CHAPTER VI

A GENERALISED PROBABILITY MODEL FOR THE MOST RECENT BIRTH INTERVAL AND ITS APPLICATION
6.1. INTRODUCTION

The problem of measurement of current potential fertility and the changes in fertility behaviour among the females in the presence or absence of any contraceptive practice have necessitated a large number of studies about spacing between births of children. Timing and frequency of births form two important dimensions of human fertility. Two couples having the same number of children may show different impact of their births on the future growth of population if the timings of their births differ. It is, therefore, desirable to analyse the interval between two consecutive births of women apart from measuring their fertility level by number of births so as to understand the future implications of their fertility behaviour. Since 1960s an increasing number of demographers are turning their attention towards the study of birth intervals, because the fertility process is itself a time dependent process and the length of the birth interval is inversely related to the number of births and hence, to the completed family size. The study of the constituents of birth intervals and their changing and interacting patterns have enabled researchers to find sensitive indices for the measurement of fertility changes.
The closed birth intervals are treated quite sensitive to reflect current and abrupt changes in the underlying fertility behaviour of women than the conventional fertility rates. The reason is that the changes in the number or timing of births may result either from changes in the duration of lactational postpartum amenorrhoea (PPA) period attributable to behavioural and psychological factors or to the use of contraception, abstinence, migration effecting sexual behaviour of women in susceptible period. The effect of these changes is well reflected in the description of birth intervals (Potter, 1963; Ryder, 1965). The analytical approaches for the study of birth intervals have extensively been used in recent years, for, the data on birth intervals collected through retrospective fertility surveys are quite useful for a variety of purposes (Singh and Singh, 1991). Data on closed birth intervals can be used for the estimation of various biological parameters such as fecundability, postpartum amenorrhoea and /or incidence of foetal wastage through suitable probability models.

In under developed countries, where literacy rate is very low, the information necessary to compute the usual measure of fertility are not generally available. So the collection and analysis of retrospective birth interval data has special significance. But, the analysis of retrospective data suffers from two main drawbacks viz., selectivity and censoring. The problem of selectivity arises due to the fact that the interval available for analysis may be a selected subset of the universe of interest, i.e. this bias occurs because all females do not get equal chance of inclusion in the study. Rindfuss *et al* (1982) suggested a method to overcome this problem. It is worthwhile to mention here that in a retrospective survey this kind of difficulty in the study of duration variable like PPA can be overcome, to some extent, by considering interval relating to last and last-but-one births (Singh *et al*, 1979b). The problem of censoring exists when incomplete information is available about the risk period because of the limited observation period. Again, in the retrospective approach, the subjects selected for study are asked to report
their past experiences. Hence, fertility data collected retrospectively is generally influenced by the non-sampling errors arising out of recall lapse on the part of the respondent (Rindfuss et al, 1982). Information on closed birth intervals related to births in the distant past are more prone to this type of errors. It has been advocated that among the closed birth intervals which may be observed retrospectively, the data related to the interval between last and last-but-one births, known as the most recent birth interval (MRBI), prior to the survey date (see Appendix II.B diagram), is considered to be least affected by the non-sampling errors due to recall lapse. Thus they will be more reliable for the analysis of fertility changes among married women compared to the other closed birth intervals (Srinivasan, 1967; Singh et al, 1988; Mukherjee et al, 1991) and the distribution of the most recent birth interval is quite sensitive to the small changes in fertility and robust with respect to non-fertility parameters (Sehgal, 1971).

A closed birth interval is considered to be generally made up of three mutually exclusive components in the absence of defective termination of pregnancy (i.e. foetal losses) viz.,(i) a period of temporary sterility caused by the postpartum amenorrhoea following the previous birth during which no conception is possible, (ii) the menstruating period, that is, the waiting time spent in the susceptible state before the occurrence of conception and (iii) the period of gestation leading to the next live birth. Except for the last component, which is generally taken to be constant equal to nine months, the other two components vary considerably with the age of the married females and with parity. However, from the point of view of the family planners, a parity dependent closed birth-interval (particularly MRBI) seems more appropriate as these components are very much affected by the desire to control or limit births which, in turn, is associated with the number of births a married women has already attained.

Consideration of the human reproduction process as a classical or markovian renewal process implies that the underlying parameters are homogeneous in time i.e.
independent of age, parity, marriage duration etc. Several empirical studies suggest that the fertility parameters are likely to be function of above mentioned biological factors. Thus, the models based on the assumption of homogeneity of fertility parameters, to describe data for long duration, may make the analysis inappropriate and inconclusive. Hence a number of models, based on the theory of heterogeneity in fertility parameters, have been formulated. Poole (1973), Sheps and Menken (1972) have outlined procedures to obtain theoretical distribution of last closed birth interval of a specific order, under the assumption that the distribution of last closed birth interval is a function of parity and marital duration. Again assuming the expected rate of achieving conception during the susceptible state and the chance of female being sterile following a birth are functions of parity and also taking probability of incurring a foetal loss as zero, Singh et al (1988) derived a probability model for the most recent birth interval, which is a case of the model proposed by Dwivedi (1985). They have not made any provision to account foetal wastage and associated rest period following it. But the assumption that there is one to one correspondence between conception and a live birth is not always true in reality. So assuming that any conception may result in either a live birth or a foetal loss, Mukherjee et al (1991) have derived a probability model for the most recent birth interval independent of parity. They have derived the model under steady state assumption that the start of the observation period is a distant point since the consummation of marriage. This attempt, however, by suggesting the start of observation period as a distant point from marriage has considered only the older women for the study. Though a number of models have already been derived, none is found to have satisfied all the real conditions and hence suitable to fit all the situations arising in different societies of the world. Hence, there is a need to develop simplified but more realistic models for the most recent birth-interval (MRBI).
In the present chapter a generalised probability model for the most recent birth-interval has been derived under some simplified assumptions relating to fertility process of married females, who has had a specific duration of married life. However, it should be mentioned here that the present model is an extension of the model derived by Singh et al (1988). This model assumes that the expected rate of achieving conception during the susceptible state is function of parity, a conception may lead to either a live birth or a foetal loss, and the non susceptible period associated with a foetal loss follows exponential distribution. First a parity dependent model is derived under the assumptions given in section 6.2. Later on, a model is developed regardless of parity. The latter model has been applied to a set of observed data with specific marital duration.

6.2. ASSUMPTIONS OF THE MODEL

Consider a married female of parity n, who has lived for time T since her marriage. Let the successive live births occurred at times $(X_0, X_0+X_1, X_0+X_1+X_2, \ldots)$ i.e., $X_0$ be the time of first live birth since marriage and $X_i$ be the interval between $i$th and $(i+1)$th live births. Denoting the most recent birth interval (MRBI), i.e., the interval between $(n-1)$th and $n$th live births by $X_{n-1}$, the probability distribution of $X_{n-1}$ is derived under the following assumptions:

1. A female is either fecund at the time of marriage or primarily sterile, i.e. incapable of conceiving throughout her life. Let $\alpha_0$ and $(1-\alpha_0)$ be the respective probabilities. Further let $(1-\alpha_i)$, $i \geq 1$ be the probability that the female becomes sterile or chooses to be so following the termination of $i$th pregnancy.

2. All fecund women are susceptible to conception at marriage and have led a married life throughout the period of observation.
The conditional instantaneous risk of conception, at any time \( t > 0 \), to a fecund woman of parity \( j \), who is susceptible at time \( t \), be \( m'_j \), with \( m'_j > 0 \), \( j = 0, 1, 2 \ldots \).

A conception may end in either a live birth or a foetal loss. Let \( (1 - \theta_j) \) be the probability that a conception to a woman of parity \( j \) will end in a foetal loss, \( 0 < \theta_j \leq 1 \), \( j = 0, 1, 2 \ldots \).

\( h_j \) be the length of the non-susceptible period, comprising the duration of pregnancy (say \( g \)) and postpartum amenorrhoea (PPA), associated with a conception leading to the \( j \)th live birth, where \( h_j \) is a constant, \( j = 1, 2, \ldots \).

Under this assumption, the maximum number of live births to a female during \( (0, T) \), \( T > g \) can not be more than \( p \), defined by the value of \( k \) satisfying the inequality

\[
h_0 + h_1 + \ldots + h_{k-1} + g < h_0 + h_1 + \ldots + h_k \quad \text{(6.1)}
\]

with \( h_0 = 0 \), \( k = 1, 2, 3, \ldots \).

The length of the non-susceptible period associated with a foetal loss occurring between \( j \)th and \( (j+1) \)th live births is a random variable which follows an exponential distribution with mean \( 1/c_j \), \( c_j > 0 \), \( j = 0, 1, 2, \ldots \).

Waiting times for live births are mutually independent.

Under the above assumptions, any interval between two consecutive live births for a fecund married female can be considered to be made up of the following components.

a) The period of PPA following the previous live birth.
b) The total duration of menstruating interval between two live births.
c) The period of gestation and PPA (if any) associated with a foetal loss intervening between two live births.
d) The period of gestation associated with the latter live birth.
In case of the interval between marriage and the first live birth the first component will be absent.

Considering the above assumptions of the model, the distribution function of the waiting time from marriage to first live birth and its p.d.f. are given respectively by

$$F_0(x) = 1 - \gamma_0 e^{-\nu_0(x-g)} - (1-\gamma_0) e^{-\nu_1'(x-g)}$$
$$f_0(x) = \gamma_0 \nu_0' e^{-\nu_0(x-g)} + (1-\gamma_0) \nu_1' e^{-\nu_1'(x-g)} ; x>g$$

Similarly the distribution function and the p.d.f. of the waiting time between rth and (r+1)th live births, for \( r \geq 1 \) are given respectively by

$$F_r(x) = 1 - \gamma_r e^{-\nu_{2r}(x-h_r)} - (1-\gamma_r) e^{-\nu_{2r+1}(x-h_r)}$$
$$f_r(x) = \gamma_r \nu_{2r} e^{-\nu_{2r}(x-h_r)} + (1-\gamma_r) \nu_{2r+1}' e^{-\nu_{2r+1}'(x-h_r)} ; x>h_r$$

[See Appendix I : (1.2), (1.3)]

where

\[
\begin{align*}
\gamma_j &= \frac{c_j \nu_{2j+1}' - \nu_{2j} \nu_{2j+1}'}{c_j \nu_{2j+1}' - c_j \nu_{2j}'} ; j = 0, 1, 2, \ldots \\
\nu_{2j}' &= \frac{1}{2} \left[ (m_j' + c_j) + \sqrt{(m_j' + c_j)^2 - 4m_j' c_j \theta_j} \right] \\
\text{and} \quad \nu_{2j+1}' &= \frac{1}{2} \left[ (m_j' + c_j) - \sqrt{(m_j' + c_j)^2 - 4m_j' c_j \theta_j} \right] ; j \geq 0
\end{align*}
\]

Now let \( K_{r+1}(T) \) denotes the distribution function of the waiting time from marriage to (r+1)th live births. Suppose among the 2(r+1) values of \( \nu'_0, \nu'_1, \ldots, \nu'_{2r+1} \) only \( k \) are distinct. Let us denote them by \( \nu_1, \nu_2, \ldots, \nu_k \) where \( \nu_j \) is repeated \( n_j \) times such that \( \sum_{j=1}^{k} n_j = 2(r + 1) \). Under the assumptions (2) to (7) Bhattacharya and Nath (1984) have shown that
\[ K_{r+1}(T) = \begin{cases} 
\sum_{i=1}^{k} \sum_{j=1}^{n_i} A_{r+1}(i, j) \cdot G_{r+1}(j, v, T) & ; \quad T \geq g + h_1 + \ldots + h_r \\
0 & ; \quad \text{otherwise} 
\end{cases} \]

where

\[ A_{r+1}(i, n_i - j) = \frac{v_i^j}{j!} \frac{d^i}{ds^i} \left[ \frac{U_{r+1}(0, s)}{V_{r+1}(i, s)} \right]_{s = -v_i} \]

\[ U_{r+1}(0, s) = \prod_{j=0}^{r} \left( 1 + \frac{s}{c_j} \right) \]

\[ V_{r+1}(i, s) = \prod_{j=1 \atop j \neq i}^{k} \left( 1 + \frac{s}{v_j} \right)^{n_i} \]

and

\[ G_{r+1}(j, v, T) = 1 - e^{-v_i(T-g-h_1-\ldots-h_r)} \sum_{u=0}^{j-1} \frac{(v_i(T-g-h_1-\ldots-h_r))^u}{u!} \]

Now, a female with parity \( r \) is either fecund or sterile. From the assumption (1) the probability that she is fecund is given by

\[ \prod_{j=0}^{r} \alpha_j \]

and that she is not by \( \prod_{j=0}^{r-1} \alpha_j (1-\alpha_r) \)

Thus the probability that the female will have at least \((r+1)\) live births during \((0,T)\) is

\[ H_{r+1}(T) = \begin{cases} 
\left( \prod_{i=0}^{r} \alpha_i \right) K_{r+1}(T) & ; \quad r = 0, 1, 2, \ldots, (p-1) \\
0 & ; \quad \text{otherwise} 
\end{cases} \]

with \( H_0(T) = 1 \)

Hence the probability of exactly \( r \) live births during \((0, T)\) is

\[ P_r(T) = \begin{cases} 
H_r(T) - H_{r+1}(T) & ; \quad r = 0, 1, 2, \ldots, (p-1) \\
H_r(T) & ; \quad r = p 
\end{cases} \]

6.3. MODEL M1

Under the assumptions (1) to (7), the probability density function of the waiting time between last and last-but-one live births with parity \( n=1 \) is given by
\[
g_1(x/T) = \begin{cases} 
0 & ; \ 0 < x < g \\
\frac{\alpha_0 f_0(x) \left[ 1 - \alpha_1 F_1(T-x) \right]}{P_1(T)} & ; \ g < x < T-h_1 \\
\frac{\alpha_0 f_0(x)}{P_1(T)} & ; \ T-h_1 < x < T
\end{cases}
\] ....(6.7)

Again according to Poole (1973), the density function of \( X_{n-1} \) for \( n \geq 2 \) is given by

\[
g_n(x/T) = \frac{P_r \left[ S_{n-1} \leq T \cap S_n > T / X_{n-1} = x \right] f_{n-1}(x)}{P_n(T)}
= \frac{P_r \left[ S_{n-2} \leq T-x \cap S_{n-2} + X_n > T-x \right] f_{n-1}(x)}{P_n(T)}
= \frac{P_r \left[ S_{n-2} < T-x \cap S_{n-2} > T-x - X_n \right] f_{n-1}(x)}{P_n(T)}
= \frac{P_r \left[ T-x - X_n < S_{n-2} < T-x \right] f_{n-1}(x)}{P_n(T)}
= \frac{[P_r \left[ S_{n-2} \leq T-x \right] - P_r \left[ S_{n-2} + X_n \leq T-x \right]] f_{n-1}(x)}{P_n(T)}
= \frac{[K_{n-1}(T-x) - K^{*}_{n+1}(T-x)] f_{n-1}(x)}{P_n(T)}
\] ....(6.8)

where \( S_{n-2} = X_0 + X_1 + \cdots + X_{n-2} \)

\[K^{*}_{n+1}(T-x) = P_r \left\{ S_{n-2} + X_n \leq T-x \right\} \]

[See Appendix II A : (II.1)]
Hence, under the assumptions of the model, the density function of the MRBI for \( n \geq 2 \) is given by

\[
g_n(x/T) = \begin{cases} 
0 & ; 0 < x < h_{n-1} \\
\prod_{j=0}^{n-2} \frac{[K_{n-1}(T-x) - \alpha_n K^{n+1}(T-x)] f_{n-1}(x)}{P_n(T)} & ; h_{n-1} \leq x < T - g - \sum_{i=1}^{n-1} h_i \\
\prod_{j=0}^{n-2} \frac{K_{n-1}(T-x) f_{n-1}(x)}{P_n(T)} & ; T - g - \sum_{i=1}^{n-1} h_i < x < T - \sum_{i=1}^{n-2} h_i \\
0 & ; T - g - \sum_{i=1}^{n-2} h_i < x < T 
\end{cases}
\]

6.4. MODEL M2

Let us now consider the length of the interval between last and last-but-one births, \( X \), regardless of parity. The p.d.f. of \( X \) is given by

\[
g(x) = \sum_{n=1}^{p} g_n(x/T) \frac{P_n(T)}{P(T)} \quad \text{...............(6.10)}
\]

where \( p \) is the maximum number of live births during \((0, T)\).

For practical application we assume that \( h_i = h \quad \forall \ i \quad \text{and} \quad \alpha_i = 1 \neq \alpha_0, \quad i \geq 1, \quad m'_1 = m_1 \quad \text{and} \quad m'_1 = m_2, \quad j \geq 1 \) such that \( m_1 \neq m_2 \). Now Model M2 reduces to

\[
g(x) = \begin{cases} 
\alpha_0 \{ \gamma_0 \nu_1 e^{-\nu_1(x-g)} + (1-\gamma_0) \nu_2 e^{-\nu_2(x-g)} \} \{ \gamma_1 e^{-\nu_3(x-h)} + (1-\gamma_1) e^{-\nu_4(x-h)} \} ; g < x < h \\
\alpha_0 \{ \gamma_0 \nu_1 e^{-\nu_1(x-g)} + (1-\gamma_0) \nu_2 e^{-\nu_2(x-g)} \} \{ \gamma_1 e^{-\nu_3(x-h)} + (1-\gamma_1) e^{-\nu_4(x-h)} \} \\
\quad + \alpha_0 \{ \gamma_1 \nu_3 e^{-\nu_3(x-h)} + (1-\gamma_1) \nu_4 e^{-\nu_4(x-h)} \} \\
\quad \{ \gamma_0 (1-e^{-\nu_1(T-x-g)}) + (1-\gamma_0) (1-e^{-\nu_2(T-x-g)}) \} \} ; h < x < T - h \\
\alpha_0 \{ \gamma_0 \nu_1 e^{-\nu_1(x-g)} + (1-\gamma_0) \nu_2 e^{-\nu_2(x-g)} \} + \alpha_0 \{ \nu_1 \nu_3 e^{-\nu_3(x-h)} \\
\quad + (1-\gamma_1) \nu_4 e^{-\nu_4(x-h)} \} \{ \gamma_0 (1-e^{-\nu_1(T-x-g)}) \\
\quad + (1-\gamma_0) (1-e^{-\nu_2(T-x-g)}) \} \} ; T - h < x < T 
\end{cases}
\]

...............(6.11)
6.5. ESTIMATION

There are a large number of parameters in the model. With few confinements, it is not possible to estimate all the parameters. On the assumption that \( h_j, \theta_j, c_j \) \((j=0,1,2,\ldots)\) are known, a procedure to obtain the m.l. estimate of \( m_l, m_2,\ldots,m_k \) (where \( k \) is the number of distinct values of \( m'_o, m'_1,\ldots,m'_{n-1}, k<n \)) is now described. Let \( X_i \) \((i=1,\ldots,n)\) be a random sample of size \( N \) from the distribution \( g_r(x) \), where \( X_i=r \) denotes the length of the MRBI to the \( i \)th female. Let \( N_r \) \((r=1,2,\ldots,n)\) be the observed frequency of a female having MRBI of length \( r \), so that \( \sum_{i=1}^n N_r = N \). The likelihood function is given by

\[
L = \frac{N!}{\prod_{r=1}^n N_r^r} \prod_{r=1}^n g_r^{N_r} \quad \text{where } g_r = P_r [X_i = r] \quad \text{(6.12)}
\]

The maximum likelihood estimators of \( (m_l, m_2,\ldots,m_k) \) are the solution to the following equation

\[
\left[ \frac{d}{dm_1} \log L, \frac{d}{dm_2} \log L, \ldots, \frac{d}{dm_k} \log L \right] = \left[ 0, 0, \ldots, 0 \right] \quad \text{(6.13)}
\]

For this, the differentiations of \( g_r \) with respect to the unknown parameters are required. Since the explicit solution of the equation (6.13.) is not possible, the scoring method can be used to obtain m.l. estimates of the unknown parameters. The pilot values of \( z \) unknown parameters which are required for scoring method can be calculated by equating the relative frequencies of \((z+1)\) cells of the observed distribution having significant number of observations to their respective theoretical expressions. These \((z+1)\) equations can be solved by using the Newton-Raphson iteration procedure. The variance-covariance matrix of the estimators is the inverse of the information matrix. For application purpose, we have used equation (6.11). Here we have to calculate the parameters \( h, \alpha_o, \theta_j, c_j \) and \( m_j, j=1,2 \). Assuming that \( h, \alpha_o, \theta_j, c_j \) are known (as given in section 6.6.) we have to
estimate only $m_1$ and $m_2$. For this, differentiation of $g(x)$ with respect to $m_1$ and $m_2$ are only needed, which are given as follows

\[
\frac{dg(x)}{dm_1} = \gamma_1 e^{-v_3(x-h)} + (1-\gamma_1) e^{-v_4(x-h)} \left[ \frac{dy_0}{dm_1} \left\{ v_1 e^{-v_1(x-g)} \right\} \right.
\]

\[
+ \frac{dy_1}{dm_1} \left\{ \gamma_0 e^{-v_1(x-g)} \right\} + \frac{de^{-v_1(x-g)}}{dm_1} \left( \gamma_0 v_1 \right) + \frac{d(1-\gamma_0)}{dm_1} \left\{ v_2 e^{-v_2(x-g)} \right\} \]

\[
+ \frac{dv_2}{dm_1} \left\{ (1-\gamma_0) e^{-v_2(x-g)} \right\} + \frac{de^{-v_2(x-g)}}{dm_1} \left\{ (1-\gamma_0) v_2 \right\} \]

\[
\frac{dg(x)}{dm_1} = \gamma_1 e^{-v_3(x-h)} + (1-\gamma_1) v_2 e^{-v_2(x-g)} \left\{ \frac{dy_1}{dm_2} e^{-v_3(x-h)} + \frac{de^{-v_3(x-h)}}{dm_2} \gamma_1 \right\}
\]

\[
+ \frac{d(1-\gamma_1)}{dm_2} e^{-v_4(x-h)} + \frac{de^{-v_4(x-h)}}{dm_2} (1-\gamma_1) \]

\[
; g < x < h
\]

\[
\frac{dg(x)}{dm_1} = \alpha_0 \left\{ \gamma_1 e^{-v_3(x-h)} + (1-\gamma_1) e^{-v_4(x-h)} \right\} \left[ \frac{dy_0}{dm_1} \left\{ v_1 e^{-v_1(x-g)} \right\} \right.
\]

\[
+ \frac{dy_1}{dm_1} \left\{ \gamma_0 e^{-v_1(x-g)} \right\} + \frac{de^{-v_1(x-g)}}{dm_1} \left( \gamma_0 v_1 \right) + \frac{d(1-\gamma_0)}{dm_1} \left\{ v_2 e^{-v_2(x-g)} \right\} \]

\[
+ \frac{dv_2}{dm_1} \left\{ (1-\gamma_0) e^{-v_2(x-g)} \right\} + \frac{de^{-v_2(x-g)}}{dm_1} \left\{ (1-\gamma_0) v_2 \right\} \]

\[
+ \alpha_0 \left\{ \gamma_1 v_3 e^{-v_3(x-h)} + (1-\gamma_1) v_4 e^{-v_4(x-h)} \right\} \left[ \frac{dy_0}{dm_1} \left\{ 1-e^{-v_1(T-x-g)} \right\} \right.
\]

\[
+ \frac{d\left\{ 1-e^{-v_1(T-x-g)} \right\}}{dm_1} \gamma_0 + \frac{d(1-\gamma_0)}{dm_1} \left\{ 1-e^{-v_2(T-x-g)} \right\} + \frac{d\left\{ 1-e^{-v_2(T-x-g)} \right\}}{dm_1} (1-\gamma_0) \right\}
\]
\[
\frac{dg(x)}{dm_2} = \alpha_0 \left\{ \gamma_0 \left( \frac{d\gamma_1}{dm_2} e^{-v_1(x-h)} + (1-\gamma_0) \frac{d\gamma_1}{dm_2} e^{-v_2(x-h)} \right) \right\} + \frac{d\gamma_1}{dm_2} e^{-v_3(x-h)} + \frac{de^{-v_3(x-h)}}{dm_2} \gamma_1 \\
+ \frac{d(1-\gamma_1)}{dm_2} e^{-v_4(x-h)} + \frac{de^{-v_4(x-h)}}{dm_2} (1-\gamma_1) + \alpha_0 \left\{ \gamma_0 \left( 1-e^{-v_1(T-x-g)} \right) \right\} \\
+ (1-\gamma_0) \left\{ 1-e^{-v_2(T-x-g)} \right\} \left\{ \frac{d\gamma_1}{dm_2} \left( \frac{d\gamma_1}{dm_2} e^{-v_3(x-h)} \right) + \frac{d\gamma_1}{dm_2} \left( \frac{d\gamma_1}{dm_2} e^{-v_4(x-h)} \right) \right\} \\
+ \frac{d\gamma_0}{dm_2} \left( 1-e^{-v_1(T-x-g)} \right) + \frac{d\gamma_1}{dm_2} \left( 1-e^{-v_2(T-x-g)} \right) \\
\frac{(d\gamma_0 + (1-\gamma_0) \frac{d\gamma_1}{dm_2}) \left( \gamma_1 \frac{d\gamma_1}{dm_2} \right)}{(1-\gamma_1) \frac{d\gamma_0}{dm_2}} \\
\frac{\gamma_1 \frac{d\gamma_1}{dm_2} + (1-\gamma_1) \frac{d\gamma_0}{dm_2}}{(1-\gamma_1) \frac{d\gamma_0}{dm_2}} \right\} \right\}; h<x<T-h
\]

\[
\frac{dg(x)}{dm_1} = \alpha_0 \left\{ \frac{d\gamma_0}{dm_1} \left( \frac{d\gamma_1}{dm_1} e^{-v_1(x-h)} + (1-\gamma_0) \frac{d\gamma_1}{dm_1} e^{-v_2(x-h)} \right) \right\} + \frac{d\gamma_1}{dm_1} e^{-v_3(x-h)} + \frac{de^{-v_3(x-h)}}{dm_1} \gamma_1 \\
+ \frac{d(1-\gamma_0)}{dm_1} \left\{ \frac{d\gamma_1}{dm_1} e^{-v_2(x-h)} \right\} + \frac{d\gamma_1}{dm_1} \left\{ (1-\gamma_0) e^{-v_2(x-h)} \right\} \\
+ \frac{de^{-v_2(x-h)}}{dm_1} \left\{ (1-\gamma_0) \frac{d\gamma_1}{dm_1} \right\} + \alpha_0 \left\{ \gamma_1 \frac{d\gamma_1}{dm_1} e^{-v_3(x-h)} + (1-\gamma_1) \frac{d\gamma_1}{dm_1} e^{-v_4(x-h)} \right\} \\
+ \left\{ \frac{d\gamma_0}{dm_1} \left( 1-e^{-v_1(T-x-g)} \right) + \frac{d(1-\gamma_0)}{dm_1} \left( 1-e^{-v_2(T-x-g)} \right) \right\} \\
\frac{\gamma_1 \frac{d\gamma_1}{dm_1} + (1-\gamma_1) \frac{d\gamma_0}{dm_1}}{(1-\gamma_1) \frac{d\gamma_0}{dm_1}} \right\} \right\}; h<x<T-h
\]

\[
\frac{dg(x)}{dm_2} = \alpha_0 \left\{ \gamma_0 \left\{ 1-e^{-v_1(T-x-g)} \right\} + (1-\gamma_0) \left\{ 1-e^{-v_2(T-x-g)} \right\} \right\} \left\{ \frac{d\gamma_1}{dm_2} \left( \frac{d\gamma_1}{dm_2} e^{-v_3(x-h)} \right) + \frac{d\gamma_1}{dm_2} \left( \frac{d\gamma_1}{dm_2} e^{-v_4(x-h)} \right) \right\} \\
+ \frac{d\gamma_1}{dm_2} \left\{ \gamma_1 \frac{d\gamma_1}{dm_2} e^{-v_3(x-h)} + (1-\gamma_1) \frac{d\gamma_1}{dm_2} e^{-v_4(x-h)} \right\} \\
+ \frac{d\gamma_1}{dm_2} \left\{ (1-\gamma_1) e^{-v_4(x-h)} \right\} + \frac{d(1-\gamma_1)}{dm_2} \left\{ (1-\gamma_1) \frac{d\gamma_1}{dm_2} \right\} ; T-h<x<T
\]
where

\[
\frac{dv_1}{dm_1} = \frac{1}{2} \left[ 1 + \frac{((m_1+c)-2c\theta)}{\sqrt{(m_1+c)^2-4m_1c\theta}} \right]
\]

\[
\frac{dv_2}{dm_1} = \frac{1}{2} \left[ 1 - \frac{((m_1+c)-2c\theta)}{\sqrt{(m_1+c)^2-4m_1c\theta}} \right]
\]

\[
\frac{dv_3}{dm_2} = \frac{1}{2} \left[ 1 + \frac{((m_2+c)-2c\theta)}{\sqrt{(m_2+c)^2-4m_2c\theta}} \right]
\]

\[
\frac{dv_4}{dm_2} = \frac{1}{2} \left[ 1 - \frac{((m_2+c)-2c\theta)}{\sqrt{(m_2+c)^2-4m_2c\theta}} \right]
\]

\[
\frac{dy_1}{dm_1} = - \frac{(\sqrt{(m_1+c)^2-4m_1c\theta})}{(m_1+c)^2-4m_1c\theta} \left[ \frac{1}{2} \left( 1 - \frac{((m_1+c)-2c\theta)}{\sqrt{(m_1+c)^2-4m_1c\theta}} \right) - \frac{1}{2} \left( \frac{(m_1+c) - \sqrt{(m_1+c)^2-4m_1c\theta}}{\sqrt{(m_1+c)^2-4m_1c\theta}} \right) - m_1\theta \right] \frac{(m_1+c)-2c\theta}{\sqrt{(m_1+c)^2-4m_1c\theta}}
\]

\[
\frac{dy_2}{dm_1} = - \frac{(\sqrt{(m_2+c)^2-4m_2c\theta})}{(m_2+c)^2-4m_2c\theta} \left[ \frac{1}{2} \left( 1 - \frac{((m_2+c)-2c\theta)}{\sqrt{(m_2+c)^2-4m_2c\theta}} \right) - \frac{1}{2} \left( \frac{(m_2+c) - \sqrt{(m_2+c)^2-4m_2c\theta}}{\sqrt{(m_2+c)^2-4m_2c\theta}} \right) - m_2\theta \right] \frac{(m_2+c)-2c\theta}{\sqrt{(m_2+c)^2-4m_2c\theta}}
\]

6.6. APPLICATION

For illustration, the model has been applied to a set of data on MRBI with marriage duration \(T=7\) years, taken from the survey entitled `Survey on status of women and their fertility histories in urban Assam' conducted during the year 1991-92. Couples who used any contraceptives during the last seven years preceding the survey date were excluded from the analysis. However, the incidence of contraception was found to be negligible. To overcome the selectivity bias in a retrospective survey, the guidelines of Rindfuss \textit{et al} (1982) has been followed here. A subset, which is bounded by the current (at the survey date) ages of 15-43 years and is initiated 7 years ending a year before the survey date, of the available set of retrospective data had been analysed in this study (Figure 6.1.).
For the application of the model we have to estimate \( \alpha_p, m_p, h_p, \theta_j \) and \( c_j \). Obviously the model is too much involved with regard to number of parameters. In a study based on the hospital records of Indian women, Srinivasan \textit{et al} (1986) found that median duration of gestation for mature and premature births were approximately 276 and 252 days respectively, and the percentage of premature births was 6 per cent. Hence, in our study we have assumed that the period of gestation leading to a live birth is 0.75 year. The analysis of birth interval data based on the present survey has revealed that the average length of the birth intervals, except for the first, are approximately equal. These intervals are shorter than the first. Due to social taboos and frequent visits of the females to their parents house during the early wedding years, the fecundability between marriage and the first birth will be different from that of subsequent birth intervals. Further fecundability of Indian women remains more or less constant upto the age of 30 years (Nath, 1984). So we have assumed that \( m'_0 = m_1 \) and \( m'_j = m_2 \) (\( j \geq 1 \)), where \( m_1 < m_2 \). It is also observed from earlier studies (Bhattacharya and Nath, 1984,1985; Singh \textit{et al}, 1983) that the non-susceptible period associated with a live birth or a foetal loss, as well as, the chance of terminating a conception through a foetal loss show little variations with order of birth. Hence, these parameters may be assumed to be same for all parities. We have assumed that \( \theta_j = 0.9 \) year and \( c_j = 2 \) years, for all \( j \) (Bhattacharya and Nath, 1984,1985; Singh \textit{et al}, 1983). Again, from chapter III, we have found that the average duration of PPA associated with a live birth for the mothers of this society is six months. Therefore in the application of the model we have assumed that \( h_j = 1.25 \) years (0.5 year for PPA and 0.75 year for gestation). Further, it has been observed that the incidence of secondary sterility and contraceptive practices among the females of interest are very low. Thus \( (1-\alpha_i) \), \( i \geq 1 \), can reasonably assumed to be 0 i.e. \( \alpha_i = 1 \neq \alpha_o \), \( i \geq 1 \). \( \alpha_o \) is obtained by equating the relative frequency of females with no conception to the theoretical probability of zero conception. Here, \( \hat{\alpha}_o \) is found to be 0.975. The value of \( \alpha_o \) is found to be consistent with that of Singh \textit{et al} (1974, 1988).
As in most of the surveys, in this survey also there was not sufficient number of observations on MRBI for each parity when females of a specific duration of marriage were grouped. Hence, we have applied Model M2 as an illustration to the observed data.

The proposed model is applied to the observed set of data to estimate $m_1$ and $m_2$. Fortran 77 programs have been developed for calculating probabilities, pilot values and variance-covariance matrix of the estimators. Then by using scoring method it has been found that in this population (i) the risk of conception between marriage and the first live birth i.e., $\hat{m}_1=0.34$ and (ii) the risk of other conception (Except the first) i.e., $\hat{m}_2=0.45$. The values of $m_1$ and $m_2$ are found to be highly negatively correlated. The model is used to a set of observed data with $T=7$ years. The observed and expected frequencies are presented in Table 6.1. The estimated values of the parameters and their variances and covariances are shown in Table 6.2. The calculated value of (chi-square) shows that the proposed model describes the observed data adequately.

The likelihood ratio criterion (Wilks, 1962) is used to test the null hypothesis that the model considering the risk of conception $m'_j, j \geq 0$ to be same for all orders of birth against the alternative hypothesis that all these $m'_j$s except $m'_0$ are equal. For this, the calculated value of chi-square is found to be significant. This suggest that in the present data, the risk of conception depends on parity.
Table 6.1. Distribution of married females according to their most recent birth interval (T=7 years).

<table>
<thead>
<tr>
<th>Class Interval (in years)</th>
<th>Observed frequency (O)</th>
<th>Expected frequency (E)</th>
<th>(\frac{(O-E)^2}{E})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75-1.50</td>
<td>39</td>
<td>44.1</td>
<td>0.5897</td>
</tr>
<tr>
<td>1.50-2.50</td>
<td>61</td>
<td>56.6</td>
<td>0.3420</td>
</tr>
<tr>
<td>2.50-3.50</td>
<td>33</td>
<td>30.8</td>
<td>0.1571</td>
</tr>
<tr>
<td>3.50-4.50</td>
<td>13</td>
<td>15.9</td>
<td>0.5289</td>
</tr>
<tr>
<td>4.50-5.50</td>
<td>6</td>
<td>7.3</td>
<td>0.2315</td>
</tr>
<tr>
<td>5.50-7.00</td>
<td>7</td>
<td>4.3</td>
<td>1.6953</td>
</tr>
</tbody>
</table>

TOTAL                     | 159                    | 159.0                  | 3.54                

\(\chi^2\) (calculated) = 3.54
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{m}_1$</td>
<td>0.34</td>
</tr>
<tr>
<td>$\hat{m}_2$</td>
<td>0.45</td>
</tr>
<tr>
<td>$V(\hat{m}_1)\times 10^5$</td>
<td>116.00</td>
</tr>
<tr>
<td>$V(\hat{m}_2)\times 10^5$</td>
<td>118.00</td>
</tr>
<tr>
<td>$r(\hat{m}_1, \hat{m}_2)$</td>
<td>-0.84</td>
</tr>
</tbody>
</table>
Fig: 6.1 Distribution of Most Recent Birth Interval
Fig 6.1. Lexis diagram representing the subset selected for analysis.