APPENDIX-II

KINEMATICS OF THE Be$^8$ DECAY.

The Be$^8$ nucleus decays in flight into two alpha particles just after emerging from the disintegrating nucleus. The initial kinetic energy of the Be$^8$ and the Q-value of the reaction are shared between the product alpha particles. Let $M$ and $E$ be respectively the mass and the kinetic energy of the Be$^8$ and $P$ its initial momentum along the direction of flight.

Suppose $E_1$ and $E_2$ are the kinetic energies of the product alpha particles, and $P_1$ and $P_2$ their momenta along the direction of flight, the angle between them being $\phi$. Then

$$P^2 = P_1^2 + P_2^2 + 2P_1P_2\cos\phi.$$  

For non-relativistic cases this can be written as

$$ME = m(E_1 + E_2) + 2m\sqrt{E_1E_2}\cos\phi.$$  

Putting $M = 2m$,

$$E = \frac{E_1 + E_2}{2} + \sqrt{E_1E_2}\cos\phi \quad \text{and} \quad Q = E_1 + E_2 - E = \frac{E_1 + E_2}{2} - \sqrt{E_1E_2}\cos\phi.$$  

Case I. When the Be$^8$ decays at rest, the alpha particles will recoil from each other in opposite directions. If the line of decay is perpendicular to the direction of flight of the Be$^8$, the two alpha particles will have equal energy and the angle $\phi$ between them will have the maximum value

$$E_1 = E_2 = \frac{E + Q}{2}.$$  

From (1) \[ E = \frac{E + Q}{2} \left( 1 + \cos \phi \right) \]

(3) \[ \cos \phi = \frac{E - Q}{E + Q} \]

Case II. When the line of decay is along the direction of flight of the \( \text{Be}^8 \), \( \phi \) will be zero, and the difference in energy of the two alpha particles will be maximum.

Putting \( \phi = 0 \) in (1),
\[ 2E = E_1 + E_2 + 2\sqrt{E_1 E_2} , \]

therefore,
\[ (4) \quad \sqrt{E_1} + \sqrt{E_2} = \sqrt{2E} . \]

Also,
\[ 2( E_1 + E_2 - Q ) = E_1 + E_2 + 2\sqrt{E_1 E_2} . \]

Hence
\[ EQ = E_1 + E_2 - 2\sqrt{E_1 E_2} , \]

therefore,
\[ (5) \quad \sqrt{E_1} - \sqrt{E_2} = \sqrt{EQ} . \]

From (4) and (5) \[ E_1 - E_2 = 2\sqrt{EQ} . \]

Also since \[ E_1 + E_2 = E + Q , \]

We get
\[ E_1 = \frac{E + Q}{2} + \sqrt{EQ} \quad \text{and} \quad (6) \]
\[ E_2 = \frac{E + Q}{2} - \sqrt{EQ} . \]