CHAPTER - 3

Mathematical structure of symptoms of schizophrenia
MATHMATICAL STRUCTURE OF SYMPTOMS OF SCHIZOPHRENIA

3.1 Introduction

It has been mentioned in Chapters 1 and 2 that schizophrenia is characterized by various symptoms like sleep disturbance, irritability, suspiciousness, irrelevant speech, sudden spells of laughing and weeping, over religiousness, depression, aggression and more importantly, the presence of various types of delusions and hallucinations and so on. These symptoms in isolation are however not unique to schizophrenia. A cluster of symptoms with definite weightage lead to a diagnosis of schizophrenia (Kaplan & Sadock, 1999, Gelder et al., 1996). As such, in the early times, variability in diagnostic procedure existed in the different parts of the world. The diagnostic uncertainty and prevailing unreliability in diagnostic procedure made attempts at international comparison and compiling of data very difficult. This led to the concentrated international efforts at developing a standardized classificatory system regarding the symptoms of schizophrenia.

In this chapter, one such classification, the Diagnostic Criteria for Research (DCR), which has been worked out by WHO (Clinical Descriptions and Diagnostic Guidelines, 2002) for the diagnosis of schizophrenia, has been considered. Under this classification, the various symptoms that are observed in a schizophrenic can be broadly classified, for diagnostic convenience, into three types. An attempt has been made to mathematically structure these different types of symptoms and their interactions, find their structure functions and
ultimately evaluate the approximate probabilities of occurrence of the different categories.

3.2 Diagnostic criteria

According to the DCR, for a diagnosis of schizophrenia, at least one of the symptoms listed below under Type A or at least two of the symptoms listed under Type B should be present for most of the time during an episode of psychotic illness. All additional symptoms, some of which always accompany a Type A or Type B symptom and are observable in schizophrenic patients but are not particularly required for diagnosis of schizophrenia, are included in Type C.

3.2.1 Type A: Under this type, we have various hallucinations and delusions as follows-

A₁: Thought echo, thought insertion or withdrawal, or thought broadcasting.

A₂: Hallucinatory voices giving a running commentary on the patient's behaviour, or discussing the patient amongst themselves, or other types of hallucinatory voices coming from some part of the body.

A₃: Delusions of control, influence or passivity, clearly referred to body or limb movements of specific thoughts, actions or sensations; delusional perception.

A₄: Persistent delusions of other kinds that are culturally inappropriate and completely impossible (eg. Being able to control the weather, to communicate with aliens of another world etc.)
3.2.2 Type B: Under this type, we have,

B₁: Persistent hallucinations when accompanied by fleeting delusions without clear affective content, or when accompanied by persistent overvalued ideas.

B₂: Neologisms, breaks or interpolations in the train of thought resulting in incoherent or irrelevant speech.

B₃: Catatonic behaviour such as excitement including aggression, abusiveness and laughing and crying to self, posturing, waxy flexibility, negativism, mutism and stupor.

B₄: Negative symptoms such as marked apathy, paucity of speech and blunting or incongruity of emotional responses. This includes depression, deterioration of role functioning and lack of concentration.

3.2.3 Type C: Under this category, we have

C₁: Sleep disturbance

C₂: Irritability

C₃: Lack of inhibition

C₄: Wandersome attitude.

C₅: Increased religiousness

C₆: Deterioration of personal hygiene.

In general, these six symptoms are considered common in schizophrenia though they, by themselves are not considered important for the diagnosis of
It has been observed that any one or more of these additional symptoms are often manifested in a schizophrenic patient.

### 3.3 Structure functions of different categories of symptoms

Given below are the structure functions of the above mentioned categories of symptoms of schizophrenia with respect to their presence at the time of diagnosis.

#### 3.3.1 Structure function for the Type A category of symptoms of schizophrenia

Let $A_i = 1$, if the $i^{th}$ Type A symptom is present, $i = 1, 2, 3, 4$

$= 0$, otherwise.

..............................(3.1)

As the diagnostic criteria states that at least one of the Type A symptom is sufficient for a diagnosis of schizophrenia, the structure of the Type A category can be diagrammatically represented in parallel connection, which may be called a rope model, as follows.

**Fig 3.1** Structure of symptoms of Type A

![Diagram of Type A symptoms structure]

Patient $\rightarrow A_3 \rightarrow A_4 \rightarrow$ Schizophrenia
Define,

$$\Phi(A) = \begin{cases} 
1, & \text{if Type A symptoms are present} \\
0, & \text{otherwise.} 
\end{cases} \quad \text{(3.2)}$$

where $\Phi(A) = \max (A_1, A_2, A_3, A_4)$

\[\therefore \text{Probability of presence of schizophrenia with Type A symptoms is given by} \]

$$\psi(A) = P(\Phi(A) = 1)$$

$$= P\{1 - (1 - A_1)(1 - A_2)(1 - A_3)(1 - A_4)\}$$

$$= 1 - \prod_{i=1}^{4} P(1 - A_i) \quad \text{.........(3.3)}$$

3.3.2 Structure function for the Type B category of symptoms of schizophrenia

Let $B_j = \begin{cases} 
1, & \text{if the } j^{th} \text{ Type B symptom is present, } j = 1,2,3,4 \\
0, & \text{otherwise.} 
\end{cases} \quad \text{.........(3.4)}$

As the diagnostic criteria states that at least two of the Type B symptoms need to be present for a diagnosis of schizophrenia, the structure of the Type B category can be diagrammatically represented in a rope model as follows.
Define, \( \Phi(B) = 1 \), if two Type B symptoms are present
\( = 0 \), otherwise.  
...............(3.5)

where \( \Phi(B) = \max (B_1B_2, B_1B_3, B_1B_4, B_2B_3, B_2B_4, B_3B_4) \)
\( = \max (\min (B_i, B_j)) \)

\[ \therefore \text{Probability of presence of schizophrenia with Type B symptoms is given by} \]

\[ \psi(B) = P(\Phi(B) = 1) \]
\[ = P\{\max (\min (B_i, B_j))\} \]
\[ = 1 - \prod_{i \neq j = 1}^{4} P(1 - B_iB_j) \]
...............(3.6)

3.3.3 Structure function for the Type C category of symptoms

Let \( C_k = 1 \), if the \( k^{th} \) Type C symptom is present, \( k = 1,2,3,4,5,6 \)
\( = 0 \), otherwise.  
...............(3.7)
Any one or more of these Type C symptoms can be present with the Type A and Type B symptoms described in 2.1 and 2.2 above. The structure of the Type C category can be diagrammatically represented as follows.

**Fig 3.3 Structure of symptoms of Type C**

Define,

\[ \Phi(C) = 1, \text{ if a Type C symptom is present} \]
\[ = 0, \text{ otherwise.} \]  

where \( \Phi(C) = \max (C_1, C_2, C_3, C_4, C_5, C_6) \)
\[ = \max (C_k, k = 1, 2, ..., 6) \]

\[ \vdots \text{ Probability of presence of Type C symptoms is given by} \]

\[ \Psi(C) = P(\Phi(C) = 1) \]
\[ = P \{ \max (C_k) \} \]
\[ = 1 - \prod_{k=4}^{6} P(1 - C_k) \]  

\[ \text{........(3.9)} \]
3.3.4  **Structure function for simultaneous presence of symptoms of Types A & B**

A schizophrenic patient can show presence of both Type A and Type B symptoms simultaneously. Then the patient can have any one or more of Type A and any one or more of Type B symptoms. In this case, there need not be at least two Type B symptoms as the diagnosis has been made possible by the presence of the Type A symptom.

Let  

\[ A_{ij} = 1, \quad \text{if the } i^{th} \text{ symptom of Type A and the } j^{th} \text{ symptom of Type B are present.} \quad (i = 1, 2, 3, 4; \ j = 1, 2, 3, 4) \]

\[ = 0, \quad \text{otherwise.} \quad \ldots \ldots \ldots (3.10) \]

The structure function of simultaneous occurrence of symptoms of Types A and B in schizophrenia can be diagrammatically represented in a chain model (series connection), while the individual structures of Type A and Type B are in rope structure (parallel connection), as follows.

**Fig 3.4**  Structure showing simultaneous occurrence of symptoms of Types A & B
Define

\[ \Phi(AB) = 1, \text{ if symptoms of Types A and B are present} \]
\[ = 0, \text{ otherwise.} \]  
..........\( (3.11) \)

where \( \Phi(AB) = \min (\Phi(A), \Phi'(B)) \)

where \( \Phi'(B) = \max (B_1, B_2, B_3, B_4) \)

and, \( \Phi'(B) = 1, \text{ if any one or more Type B symptom is present} \)
\[ = 0, \text{ otherwise} \]  
..........\( (3.12) \)

\[ \therefore \quad \psi'(B) = P(\Phi'(B) = 1) = 1 - \prod_{i=1}^{4} P(1-B_i) \]  
..........\( (3.13) \)

\[ \therefore \text{ Probability of presence of schizophrenia with symptoms of Types A and B is given by} \]
\[ \psi(AB) = P(\Phi(AB) = 1) = P(\Phi(A) = 1) P(\Phi'(B) = 1) \]
\[ = \psi(A) \psi'(B) \]  
..........\( (3.14) \)

3.4 Structure functions of situations manifested in schizophrenic patients

As mentioned earlier, a patient diagnosed as a schizophrenic will have either Type A symptoms or Type B symptoms or can have both Type A and Type B symptoms together. In addition, some Type C symptoms will also be manifested simultaneously in every situation (Gelder et al., Case studies). Thus there could be three situations for a patient suffering from schizophrenia. The structure functions of these three cases are discussed below.
3.4.1 Case I: Suppose that a patient shows the presence of Type A symptoms alone or Type A along with some Type C symptoms. This situation can be diagrammatically represented as follows.

![Fig 3.5](structure_of_symptoms.png) Structure of schizophrenia with symptoms of Type A

As shown in 3.3.4, we can define,

\[ \Phi(AC) = 1, \text{ if symptoms of Types A and C are present} \]
\[ = 0, \text{ otherwise.} \] .........(3.15)

where \( \Phi(AC) = \min(\Phi(A), \Phi(C)) \)

\[ = \text{presence of symptoms of both Types A and C} \]

\[ \therefore \text{Probability of presence of schizophrenia with symptoms of Types A and C is given by} \]

\[ \psi(AC) = \Pr(\Phi(AC) = 1) \]
\[ = \Pr(\Phi(A) = 1) \Pr(\Phi(C) = 1) \]
\[ = \psi(A) \psi(C) \] .........(3.16)
Also define,

$$\Phi(I) = \begin{cases} 1, & \text{if either Type A alone or both Types A and C are present} \\ 0, & \text{otherwise} \end{cases} \quad \text{.........(3.17)}$$

where \( \Phi(I) = \max(\Phi(A), \Phi(AC)) \)

$$\psi(I) = P(\text{Case I}) = P(\Phi(I) = 1)$$

= Probability of presence of Type A alone or Type A along

. with Type C

= 1 - \left\{ 1 - P(\Phi(A) = 1) \right\} \left\{ 1 - P(\Phi(AC) = 1) \right\} 

= 1 - \left\{ 1 - \psi(A) \right\} \left\{ 1 - \psi(AC) \right\} 

= \psi(A) + \psi(AC) - \psi(A) \cdot \psi(AC) \quad \text{.........(3.18)}$$

### 3.4.2 Case II:

Suppose that a patient shows the presence of Type B symptoms alone or Type B along with some Type C symptoms. This situation can be diagrammatically represented as follows.

**Fig 3.6** Structure of schizophrenia with symptoms of Type B
As shown in 3.3.4, we can define,
\[ \Phi(BC) = 1, \text{ if symptoms of Types B and C are present} \]
\[ = 0, \text{ otherwise.} \] ...............(3.19)

where \( \Phi(BC) = \min (\Phi(B), \Phi(C)) \)

\[ = \text{presence of symptoms of both Types B and C} \]

\[ \therefore \text{Probability of presence of schizophrenia with symptoms of Types B and C is } \]
given by
\[ \psi(BC) = P(\Phi(BC) = 1) = P(\Phi(B) = 1) P(\Phi(C) = 1) \]
\[ = \psi(B) \psi(C) \] ...............(3.20)

Also define,
\[ \Phi(II) = 1, \text{ if either Type B alone or both Types B and C are present} \]
\[ = 0, \text{ otherwise} \] ...............(3.21)

where \( \Phi(II) = \max (\Phi(B), \Phi(BC)) \)

\[ \therefore \psi(II) = P(\text{Case II}) = P(\Phi(II) = 1) \]
\[ = \text{Probability of presence of Type B alone or Type B along with Type C} \]
\[ = 1 - [ \{1 - P(\Phi(B) = 1)\} \{1 - P(\Phi(BC) = 1)\} ] \]
\[ = 1 - [ \{1 - \psi(B)\} \{1 - \psi(BC)\}] \]
\[ = \psi(B) + \psi(BC) - \psi(B) \cdot \psi(BC) \] ...............(3.22)
3.4.3 Case III: Suppose that a patient shows the presence of one or more symptoms of both Types A and B or of Types A and B along with one or more Type C symptoms. This situation can be diagrammatically represented as follows.

**Fig 3.7** Structure of schizophrenia with symptoms of Types A & B

As shown in 3.3.4, we can define,

\[ \Phi(ABC) = \begin{cases} 1, & \text{if symptoms of Types A, B and C are present} \\ 0, & \text{otherwise.} \end{cases} \] ........(3.23)

where \( \Phi(ABC) \) = presence of symptoms of Types A, B and C

\[ = \min (\Phi(AB), \Phi(C)) \] [where \( \Phi(AB) = \min (\Phi(A), \Phi(B)) \)]

\[ \therefore \text{Probability of presence of schizophrenia with symptoms of Types A, B and C is given by} \]

\[ \psi(ABC) = P(\Phi(ABC) = 1) = P(\Phi(AB) = 1) P(\Phi(C) = 1) \]

\[ = \psi(AB) \psi(C) \] ........(3.24)

[where \( \psi(AB) \) is as defined in section 3.3.4]
Also define,

\[ \Phi(III) = \begin{cases} 1, & \text{if either Types A and B alone or all of Types A, B & C are present} \\ 0, & \text{otherwise} \end{cases} \] ...........(3.25)

where \( \Phi(III) = \max (\Phi(AB), \Phi(ABC)) \)

\[ \therefore \psi(III) = P(\text{Case III}) = P(\Phi(III) = 1) \]

= Probability of presence of Types A and B alone or all of Types A, B & C

\[ = 1 - \left[ (1 - P(\Phi(AB) = 1))(1 - P(\Phi(ABC) = 1)) \right] \]

\[ = 1 - \left[ (1 - \psi(AB))(1 - \psi(ABC)) \right] \]

\[ = \psi(AB) + \psi(ABC) - \psi(AB) \cdot \psi(ABC) \] ...........(3.26)

3.4.4 Final structure function of a patient diagnosed as schizophrenic

Cases I, II and III constitute the three mutually exclusive forms of occurrence of symptoms in a schizophrenic patient. However, it cannot be said that they are exhaustive as there could be other situations which have not yet been recognized or explored. Taking these three cases into consideration, the final structure of the symptoms of a patient diagnosed with schizophrenia can be diagrammatically represented as follows.
Define

$$\Phi(\text{DS}) = 1, \text{ if a patient is diagnosed as a schizophrenic}$$

$$= 0, \text{ otherwise} \quad \text{.........(3.27)}$$

$$\therefore \quad \Psi(\text{DS}) = P(\Phi(\text{DS}) = 1)$$

$$= \max \{ P(\Phi(\text{I}) = 1), P(\Phi(\text{II}) = 1), P(\Phi(\text{III}) = 1) \}$$

$$= 1 - \left[ \{1 - P(\Phi(\text{I}) = 1)\}\{1 - P(\Phi(\text{II}) = 1)\}\{1 - P(\Phi(\text{III}) = 1)\} \right]$$

$$= 1 - \left[ \{1 - \Psi(\text{I})\}\{1 - \Psi(\text{II})\}\{1 - \Psi(\text{III})\} \right] \quad \text{.........(3.28)}$$

$$\Rightarrow \quad \Psi(\text{DS}) \text{ is a function of } \Psi(\text{I}), \Psi(\text{II}) \text{ and } \Psi(\text{III})$$

### 3.5 Calculation of the various probabilities defined in section 3

It is to be noted that at the initial stage of contact with a treatment agency, every symptom, whether it belongs to category A or B or C, has equal chance of being present in a patient. It is a medically accepted fact that there is no reason to expect the presence of any symptom over another in a patient. (Freedman et al., 2002)
Let us take \( w \) as the probability of initial occurrence of a symptom i.e.,

\[
W = P(A_i = 1) = P(B_j = 1) = P(C_k = 1)
\] ........(3.29)

From (3.3), we have

\[
\psi(A) = 1 - \prod_{i=1}^{4} P(1 - A_i)
\]

\[
= 1 - \{ (1 - P(A_1)) (1 - P(A_2)) (1 - P(A_3)) (1 - P(A_4)) \}
\]

\[
= 1 - \{ (1-w) (1-w) (1-w) (1-w) \}
\]

\[
= 1 - (1-w)^4
\] ........(3.30)

From (3.6), we have

\[
\psi(B) = 1 - \prod_{i=1}^{4} P(1 - B_i B_j)
\]

\[
= 1 - \{ (1 - P(B_1 B_2)) (1 - P(B_1 B_3)) ... (1 - P(B_3 B_4)) \}
\]

\[
= 1 - \{ (1-w^2) (1-w^2) ... (1-w^2) \}
\]

\[
= 1 - (1-w^2)^6
\] ........(3.31)

From (3.9), we have

\[
\psi(C) = 1 - \prod_{k=1}^{6} P(1 - C_k)
\]

\[
= 1 - \{ (1 - P(C_1)) (1 - P(C_2)) (1 - P(C_3)) ... (1 - P(C_6)) \}
\]

\[
= 1 - \{ (1-w) (1-w) ... (1-w) \} = 1 - (1-w)^6
\] ........(3.32)
From (3.14), (3.13) and (3.30), we have

\[ \psi (AB) = \psi (A) \psi'(B) \]

\[ = \{1 - (1-w)^4\} \{1 - (1-w)^4\} = \{1 - (1-w)^4\}^2 \] .......(3.33)

From (3.16), (3.30) and (3.32), we have

\[ \psi (AC) = \psi (A) \psi(C) \]

\[ = \{1 - (1-w)^4\} \{1 - (1-w)^6\} \] .......(3.34)

From (3.20), (3.31) and (3.32), we have

\[ \psi (BC) = \psi (B) \psi(C) \]

\[ = \{1 - (1-w^2)^6\} \{1 - (1-w)^6\} \] .......(3.35)

From (3.24), (3.32) and (3.33), we have

\[ \psi (ABC) = \psi (AB) \psi(C) \]

\[ = \{1 - (1-w)^4\}^2 \{1 - (1-w)^6\} \] .......(3.36)

\( \psi (I), \psi (II), \psi (III) \) and \( \psi (DS) \) can be evaluated by substituting the above values in their respective formulae given in (3.18), (3.22) and (3.26).

3.6 Results and interpretations of numerical work done

Since \( w \) is the occurrence probability of a symptom, therefore \( 0 < w < 1 \). Taking different values of \( w \) from 0.05 to 0.95, the probabilities defined in section 3.5
have been evaluated using Microsoft Office Excel 2003. The results obtained
(Annexure 1) are presented graphically along with their interpretations.

3.6.1 Behaviour of symptoms of Types A, B and AB as functions of \( w \)

**Fig 3.9** Graph showing behaviour of symptoms of Types A, B and AB
as functions of occurrence probability \( w \)

Interpretation of the graph shown in Fig. 3.9

In the above figure, the different values of \( w \) are taken along the x-axis and
probabilities of presence of schizophrenia are taken along the y-axis. As marked,
the different coloured lines represent the probabilities of presence of symptoms
of Types A, B and AB in schizophrenia corresponding to different values of the probability of occurrence of a symptom at the initial stage. The line graph indicates that the presence of Type A symptoms is highest and in fact, from relevant data as mentioned in Chapter 7, it is seen that the Type A symptoms are more commonly noticed in patients. Again, Type AB curve is higher than Type B curve implying a greater occurrence of Types A and B together than Type B alone. Infact, while studying the data sheets, it has been noticed that there have been comparatively fewer patients diagnosed with schizophrenia having two or more of Type B symptoms in absence of any Type A symptom. The occurrence of both types together has been found to be more common and this has been reflected well in the figure. This has been dealt with in greater detail in Chapter 7.

All the three lines are smooth curves and the overall trend for all the curves is increasing and rightly so, as this indicates increase in the probability of occurrence of schizophrenia in every category of symptoms along with an increase in the initial probability of occurrence of a symptom i.e., \( w \). Mathematically, as \( w \) increases, \( \psi (.) \) also increases. It is also seen that \( \psi (A) \), \( \psi (B) \) and \( \psi (AB) \) are well behaved functions and converges to unity as the occurrence probability \( w \) increases.
3.6.2 Behaviour of symptoms of Types AC, BC, and ABC as functions of \( w \)

**Fig 3.10** Graph showing behaviour of symptoms of Types AC, BC, and ABC as functions of occurrence probability \( w \)

Interpretation of the graph shown in Fig. 3.10

In the above figure, the different values of \( w \) are taken along the x-axis and probabilities of presence of schizophrenia are taken along the y-axis. As marked, the different coloured lines represent the probabilities of presence of schizophrenia with symptoms of Types A and C together, Types B and C together and all of Types A, B and C together corresponding to different values of the probability of occurrence of a symptom at the initial stage. The line graph indicates that the presence of symptoms of Types A and C together is highest
followed by presence of Types A, B and C together which shows the prevalence of Type A over Type B as noticed in the previous figure also.

The overall trend for all the curves is increasing and rightly so, as this indicates increase in the probability of occurrence of schizophrenia in every category of symptoms along with an increase in the initial probability of occurrence of a symptom i.e., $w$. In this case also it is seen that $\psi(AC)$, $\psi(BC)$ and $\psi(ABC)$ are well behaved functions and converges to unity as the occurrence probability $w$ increases.

3.6.3 Behaviour of the possible cases of schizophrenia as functions of $w$

Fig 3.11  Graph showing different cases of schizophrenia as functions of occurrence probability $w$
Interpretation of the graph shown in Fig. 3.11

As mentioned in section 3.4, $\Psi$ (I), $\Psi$ (II) and $\Psi$ (III) are functions of $w$. In the above figure, the different values of $w$ are taken along the x-axis and probabilities of presence of schizophrenia are taken along the y-axis. As marked, the different coloured lines represent the probabilities of presence of schizophrenia under Cases I, II and III mentioned in section 3.4 corresponding to different values of $w$. The line graph indicates that the occurrence of Case I is highest followed by Case III which again emphasizes the prevalence of symptoms of Type A over the prevalence of Type B, in presence of Type C.

The overall trend for all the curves is increasing and rightly so, as this indicates increase in the probability of occurrence of schizophrenia in every case along with an increase in the initial probability of occurrence of a symptom i.e., $w$.

3.6.4 Probability of diagnosis of schizophrenia as a function of $w$

Fig 3.12  Graph showing probability of diagnosis of schizophrenia as a function of occurrence probability $w$
Interpretation of the graph shown in Fig. 3.12

It has already been mentioned in section 3.4.4 that $\psi$ (DS) is a function of $\psi$ (I), $\psi$ (II) and $\psi$ (III) which in turn are functions of $w$. In the above figure, the different values of $w$ are taken along the x-axis and probabilities of diagnosis of schizophrenia (DS) are taken along the y-axis. The line graph represents the probability of diagnosing a patient as a schizophrenic corresponding to different values of the initial occurrence of a symptom. It is seen that as long as $w$ is small, the desired probability $\psi$ (DS) is also low and gradually increasing. As $w$ takes values from 0.05 to 0.2, the graph shows a gradual rise more or less at a gradient of 45 degrees and for $w$ from 0.3 to 0.9, the probability is more or less stable at unity which implies that whatever the probability of initial occurrence of a symptom might be starting from 0.3, a person with symptoms of Types A or/and B with C, will always be diagnosed as a schizophrenic.

3.7 Conclusion

In this chapter we have attempted to find the structure functions of the different categories of symptoms of schizophrenia and thereby evaluate the occurrence probabilities of the different categories at the time of diagnosis. The modeling has been done on the basis of the diagnostic criteria accepted by the medical fraternity. It has been observed that the occurrence of symptoms in real life, as seen from the collected data (details in Chapter 7), is more or less in accordance
with the evaluated probabilities. However, a properly designed clinical assessment or evaluation will probably lead to a clearer pattern in the occurrence of symptoms. Our next endeavour would be to study the behaviour of the symptoms present at the time of diagnosis, during the course of treatment.