ in the cooling case. At $\eta = -1$, it experiences an enhancement with $x$ in both heating and cooling cases (Table 5 & 10).

The Sherwood number ($Sh$) which measures the rate of mass transfer at the boundaries is shown in tables 11-22 for different parameters $G$, $D^1, M, \alpha, S_0, S_m, k$ and $x$. The variation of $Sh$ with Grashoff number $G$ shows that the rate of mass transfer depreciates with increase in $G > 0$ at $\eta = \pm 1$ while an increase in $G < 0$ enhances $|Sh|$ at $\eta = +1$ and reduces at $\eta = -1$. Lesser the permeability of the porous medium lesser $|Sh|$ at $\eta = +1$ for $G > 0$ and for $G < 0$, lesser the magnitude of $Sh$ and for further lowering of the permeability smaller $|Sh|$. At $\eta = -1$, smallest $|Sh|$ and larger $|Sh|$ for still lowering of the permeability for $G > 0$ and for $G < 0$, larger $|Sh|$ at $\eta = -1$ (Table 11&17). The variation of $Sh$ with $\alpha$ is exhibited in tables 12 & 18. An increase in the strength of the heat source enhances $|Sh|$ at $\eta = +1$ in both heating and cooling of the channel walls and at $\eta = -1$, it depreciates with $\alpha > 0$ for $G > 0$ and enhances while $\alpha$ for $\alpha < 0$. Also an increment in $\alpha < 0$ enhances $|Sh|$ for $G > 0$ and for $G < 0$, it depreciates $|Sh|$ at $\eta = 1$, while at $\eta = -1$, it depreciate with $|\alpha| \leq 4$ and enhances with $|\alpha| > 6$ for all $|G| (<> 0)$. With reference to variation of $Sh$ with Schmidt number $S_c$ we find that the rate of heat transfer at $\eta = 1$, enhances with $S_c \leq 0.6$ and depreciates with higher $S_c \geq 2.01$. at $G = 10^3$ and higher $G \geq 3 \times 10^3$, $|Sh|$ reduces with $S_c \leq 0.6$ and enhances with higher $S_c \geq 2.01$ in the heating case while in the cooling case it reduces with $S_c \leq 1.3$ and enhances with $S_c \geq 2.01$. At $\eta = -1$, lesser the molecular diffusivity smaller $|Sh|$ and for further lowering of the diffusivity smaller $|Sh|$ for $G > 0$ and for $G < 0$, small $|Sh|$. An increase in $S_0 > 0$ enhances $|Sh|$ at $\eta = 1$ and reduces at $\eta = -1$ for all $|G| (\geq 0)$ while an increase in $|S_0| (< 0)$ enhances $|Sh|$ at $\eta = \pm 1$ for $G > 0$ and depreciates it for $G < 0$. (Tables 13&19). When the molecular buoyancy force dominates over the thermal buoyancy
force the rate of mass transfer reduces at $\eta=+1$ and enhances at $\eta=-1$ for $G>0$ and for $G<0$ enhances at $\eta=\pm1$ when the buoyancy forces act in the same direction and for the forces acting in opposite directions $|Sh|$ enhances for $G>0$ and depreciates for $G<0$ at $\eta=+1$ and at $\eta=-1$, it reduces for all $G$ (table 14&20). The effect of waviness of the boundary on $Sh$ is shown in tables 15&21. Higher the dilation larger $|Sh|$ at $\eta=1$ for all $G$ while at $\eta=-1$ smaller $|Sh|$ at $G=10^3$ and at higher $G \geq 2 \times 10^3$, smaller $|Sh|$ and for higher dilation larger $|Sh|$ and for $G<0$, smaller $|Sh|$. For an increase in the chemical reaction parameter $k$ we find that $|Sh|$ at $\eta=+1$ enhances with $k \leq 1.0$ and depreciates with $k \geq 2.0$ for all $G$ and for $G<0$, $|Sh|$ experiences an enhancement with $k$ for all $G$. Moving along the axial direction the rate of mass transfer at $\eta=+1$ depreciates for $G>0$ and for $G<0$ it depreciates with $x<\pi/2$ and enhances with $x \geq \pi$ at $|G|=10^3$ and at higher $|G| \geq 2 \times 10^3$, depreciates with $x$. At $\eta=-1$ it enhances with $x \leq \pi/2$ and reduces with higher $x \geq \pi$ in the heating case and enhances with $x$ in the cooling case (Tables 16&20).

7. REFERENCES


180
\[ a_1 = \frac{1}{2} \]
\[ a_3 = \frac{a_1}{\beta_1 \cosh \beta_1} \]
\[ a_5 = \frac{G}{4\beta_1^2} \]
\[ a_7 = \frac{1}{\beta_1^2 (\beta_1^2 - \beta_2^2)} \]
\[ a_9 = \frac{-a_{14}}{\beta_2 \sinh \beta_2} \]
\[ a_{11} = -a_{13} - \beta_2 a_{10} \cosh \beta_2 \]
\[ a_{14} = \frac{1}{2} \left[ -a_6' \cosh \beta_2 + a_5 \frac{\beta_2}{f} + 1 \sinh \beta_2 + \cosh \beta_2 + \frac{a_7 \beta_3 f' \sinh \beta_1}{f} + a'_5 - \frac{2a_5 f''}{f} \right] \]
\[ a_{15} = \alpha f^2 x + 0.5 - a_{10} \sinh \beta_2 + \frac{a_{10} \beta_2}{f} + \frac{1}{f} \cosh \beta_2 + \frac{a_7 \beta_3 f' \sinh \beta_1}{f} + a'_5 - \frac{2a_5 f''}{f} \]
\[ a_{16} = \alpha f^2 x + 0.5 \left( -a'_6 - \frac{2a_5 f''}{f} \right) \]
\[ a_{17} = \alpha f^2 x \left( -a'_5 - \frac{2a_5 f''}{f} \right) \]
\[ a_{18} = \alpha f^2 x \left( -a'_6 - \frac{3a_6 f''}{f} \right) \]
\[ a_{19} = 0.5 \{ a'_6 \} \]
\[ a_{20} = 0.5 \left( -a_7 \beta_3 f' + \alpha f^2 a'_6 \right) \]
\[ a_{21} = 0.5 \{ a'_6 \} \]
\[ a_{22} = 0.5 \left( -a_8 \beta_3 f' + \alpha f^2 a'_6 \right) \]
\[ a_{23} = \alpha f^2 \frac{a_8 \beta_3 f'}{f} \]

181
\[ a_{34} = -\alpha f' + \frac{\alpha_s \beta_2 f'}{f} \quad a_{35} = 0.5 \times a'_6 \]
\[ a_{36} = 0.5 \times a'_{10} \quad a_{37} = \alpha f' a'_9 + 0 + \frac{a_{10} \beta_2 f'}{f} \]
\[ a_{38} = 0.5 \times \frac{\alpha_s \beta_2 f'}{f} \quad a_{39} = \alpha f' x + \frac{a_{10} \beta_2 f'}{f} \]
\[ a_{40} = \alpha f' \left( -\beta_s a'_{10} \cosh \beta_2 + 3 a_6 + \beta_s a_6 \cosh \beta_2 \right) + \alpha f' (2 a_t) \]
\[ a_{41} = \frac{\pm f'}{f} (\beta_s a'_t) \quad a_{42} = \frac{+ f'}{f} (\beta_s a_8) \]
\[ a_{43} = \frac{+ f'}{f} (\beta_s a_t) \quad a_{44} = -2 a f' \times (\beta_s a_8) \]
\[ a_{45} = 2 a f' \beta_s a_4 \quad a_{46} = 2 a f' \beta_s a_t \]
\[ a_{47} = f p (a_{44} - a_{35}) \quad a_{48} = f p (a_{45} - a_{46}) \]
\[ a_{49} = f p (a_{46} - a_{35}) \quad a_{50} = f p (a_{47} - a_{35}) \]
\[ a_{51} = f p (a_{48} - a_{46}) \quad a_{52} = f p (a_{49} - a_{46}) \]
\[ a_{53} = f p (a_{47} - a_{46}) \]
\[
a_{44} = fp(a_{26} - a_{37}) \\
a_{56} = fp(a_{21} - a_{39}) \\
a_{58} = fp(a_{28} - a_{41}) \\
a_{60} = fp(a_{36} - a_{43}) \\
a_{62} = fp(a_{34} - a_{47}) \\
a_{64} = fp(a_{38} - a_{45}) \\
a_{66} = \frac{a_{39}}{6} \\
a_{68} = \frac{a_{31}}{20} \\
a_{70} = \frac{a_{33}}{\beta_2^2} \\
a_{72} = \frac{a_{35}}{\beta_2^2} \\
a_{76} = \frac{a_{33}}{\beta_2^2} - \frac{2a_{57}}{\beta_1^2} + \frac{6a_{63}}{\beta_1^4} \\
a_{72} = \frac{a_{35}}{\beta_2^2} - \frac{2a_{59}}{\beta_1^2} + \frac{6a_{63}}{\beta_1^4} \\
a_{74} = \frac{a_{37}}{\beta_2^2} - \frac{4a_{64}}{\beta_1^4} \\
a_{76} = \frac{a_{39}}{\beta_2^2} - \frac{4a_{62}}{\beta_1^4} \\
a_{78} = \frac{a_{61}}{\beta_2^2} \\
a_{80} = \frac{a_{61}}{\beta_2^2} \\
a_{1100} = -\beta_2 Ch \beta_2 a_{109} + 3a_{6} + \beta_1 Ch \beta_1 a_{6} \\
a_{1101} = -2a_{3} \\
a_{1102} = -3a_{6} \\
a_{1104} = \beta_2 a_{10} \\
a_{64} = \frac{a_{48}}{2} \\
a_{67} = \frac{a_{50}}{12} \\
a_{69} = \frac{a_{52}}{70} \\
a_{71} = \frac{a_{54}}{\beta_2^2} \\
a_{73} = \frac{a_{56}}{\beta_2^2} \\
a_{75} = \frac{a_{44}}{\beta_2^2} \\
a_{77} = \frac{2a_{58}}{\beta_2^2} + \frac{6a_{64}}{\beta_2^2} \\
a_{79} = \frac{2a_{60}}{\beta_2^2} + \frac{6a_{62}}{\beta_2^2} \\
\]
\[ a_{105} = \beta_1 a_7 \frac{Sh \beta_1}{Sh \beta_2} + \frac{2a_s}{Sh \beta_2} \]
\[ a_{106} = -\beta_1 a_7 \]
\[ a_{107} = 2a_{102} \]
\[ a_{108} = \beta_2 a_{103} \]
\[ a_{109} = \beta_2 a_{104} \]
\[ a_{110} = \beta_2 a_{105} \]
\[ a_{111} = \beta_2 a_{106} \]
\[ a_{112} = a'_{101} \]
\[ a_{113} = 2a'_{102} - \frac{2a_{102}f'}{f} \]
\[ a_{114} = \beta_2 a'_{103} \]
\[ a_{115} = \frac{\beta_2 f'}{f^2} a'_{103} \]
\[ a_{116} = \beta_2 a'_{104} \]
\[ a_{117} = \frac{\beta_2 f'}{f^2} a'_{104} \]
\[ a_{118} = \beta_2 a'_{105} \]
\[ a_{119} = -\beta_2 a'_{105} \frac{f'}{f} \]
\[ a_{120} = \beta_2 a'_{106} \]
\[ a_{121} = \frac{\beta_2 a'_{106} f'}{f} \]
\[ a_{122} = a_{112} a_{106} \]
\[ a_{123} = a_{112} a_{101} + a_{100} a_{113} \]
\[ a_{124} = a_{112} a_{102} + a_{113} a_{101} \]
\[ a_{125} = a_{113} a_{102} \]
\[ a_{126} = (a_{112} a_{103} + a_{100} a_{120}) \]
\[ a_{127} = a_{112} a_{104} + a_{100} a_{118} \]
\[ a_{128} = a_{112} a_{106} + a_{100} a_{114} \]
\[ a_{129} = a_{112} a_{105} + a_{100} a_{116} \]
\[ a_{130} = a_{113} a_{103} + a_{101} a_{130} \]
\[ a_{131} = a_{113} a_{104} + a_{101} a_{118} \]
\[ a_{132} = a_{113} a_{106} + a_{101} a_{114} \]
\[ a_{133} = a_{116} a_{101} + a_{113} a_{105} \]
\[ a_{134} = a_{114} a_{102} \]
\[ a_{135} = a_{102} a_{116} \]
\[ a_{136} = a_{102} a_{120} \]
\[ a_{137} = a_{102} a_{118} \]
\[ a_{138} = a_{102} a_{121} \]
\[ a_{139} = a_{115} a_{102} \]
\[ a_{140} = \frac{a_{114} a_{103}}{2} + \frac{a_{130} a_{106}}{2} \]
\[ a_{44} = \frac{a_{114}a_{106}}{2} \]  
\[ a_{43} = \frac{a_{121}a_{106}}{2} \]  
\[ a_{45} = \frac{a_{115}a_{103}}{2} \]  
\[ a_{47} = \frac{a_{116}a_{105}}{2} \]  
\[ a_{49} = \frac{a_{118}a_{104}}{2} \]  
\[ a_{51} = a_{114}a_{105} \]  
\[ a_{53} = a_{121}a_{105} \]  
\[ a_{55} = a_{120}a_{105} \]  
\[ a_{57} = a_{115}a_{105} \]  
\[ a_{58} = a_{114} + a_{141} + a_{143} + a_{144} + a_{145} + a_{147} + a_{149} \]  
\[ a_{59} = -a_{144} + a_{149} \]  
\[ a_{60} = -a_{143} + a_{145} \]  
\[ a_{61} = -a_{147} + a_{149} \]  
\[ a_{62} = -a_{146} + a_{148} \]  
\[ a_{63} = \left(\frac{a_{150} + a_{155}}{2}\right) \]  
\[ a_{64} = \frac{a_{152} + a_{157}}{2} \]  
\[ a_{65} = \frac{a_{150} + a_{155}}{2} \]  
\[ a_{66} = \frac{-a_{152} + a_{157}}{2} \]  
\[ a_{67} = \frac{a_{151} + a_{154}}{2} \]  
\[ a_{68} = \frac{-a_{153} + a_{156}}{2} \]  
\[ a_{69} = \frac{-a_{151} + a_{154}}{2} \]  
\[ a_{70} = \frac{-a_{153} + a_{156}}{2} \]  
\[ a_{71} = a_{10} C h \beta_2 + \frac{a_{10} \beta_2 f'}{f} S h \beta_2 - a_1 C h \beta_2 + \frac{a_1 \beta_2 f'}{f} S h \beta_1 - a_1 + 2 a_1 f' \]
\[ a_{172} = -a_{10}^{'} \beta_{2} + \frac{a_{10} \beta_{2} f'}{f} \ Ch \beta_{2} - a_{4}^{'} \beta_{1} \ Ch \beta_{1} - a_{0}^{'} + \frac{3a_{0}^{'} f'}{f} \]

\[ a_{173} = -a_{5}^{'} - \frac{2a_{5} f'}{f} \]

\[ a_{174} = -a_{6}^{'} - \frac{3a_{6} f'}{f} \]

\[ a_{175} = -a_{7}^{'} \]

\[ a_{176} = -\frac{a_{8} f^{'2}}{f} \]

\[ a_{177} = a_{8}^{'} \]

\[ a_{178} = \frac{-a_{10} \beta_{2} f'}{f} \]

\[ a_{179} = a_{9}^{'} \]

\[ a_{180} = -a_{10} \beta_{1} f' \]

\[ a_{181} = a_{10}^{'} \]

\[ a_{182} = -\frac{a_{10}^{'} \beta_{2} f'}{f} \]

\[ a_{183} = a_{171} a_{107} \]

\[ a_{184} = a_{172} a_{108} + a_{107} a_{176} + a_{172} a_{109} \]

\[ a_{185} = a_{173} a_{108} + a_{107} a_{176} \]

\[ a_{186} = a_{172} a_{107} + a_{171} a_{109} \]

\[ a_{187} = a_{174} a_{108} \]

\[ a_{188} = a_{172} a_{108} \]

\[ a_{189} = a_{174} a_{108} + a_{172} a_{109} \]

\[ a_{190} = a_{173} a_{109} \]

\[ a_{191} = a_{174} a_{109} + a_{172} a_{111} + a_{180} a_{107} \]

\[ a_{192} = a_{174} a_{111} + a_{107} a_{179} \]

\[ a_{193} = a_{172} a_{111} + a_{180} a_{107} \]

\[ a_{194} = a_{173} a_{111} \]

\[ a_{195} = a_{174} a_{111} + a_{172} a_{111} + a_{180} a_{107} \]

\[ a_{196} = a_{174} a_{111} + a_{171} a_{1110} \]

\[ a_{197} = a_{182} a_{107} + a_{110} a_{172} \]

\[ a_{198} = a_{173} a_{110} \]

\[ a_{199} = a_{174} a_{110} \]

\[ a_{200} = a_{175} a_{108} \]

\[ a_{201} = \frac{a_{108} a_{176}}{2} \]

\[ a_{202} = a_{175} a_{108} \]

\[ a_{203} = \frac{a_{176} a_{108}}{2} \]

\[ a_{204} = a_{170} a_{108} + a_{175} a_{111} \]

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\[
\begin{align*}
a_{203} &= \frac{a_{108}a_{180} + a_{176}a_{111}}{2} \\
a_{207} &= \frac{a_{182}a_{108}}{2} \\
a_{209} &= \frac{a_{170}a_{109}}{2} \\
a_{211} &= \frac{a_{170}a_{109}}{2} \\
a_{213} &= \frac{a_{180}a_{109}}{2} \\
a_{215} &= \frac{a_{180}a_{109} + a_{176}a_{110}}{2} \\
a_{217} &= \frac{a_{176}a_{110}}{2} \\
a_{219} &= \frac{a_{180}a_{110}}{2} \\
a_{221} &= \frac{a_{182}a_{110}}{2} \\
a_{223} &= \frac{a_{176}a_{111}}{2} \\
a_{225} &= \frac{a_{180}a_{111}}{2} \\
a_{227} &= \frac{a_{182}a_{182}}{2} \\
a_{229} &= a_{172} + a_{201} + a_{211} + a_{221} - a_{223} \\
a_{231} &= a_{201} + a_{225} \\
a_{233} &= a_{211} + a_{221} \\
a_{235} &= a_{203} + a_{209} + a_{219} + a_{227} \\
a_{237} &= a_{183} + a_{200} + a_{210} + a_{220} - a_{224} \\
a_{239} &= a_{172} + a_{201} + a_{211} + a_{221} - a_{223} \\
a_{241} &= a_{201} + a_{225} \\
a_{243} &= a_{211} + a_{221} \\
a_{245} &= a_{203} + a_{209} + a_{219} + a_{227} \\
a_{247} &= a_{183} + a_{200} + a_{210} + a_{220} - a_{224} \\
a_{249} &= a_{172} + a_{201} + a_{211} + a_{221} - a_{223} \\
a_{251} &= a_{201} + a_{225} \\
a_{253} &= a_{211} + a_{221} \\
a_{255} &= a_{203} + a_{209} + a_{219} + a_{227} \\
a_{257} &= a_{183} + a_{200} + a_{210} + a_{220} - a_{224} \\
a_{259} &= a_{172} + a_{201} + a_{211} + a_{221} - a_{223} \\
\end{align*}
\]
\begin{align*}
    a_{217} &= a_{203} + a_{209} - a_{219} - a_{227} \\
    a_{239} &= a_{207} + a_{213} + a_{217} + a_{223} \\
    a_{241} &= -a_{207} + a_{213} - a_{217} + a_{223} \\
    a_{243} &= f(a_{239} - a_{123}) \\
    a_{245} &= f(a_{174} - a_{125}) \\
    a_{247} &= f(a_{185} - a_{130}) \\
    a_{249} &= (a_{187} - a_{131})f \\
    a_{251} &= (a_{225} - a_{168})f \\
    a_{253} &= (a_{237} - a_{170})f \\
    a_{255} &= (a_{189} - a_{139})f \\
    a_{257} &= (a_{191})f \\
    a_{259} &= (a_{231} - a_{160})f \\
    a_{261} &= (a_{233})f \\
    a_{263} &= (a_{193} - a_{132})f \\
    a_{265} &= (a_{195} - a_{138})f \\
    a_{267} &= (a_{197} - a_{133})f \\
    a_{269} &= (a_{199})f \\
    a_{271} &= (a_{305} - a_{142})f \\
    a_{273} &= (a_{214} - a_{162})f \\
    a_{274} &= (a_{238} - a_{163})f \\
    a_{276} &= (a_{240} - a_{165})f \\
    a_{278} &= -a_{66} - a_{68} - a_{74} \text{ Ch } \beta_2 - a_{71} \text{ Sh } \beta_1 - a_{76} \text{ Ch } \beta_1 - a_{73} \text{ Sh } \beta_1 - a_{79} \text{ Sh } \beta_2 \\
    a_{279} &= -2 a_{65} - a_{81} \text{ Sh } \beta_1 \\
    a_{280} &= 6 a_{69}
\end{align*}
\[ a_{281} = \alpha_7 + \beta_1 \alpha_{73} \]
\[ a_{283} = \beta_1 a_{81} \]
\[ a_{285} = \alpha_{70} \beta_1 + a_{81} \]
\[ a_{287} = \beta_2 (a_{81} + a_{80}) \]
\[ a_{288} = N \left( \begin{array}{c}
- b_{27} \beta_1 - \beta_1 b_{66} - \beta_1 b_{66} Ch \beta_2 - \beta_1 b_{70} Ch \beta_3 - \beta_1 b_{73} Sh \beta_2 \\
Ch \beta_1 - Ch \beta_1 - Ch \beta_1 - Ch \beta_1 - Ch \beta_1
\end{array} \right) \]
\[ a_{289} = N (b_{76} + 2 b_{81}) \]
\[ a_{290} = N \left( \begin{array}{c}
\beta_2 \beta_1 + 3 b_8 \\
Ch \beta_1
\end{array} \right) \]
\[ a_{291} = N \beta_1 b_{74} \]
\[ a_{292} = N (2 b_{92} \beta_1 + b_{93}) \]
\[ a_{293} = N \beta_2 b_{94} \]
\[ a_{294} = N (\beta_2 b_{65} + b_{73}) \]
\[ a_{295} = N (\beta_2 b_{69} + 2 b_{73}) \]
\[ a_{296} = N (\beta_2 b_{70} + 3 b_{73}) \]
\[ a_{297} = N \beta_1 \beta_2 \]
\[ a_{298} = N (b_{83} \beta_2 + b_{84}) \]
\[ a_{299} = N (\beta_5 b_{87}) \]
\[ a_{300} = N (\beta_4 b_{86} + b_{90}) \]
\[ a_{301} = N \beta_1 b_{88} \]
\[ a_{302} = N \left( \begin{array}{c}
- b_{37} \beta_1 - b_{37} \beta_1 - b_{37} \beta_1 - b_{37} \beta_1 - b_{37} \beta_1 \\
Sh \beta_1 - Sh \beta_1 - Sh \beta_1 - Sh \beta_1 - Sh \beta_1
\end{array} \right) \]
\[ a_{303} = N \left( \begin{array}{c}
2 b_{71} \beta_1 + \beta_1 b_{70} \\
Sh \beta_1
\end{array} \right) \]
\[ a_{304} = N (3 b_{78} + b_{81} \beta_1) \]
\[ a_{305} = N \beta_1 b_{82} \]
\[ a_{306} = N b_{69} \]
\[ a_{307} = N (2b_{70} - \beta_3 b_{89}) \]

\[ a_{310} = N (3b_{71} - \beta_3 b_{72}) \]

\[ a_{311} = N \beta_3 b_{89} \]

\[ a_{312} = N (\beta_3 b_{88} + b_{88}) \]

\[ a_{313} = N \beta_3 b_{90} \]

\[ a_{314} = N (2\beta_1 b_{91 + b_{92}}) \]

\[ a_{315} = N 2\beta_3 b_{93} \]

\[ b_1 = \left[ -a'_4 \cosh \beta_2 + \frac{a_4 \beta_2 f'}{f} \sinh \beta_2 - a'_4 \cosh \beta_1 + \frac{a_4 \beta_1 f'}{f} \cosh \beta_2 + \frac{2nf'}{f} \right] \]

\[ b_2 = \frac{S_s S_0 f p}{N} \left( a_{10} \sinh \beta_2 - \frac{a_{10} \beta_2 f'}{f} \cosh \beta_2 + a'_5 \cosh \beta_1 - \frac{a_5 \beta_1 f'}{f} \cosh \beta_2 - a'_0 - \frac{3a_0 f'}{f} \right) \]

\[ b_3 = \frac{\alpha f^2 S_s S_0 f p}{N} \left( a'_4 \cosh \beta_2 - \frac{a_4 \beta_2 f'}{f} \sinh \beta_2 + a'_3 \cosh \beta_1 - \frac{a_3 \beta_1 f'}{f} \cosh \beta_2 - a'_0 - \frac{2a_0 f'}{f} \right) \]

\[ b_4 = \frac{\alpha f^2 S_s S_0 f p}{N} \left( a_{10} \sinh \beta_2 - \frac{a_{10} \beta_2 f'}{f} \cosh \beta_2 + a'_4 \cosh \beta_1 - \frac{a_4 \beta_1 f'}{f} \cosh \beta_2 - a'_0 - \frac{3a_0 f'}{f} \right) \]

\[ b_5 = \frac{\alpha f^2 S_s S_0 f p}{N} \left( a'_5 + \frac{3a_5 f'}{f} \right) \]

\[ b_7 = a'_6 \times \frac{S_s S_0 f p}{2N} \]

\[ b_9 = \frac{S_s S_0 f p}{2N} \times \frac{a_0 \beta_3 f'}{f} \]

\[ b_{10} = -\frac{\alpha f^2 S_s S_0 f p}{N} \times a'_4 \]

\[ b_{11} = -\frac{\alpha f^2 S_s S_0 f p}{N} \times a'_{10} \]

\[ b_{12} = +\frac{\alpha f^2 S_s S_0 f p}{N} \times a'_0 \beta_3 f' \]

\[ b_{13} = +\frac{\alpha f^2 S_s S_0 f p}{N} \times a'_4 \beta_3 f' \]
\[ h_{14} = f S_4 a_4 \times \left[ \begin{array}{c}
-a'_0 \cosh \beta_2 + \frac{a_0 \beta_2 f'}{f} \cosh \beta_2 - a'_0 \cosh \beta_1 + \frac{a_0 \beta_1 f'}{f} \cosh \beta_1 \\
+ a'_0 + \frac{2a_0 f'}{f}
\end{array} \right] - a_7 \frac{S S_6 f p}{2N} \]

\[ h_{15} = f S_5 a_5 \times \left[ \begin{array}{c}
-a'_0 \cosh \beta_2 + \frac{a_0 \beta_2 f'}{f} \sinh \beta_2 - a'_0 \cosh \beta_1 + \frac{a_0 \beta_1 f'}{f} \sinh \beta_1 \\
+ a'_0 + \frac{2a_0 f'}{f}
\end{array} \right] - a_6 \frac{S S_6 f p}{2N} \]

\[ h_{16} = a_1 f S_4 \times \left[ \begin{array}{c}
-a'_0 \cosh \beta_2 + \frac{a_0 \beta_2 f'}{f} \cosh \beta_2 - a'_0 \cosh \beta_1 + \frac{a_0 \beta_1 f'}{f} \cosh \beta_1 \\
+ a'_0 + \frac{2a_0 f'}{f}
\end{array} \right] - a'_1 \frac{S S_6 f p}{2N} \]

\[ b_{18} = - a_3 f S_6 \]

\[ b_{19} = f S_4 a_4 \times \left( a'_0 + \frac{2a_0 f'}{f} \right) - \frac{af^2 S S_6 f p}{N} \times a'_1 \]

\[ b_{20} = f S_5 a_5 \times \left( a'_0 + \frac{2a_0 f'}{f} \right) - \frac{af^2 S S_6 f p}{N} \times a'_6 \]

\[ b_{21} = - f S_4 a_4 \times \left[ \begin{array}{c}
a'_0 \sinh \beta_2 - \frac{a_0 \beta_2 f'}{f} \cosh \beta_2 + a'_0 \sinh \beta_1 - \frac{a_0 \beta_1 f'}{f} \cosh \beta_1 \\
- a'_0 - \frac{3a_0 f'}{f}
\end{array} \right] - \frac{af^2 S S_6 f p}{N} \times a'_4 \]

\[ b_{22} = f S_5 a_5 \times \left[ \begin{array}{c}
a'_0 \sinh \beta_2 - \frac{a_0 \beta_2 f'}{f} \cosh \beta_2 + a'_0 \sinh \beta_1 - \frac{a_0 \beta_1 f'}{f} \cosh \beta_1 \\
- a'_0 - \frac{2a_0 f'}{f}
\end{array} \right] + \frac{af^2 S S_6 f p}{N} \times a'_5 \beta_5 f' \]

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\[ b_{23} = \frac{a_3 f_{S_{10}} a_0}{2} + \frac{a_4 f_{S_{10}} a_0'}{2} \]
\[ b_{24} = \frac{a_3 f_{S_{10}} a_0}{2} + \frac{a_4 f_{S_{10}} a_6}{2} \]
\[ b_{25} = \frac{a_3 f_{S_{10}} a_0}{2} + \frac{a_4 f_{S_{10}} a_6'}{2} \]
\[ b_{26} = \frac{a_3 f_{S_{10}} a_0}{2} - \frac{a_3 f_{S_{10}} a_{10}}{2} \]
\[ b_{27} = \frac{a_3 f_{S_{10}} a_0'}{2} \]
\[ b_{28} = \frac{a_3 f_{S_{10}} a_6'}{2} \]
\[ b_{29} = -\frac{a_3 \beta_{0} f'}{f} \times \frac{a_3 f_{S_{10}}}{2} - \frac{a_3 \beta_{1} f''}{f} \times \frac{a_4 f_{S_{10}}}{2} \]
\[ b_{30} = \frac{a_3 \beta_{0} f'}{f} \times \frac{a_3 f_{S_{10}}}{2} - \frac{a_3 \beta_{1} f''}{f} \times \frac{a_4 f_{S_{10}}}{2} \]
\[ b_{31} = \frac{a_3 \beta_{0} f'}{f} \times \frac{a_4 f_{S_{10}}}{2} \]
\[ b_{32} = \frac{a_3 \beta_{1} f''}{f} \times \frac{a_4 f_{S_{10}}}{2} \]
\[ b_{33} = -\frac{a_3 \beta_{0} f'}{f} \times \frac{a_3 f_{S_{10}}}{2} - \frac{a_3 \beta_{1} f''}{f} \times \frac{a_4 f_{S_{10}}}{2} \]
\[ b_{34} = -\frac{a_3 \beta_{0} f'}{f} \times \frac{a_3 f_{S_{10}}}{2} - \frac{a_3 \beta_{1} f''}{f} \times \frac{a_4 f_{S_{10}}}{2} \]
\[ b_{35} = -\frac{S_{S_{0}} f_{p} N}{\beta_{1} \cosh \beta_{1}} \]
\[ b_{36} = \frac{S_{S_{0}} f_{p} N}{\beta_{1} \cosh \beta_{1}} \]
\[ b_{37} = \frac{2 S_{S_{0}} f_{p} N}{\beta_{1} \cosh \beta_{1}} \]
\[ b_{38} = \frac{a_{11} f f_{S_{10}}}{\beta_{1} \cosh \beta_{1}} \]
\[ b_{39} = \frac{f f_{S_{10}} \beta_{1}}{2 \sinh \beta_{1}} \]
\[ b_{40} = b_{35} x (\beta_{2} a_{10} \cosh \beta_{2} + 3 a_{6} + \beta_{1} a_{8} \cosh \beta_{1}) \]
\[ b_{41} = b_{36} \]
\[ b_{42} = b_{37} \]
\[ b_{43} = -2 b_{37} a_{5} - 3 a_{6} b_{36} \]
\[ b_{44} = b_{35} - \beta_{2} b_{10} \]
\[ b_{45} = \frac{b_{35} \beta_{1} a_{7} \sinh \beta_{1}}{\sinh \beta_{2}} \]
\[ b_{46} = -\beta_{1} b_{35} b_{35} \]
\[ b_{47} = -\beta_{1} b_{36} b_{35} \]
\[ b_{48} = -b_{36} \beta_{1} b_{10} \]

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\[ b_{49} = \frac{-h_5 \beta_5 a_2 \sinh \beta_5}{\sinh \beta_2} + \frac{2a_5}{\sinh \beta_2} \]

\[ b_{50} = b_{36} \beta_7 b_{38} + b_{39} \times [\beta_7 a_{10} \sinh \beta_5 + 3a_6 + \beta_1 a_8 \cosh \beta_1] \]

\[ b_{51} = b_{38} \times [-\beta_2 a_{10} + 3a_6 + \beta_2 a_8 \cosh \beta_1] + b_{36} \times (\beta_1 a_7) \]

\[ b_{52} = -b_{37} \beta_7 a_7 \]

\[ b_{53} = -b_{37} \beta_7 a_7 \]

\[ b_{54} = -3a_6 b_{39} \]

\[ b_{55} = -3a_6 b_{38} \]

\[ b_{56} = \left[ -h_1 \cosh \beta_5 a_7 \sinh \beta_1 \frac{2b_{36} a_5}{2 \sinh \beta_2} + \frac{2b_{36} a_5}{2 \sinh \beta_2} + \frac{b_{39} \beta_5 a_{10}}{2} \right] \]

\[ b_{57} = -b_{36} \beta_7 a_2 \sinh \beta_5 \frac{2b_{36} a_5}{2 \sinh \beta_2} - \frac{2b_{36} a_5}{2 \sinh \beta_2} + \frac{b_{39} \beta_5 a_{10}}{2} \]

\[ b_{58} = \left[ \frac{b_{38} \beta_5 a_{10}}{2} - \frac{\beta_7 a_7 b_{38}}{2} + b_{39} \left( \frac{2a_5 - \beta_7 a_7 \sinh \beta_1}{2 \sinh \beta_2} \right) \right] \]

\[ b_{59} = \left[ \frac{b_{38} \beta_5 a_{10}}{2} - \frac{\beta_7 a_7 b_{38}}{2} + b_{39} \left( \frac{\beta_5 a_2 \sinh \beta_1 - 2a_6}{2 \sinh \beta_2} \right) \right] \]

\[ b_{60} = -h_{39} \frac{\beta_5 a_7}{2} \]

\[ b_{61} = -b_{39} \frac{\beta_5 a_7}{2} \]

\[ b_{62} = h_3 - b_{36} - \frac{2(h_3 - h_{32})}{\beta_1^4} \frac{24h_5}{\beta_1^6} \]

\[ b_{63} = \frac{h_3 - b_{36}}{\beta_1^2} + \frac{6}{\beta_1^4} (h_4 - h_{33}) \]

\[ b_{64} = \frac{h_3 - b_{36}}{-\beta_1^2} - \frac{12h_5}{\beta_1^6} \]

\[ b_{65} = \frac{h_3 - b_{36}}{-\beta_1^2} - \frac{20h_5}{\beta_1^6} \]

\[ b_{66} = \frac{h_5}{-\beta_1^2} \]
\[ b_{67} = \frac{b_6}{\beta_i^3} - \beta_i^3 \]
\[ b_{68} = \frac{b_7 - b_{44}}{\beta_i^3 - \beta_i^3} - 2(b_6 - b_{49}) \]
\[ b_{69} = \frac{-2b_{41} - b_{44}}{\beta_i^3 - \beta_i^3} + \frac{12b_{42}}{\beta_i^3 - \beta_i^3} + \frac{6b_{43}}{\beta_i^3 - \beta_i^3} \]
\[ b_{70} = \frac{-3b_{41}}{\beta_i^3 - \beta_i^3} + \frac{b_{10}}{\beta_i^3 - \beta_i^3} \]
\[ b_{71} = \frac{b_{13}}{\beta_i^3 - \beta_i^3} \]
\[ b_{72} = \frac{b_8 - b_{49}}{\beta_i^3 - \beta_i^3} \]
\[ b_{73} = \frac{b_{13}}{\beta_i^3 - \beta_i^3} - \frac{3b_{42}}{\beta_i^3 - \beta_i^3} \]
\[ b_{74} = \frac{b_{41}}{\beta_i^3 - \beta_i^3} \]
\[ b_{75} = \frac{-2(b_8 - b_{49})}{\beta_i^3 + b_{49} - b_{52}} \]
\[ b_{76} = \frac{-b_{55} + b_{33}}{2\beta_i^3} + \frac{b_{31}}{4\beta_i^3} \]
\[ b_{77} = \frac{b_{10} - b_{55}}{2\beta_i^3} - \frac{b_{10} - b_{55}}{2\beta_i^3} \]
\[ b_{78} = \frac{b_{31} - b_{54}}{2\beta_i^3} \]
\[ b_{79} = \frac{b_{10} - b_{55}}{4\beta_i^3} \]
\[ b_{80} = \frac{b_{10} - b_{55}}{2\beta_i^3} + \frac{b_{11} - b_{18}}{2\beta_i^3} - \frac{b_{10} - b_{55}}{2\beta_i^3} + \frac{b_{10} - b_{55}}{4\beta_i^3} \]
\[ b_{81} = \frac{b_{10} - b_{55}}{2\beta_i^3} - \frac{b_{11} - b_{18}}{2\beta_i^3} \]
\[ b_{82} = \frac{b_{22} - b_{55}}{2\beta_i^3} \]
\[ b_{83} = \frac{b_{13}}{\beta_i^3 - \beta_i^3} \]
\[ b_{84} = \frac{b_{24}}{\beta_i^3 - \beta_i^3} \]
\[ b_{85} = \frac{b_{25}}{\beta_i^3 - \beta_i^3} \]
\[ b_{86} = \frac{b_{26}}{\beta_i^3 - \beta_i^3} \]
\[
b_{91} = \frac{b_{27} + 2(b_{30} - b_{61})}{3\beta^2_i + 4\beta^3_i}
\]

\[
b_{92} = \frac{b_{29} - b_{50} + b_{28} - (b_{34} - b_{50})}{4\beta^2_i + 3\beta^3_i - \beta^4_i}
\]

\[
b_{93} = \frac{b_{29} - b_{60}}{4\beta^2_i}
\]

\[
b_{94} = \frac{b_{35} - b_{61}}{4\beta^2_i}
\]

\[
b_{100} = Gf^3 (a_{278} - Nb_{63})
\]

\[
b_{102} = Gf^3 (3a_{66} - 3Nb_{65})
\]

\[
b_{104} = Gf^3 (5a_{68} + Nb_{67})
\]

\[
b_{106} = Gf^3 (a_{284} + a_{302})
\]

\[
b_{108} = Gf^3 (a_{304})
\]

\[
b_{110} = Gf^3 (a_{282} + a_{306})
\]

\[
b_{112} = Gf^3 (a_{308})
\]

\[
b_{114} = Gf^3 (a_{310})
\]

\[
b_{116} = Gf^3 (a_{312})
\]

\[
b_{118} = Gf^3 (a_{314})
\]

\[
b_{120} = Gf^3 (a_{283} + a_{286})
\]

\[
b_{122} = Gf^3 (a_{276} + a_{290})
\]

\[
b_{124} = Gf^3 (a_{292})
\]

\[
b_{126} = Gf^3 (a_{294})
\]

\[
b_{128} = Gf^3 (a_{287} + a_{296})
\]

\[
b_{130} = Gf^3 (a_{298})
\]

\[
b_{132} = Gf^3 (a_{300})
\]

\[
b_{134} = a_{242} + b_{100}
\]

\[
b_{135} = a_{243} + b_{101}
\]
\[ b_{136} = a_{244} + b_{102} \]
\[ b_{138} = a_{246} + b_{106} \]
\[ b_{140} = a_{108} + b_{254} \]
\[ b_{142} = a_{248} + b_{110} \]
\[ b_{144} = a_{256} + b_{112} \]
\[ b_{146} = a_{250} + b_{114} \]
\[ b_{148} = a_{252} + b_{116} \]
\[ b_{150} = a_{258} + b_{118} \]
\[ b_{152} = a_{262} + b_{120} \]
\[ b_{154} = a_{264} + b_{122} \]
\[ b_{156} = a_{270} + b_{124} \]
\[ b_{158} = a_{266} + b_{126} \]
\[ b_{160} = a_{268} + b_{128} \]
\[ b_{162} = a_{274} + b_{130} \]
\[ b_{164} = a_{276} + b_{132} \]
\[ b_{166} = \frac{h_{134}}{2M_i^2} \]
\[ b_{168} = \frac{h_{36}}{12M_i^2} \]
\[ h_{170} = \frac{h_{36}}{30M_i^2} \]
\[ h_{172} = \frac{h_{138}}{\beta_i^2(\beta_i^2 - M_i^2)} \]
\[ h_{174} = \frac{h_{140}}{\beta_i^2(\beta_i^2 - M_i^2)} \]
\[
\begin{align*}
  b_{176} &= \frac{b_{160}}{4\beta_1^2(4\beta_2^2 - M_1^2)} \\
  b_{178} &= \frac{b_{152}}{4\beta_1^2(4\beta_2^2 - M_1^2)} \\
  b_{180} &= \frac{b_{158}}{4\beta_1^2(4\beta_2^2 - M_1^2)} \\
  b_{182} &= \frac{b_{162}}{4\beta_1^2(4\beta_2^2 - M_1^2)} \\
  b_{184} &= \frac{2b_{140}}{2\beta_1^5} \\
  b_{186} &= \frac{b_{140}}{6\beta_1^3} \\
  b_{188} &= -\frac{5(b_{159} + b_{151})}{4\beta_1^3} + \frac{2b_{153}}{\beta_1^3} \\
  b_{190} &= \frac{2\beta_2 b_{154} - 5b_{141}}{4\beta_1^4} \\
  b_{192} &= \frac{(4b_{144} - 5\beta_2 b_{149})}{4\beta_2^3} \\
  b_{194} &= \frac{b_{144}}{6\beta_2^3} - \frac{5b_{161}}{4\beta_2^3} \\
  b_{196} &= \frac{b_{159}}{2\beta_2^3} + \frac{3b_{161}}{2\beta_2^3} - \frac{5b_{144}}{2\beta_2^3} \\
  b_{198} &= \frac{b_{161}}{8\beta_2^3} \\
  b_{200} &= -\frac{5b_{153}}{4\beta_2^3}
\end{align*}
\]
\[ b_{203} = \frac{-5h_{147}}{4\beta_3^4} \]
\[ b_{204} = \frac{h_{195}}{2\beta_3^2} \]
\[ b_{206} = \frac{h_{192}}{4\beta_3^4} \]
\[ b_{208} = \frac{-5h_{189}}{4\beta_4^4} \]
\[ b_{210} = \frac{-5h_{151}}{32\beta_4^4} \]
\[ b_{212} = \frac{a_{173}}{8\beta_2^4} \]
\[ b_{214} = \frac{a_{173}}{16\beta_2^4} \]
\[ b_{216} = \frac{\phi_1(1) - \phi_1(-1)}{2} \]
\[ b_{217} = \frac{b_{216} - b_{215}}{M_2 Ch M_1 - Sh M_1} \]
\[ e_1 = \frac{-b_{216}}{M_2 Sh M_1} \]
\[ e_2 = \frac{-b_{215}}{M_1 Ch M_1} - Sh M_1 \]
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With Best Wishes

Prof. ADEEL AHMAD
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