CHAPTER V

DETERMINATION OF EXPECTED TIME TO RECRUITMENT FOR THE BREAK DOWN THRESHOLD HAS FOUR COMPONENTS*

5.1 Introduction

Determination of the expected time to recruitment based on the magnitude of attrition at decision epochs, the likely time at which the total attrition reaches a particular level called threshold is an important aspect. If the total magnitude of manpower depletion crosses the threshold level the breakdown of the organization occurs. In this Chapter, expected time to recruitment when the breakdown threshold has four components has been discussed by using the concept of Shock model and cumulative damage process.

5.2 Assumptions

(i) The depletion of manpower due to attrition occurs at decision making epochs. The decisions are regarding pay, perquisites and the work schedule.

(ii) The policy decisions are taken at random time intervals.

(iii) The breakdown of the organization or activities occurs when the total magnitude manpower depletion crosses a particular level called the threshold.

(iv) The threshold comprises of three different components namely the basic level of manpower attrition which can be permitted, so that the normal activity will not suffer. The manpower which can be raised by transfer personnel from sister organizations and the extra time work of existing workers.

(v) The manpower depletion is measured in term of man hours.

5.3 Notations

(i) \( W = \) The threshold of manpower depletion which can be permitted and \( W = Y_1 + Y_2 + Y_3 + Y_4 \)

(ii) \( Y_1 = \) the natural level of attrition which can be permitted

(iii) \( Y_2 = \) the manpower available due to transfer of personnel

(iv) \( Y_3 = \) the manpower available due to extra time work of existing workers.

(v) \( Y_4 = \) the manpower available due to taking more frequent breaks of existing workers.

(vi) \( W \) has the probability density function \( h(.) \) and cumulative distribution function \( H(.) \).

(vii) \( X_i = \) The magnitude of manpower depletion at each decision epoch, \( i = 1,2,\ldots,k \) and with probability density function \( g(.) \) and cumulative distribution function \( G(.) \).

(viii) \( U_i = \) The interarrival times between successive decision epochs \( i = 1,2,\ldots,k \) and with probability density function \( f(.) \) and cumulative distribution function \( F(.) \).

(ix) \( T = \) A random variable denoting the time to recruitment and it has probability density function \( l(.) \) and cumulative distribution function \( L(.) \).

(x) \( F_k(.) = k \text{ fold convolution of } F(.) \).

(xi) \( g_k(.) = k \text{ fold convolution of } g(.) \)

(xii) \( g^*, F^*, f^* \) are the respective Laplace transforms.

5.4 Results

The expected time to breakdown of the organization in other words expected time to recruitment is to be determined. \( S(t) = P(T > t) = \) survivor function which indicates that the breakdown of the organization due manpower attrition does not happen in \((0,t]\).
Now, \[ p \left[ \sum_{i=1}^{k} X_i < W \right] = \int_{0}^{\infty} g_k(x) \frac{H(x)}{x} \, dx \]

= \text{pr}[\text{that the total depletion of manpower on k decision epochs does not exceed the threshold W}]

Where \[ H(x) = 1 - H(x) \]

\[ S(t) = P(T > t) = \text{the probability that the breakdown does not occur before t} \]

= \text{pr} \left[ \text{there are exactly k occasions of decision making in (0, t] \times p \left[ \sum_{i=1}^{k} X_i < W \right] } \right]

= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \text{pr} \left[ \sum_{i=1}^{k} X_i < W \right]

where \( F_k(t) - F_{k+1}(t) \) denotes the probability that there are k occasions of decision making in (0, t], by renewal theory.

Assuming that

\[ Y_1 \sim \exp (\theta_1), Y_2 \sim \exp (\theta_2), Y_3 \sim \exp (\theta_3) \text{ and } Y_4 \sim \exp (\theta_4), \]

we have by convolution formula,

\[ h(w) = \int_{0}^{\infty} \frac{\theta_1 \theta_2 \theta_3 \theta_4}{\theta_3 - \theta_2 - \theta_1} (e^{-\theta_3 z} - e^{-\theta_2 z} - e^{-\theta_1 z}) \theta_4 e^{-\theta_4 (w-z)} \, dz \]

\[ = \frac{\theta_1 \theta_2 \theta_3 \theta_4}{\theta_3 - \theta_2 - \theta_1} \int_{0}^{\infty} e^{-\theta_3 z + \theta_4 z - \theta_4 w} \, dz - \frac{\theta_1 \theta_2 \theta_3 \theta_4}{\theta_3 - \theta_2 - \theta_1} \int_{0}^{\infty} e^{-\theta_2 z + \theta_4 z - \theta_4 w} \, dz \]

\[ - \frac{\theta_1 \theta_2 \theta_3 \theta_4}{\theta_3 - \theta_2 - \theta_1} \int_{0}^{\infty} e^{-\theta_1 z + \theta_4 z - \theta_4 w} \, dz \]

\[ \ldots (5.1) \]

It can be shown by simplification that

\[ h(w) = \frac{\theta_1 \theta_2 \theta_3 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_4 - \theta_3)} \left\{ (\theta_4 - \theta_3) e^{-\theta_1 w} + (\theta_4 - \theta_2) e^{-\theta_2 w} + (\theta_4 - \theta_1) e^{-\theta_3 w} + (2\theta_4 - \theta_3 - \theta_2 - \theta_1) e^{-\theta_4 w} \right\} \]

\[ \ldots (5.2) \]

\[ H(y) = \frac{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_4 - \theta_3)(\theta_4 - \theta_2)(\theta_4 - \theta_3)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_4 - \theta_3)(\theta_4 - \theta_2)(\theta_4 - \theta_3)} \left\{ (\theta_4 - \theta_3) \int_{0}^{y} e^{-\theta_1 w} \, dw + (\theta_4 - \theta_2) \int_{0}^{y} e^{-\theta_2 w} \, dw + (\theta_4 - \theta_1) \int_{0}^{y} e^{-\theta_3 w} \, dw + (2\theta_4 - \theta_3 - \theta_2 - \theta_1) \int_{0}^{y} e^{-\theta_4 w} \, dw \right\} \]

\[ \ldots (5.3) \]
\[ H(y) = \frac{\theta_2 \theta_3 \theta_4 (1 - e^{-\theta_1 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)} + \frac{\theta_1 \theta_3 \theta_4 (1 - e^{-\theta_2 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} + \frac{\theta_1 \theta_2 \theta_4 (1 - e^{-\theta_3 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} + \frac{\theta_1 \theta_2 \theta_3 (2\theta_4 - \theta_3 - \theta_2 - \theta_1)(1 - e^{-\theta_4 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} \quad \ldots (5.4) \]

\[ \overline{H(y)} = 1 - H(y) \]

\[ = 1 - \frac{\theta_2 \theta_3 \theta_4 (1 - e^{-\theta_1 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)} + \frac{\theta_1 \theta_3 \theta_4 (1 - e^{-\theta_2 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} + \frac{\theta_1 \theta_2 \theta_4 (1 - e^{-\theta_3 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} + \frac{\theta_1 \theta_2 \theta_3 (2\theta_4 - \theta_3 - \theta_2 - \theta_1)(1 - e^{-\theta_4 y})}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} \quad \ldots (5.5) \]

\[
\begin{align*}
(1 - (\theta_2 \theta_3 \theta_4 (\theta_4 - \theta_3) + \theta_1 \theta_3 \theta_4 (\theta_4 - \theta_2) + (\theta_1 \theta_2 \theta_4 (\theta_4 - \theta_2)
+ & (\theta_1 \theta_2 \theta_3 (2\theta_4 - \theta_3 - \theta_2 - \theta_1))))
= \frac{1}{((\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(((\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)))}
\end{align*}
\]

\[ + \left\{ (\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(((\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)) \right\} \theta_2 \theta_3 \theta_4 (\theta_4 - \theta_3) e^{-\theta_4 y} + \theta_1 \theta_3 \theta_4 (\theta_4 - \theta_2) e^{-\theta_2 y} + \theta_1 \theta_2 \theta_4 (\theta_4 - \theta_2) e^{-\theta_3 y} + \theta_1 \theta_2 \theta_3 (2\theta_4 - \theta_3 - \theta_2 - \theta_1) e^{-\theta_4 y} \}
\]

\[ H(y) = E + \left\{ 1/((\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)) \right\}
\]

\[ \begin{align*}
& \frac{\theta_2 \theta_3 \theta_4 (\theta_4 - \theta_3) e^{-\theta_1 y} + \theta_1 \theta_3 \theta_4 (\theta_4 - \theta_2) e^{-\theta_2 y} + \theta_1 \theta_2 \theta_4 (\theta_4 - \theta_2) e^{-\theta_3 y} + \theta_1 \theta_2 \theta_3 (2\theta_4 - \theta_3 - \theta_2 - \theta_1) e^{-\theta_4 y} }\}
\end{align*} \quad \ldots (5.6) \]
Where

\[ E = (1 - (\theta_2 \theta_3 \theta_4 (\theta_4 - \theta_3) + \theta_1 \theta_2 \theta_4 (\theta_4 - \theta_2) + \theta_1 \theta_2 \theta_3 (2 \theta_4 - \theta_3 - \theta_2)) \frac{1}{((\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_4 - \theta_2)(\theta_4 - \theta_3))} \]

\[ s(t) = p[T > t] \]

\[ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \left\{ \int_{0}^{\infty} g_k(x)(E)dx + (\theta_2 \theta_3 \theta_4 (\theta_4 - \theta_3)) \frac{1}{((\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_2)(\theta_4 - \theta_3))} \int_{0}^{\infty} g_k(x)e^{-\theta_3 x}dx + (\theta_2 \theta_3 \theta_4 (\theta_4 - \theta_2)) \frac{1}{((\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_4 - \theta_2)(\theta_4 - \theta_3))} \int_{0}^{\infty} g_k(x)e^{-\theta_4 x}dx \right\} \]

... (5.7)

Let

\[ c_1 = \frac{\theta_2 \theta_3 \theta_4 (\theta_4 - \theta_3)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_4 - \theta_2)(\theta_4 - \theta_3)} \]

\[ c_2 = \frac{\theta_1 \theta_2 \theta_4 (\theta_4 - \theta_2)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)} \]

\[ c_3 = \frac{\theta_1 \theta_2 \theta_3 (\theta_4 - \theta_1)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)} \]

\[ c_4 = \frac{\theta_1 \theta_2 \theta_3 (2 \theta_4 - \theta_3 - \theta_2 - \theta_1)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)} \]
\[ s(t) = \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] \{E + c_1 g_k^*(\theta_1) + c_2 g_k^*(\theta_2) + c_3 g_k^*(\theta_3) + c_4 g_k^*(\theta_4)\} \]

\[ = E + c_1 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_1) + c_2 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_2) \]

\[ + c_3 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_3) + c_4 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_4) \]

\[ = E + c_1 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_1)^k + c_2 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_2)^k \]

\[ + c_3 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_3)^k + c_4 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_4)^k \]

\[ \ldots (5.8) \]

Now, \( c_1 \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)] g_k^*(\theta_1)^k \)

\[ = c_1 \{[F_0(t) - F_1(t)] g^*(\theta_1)^0 + [F_1(t) - F_2(t)] g^*(\theta_1)^1 + [F_2(t) \]

\[ - F_3(t)] g^*(\theta_1)^2 + \ldots\} \]

\[ = c_1 \{F_0(t) g^*(\theta_1)^0 - F_1(t) g^*(\theta_1)^0 + F_1(t) g^*(\theta_1)^1 - F_2(t) g^*(\theta_1)^1 - F_2(t) g^*(\theta_1)^2 \]

\[ - F_3(t) g^*(\theta_1)^2 + \ldots\} \]

\[ = c_1 \{1 - F_1(t)[1 - g^*(\theta_1)] - F_2(t) g^*(\theta_1) + F_1(t) g^*(\theta_1)^1 - F_2(t) g^*(\theta_1)^1 \]

\[ - F_2(t) g^*(\theta_1)^2 - F_3(t) g^*(\theta_1)^2 + \ldots\} \]

\[ = c_1 - [1 - g^*(\theta_1)] c_1 \sum_{k=1}^{\infty} F_k(t) g^*(\theta_1)^{k-1} \]
Similarly,

\[
c_2 - [1 - g^*(\theta_2)]c_2 \sum_{k=1}^{\infty} F_k(t)g^*(\theta_2)^{k-1}
\]

\[
c_3 - [1 - g^*(\theta_3)]c_3 \sum_{k=1}^{\infty} F_k(t)g^*(\theta_3)^{k-1}
\]

\[
c_4 - [1 - g^*(\theta_4)]c_4 \sum_{k=1}^{\infty} F_k(t)g^*(\theta_4)^{k-1}
\]

\[
s(t) = E + c_1 + c_2 + c_3 + c_4 - \{c_1(1 - g^*(\theta_1)) \sum_{k=1}^{\infty} F_k(t)g^*(\theta_1)^{k-1} + c_2(1 - g^*(\theta_2)) \sum_{k=1}^{\infty} F_k(t)g^*(\theta_2)^{k-1} + c_3(1 - g^*(\theta_3)) \sum_{k=1}^{\infty} F_k(t)g^*(\theta_3)^{k-1} + c_4(1 - g^*(\theta_4)) \sum_{k=1}^{\infty} F_k(t)g^*(\theta_4)^{k-1}\}
\]

\[\ldots (5.9)\]

\[
L(t) = 1 - S(t)
\]

Taking Laplace transform of both sides

\[
L^*(s) = 1 - (E + c_1 + c_2 + c_3 + c_4) + \{c_1(1 - g^*(\theta_1)) \sum_{k=1}^{\infty} \frac{F^*(s)^*}{s} g^k(\theta_1)^{k-1} + c_2(1 - g^*(\theta_2)) \sum_{k=1}^{\infty} \frac{F^*(s)^*}{s} g^k(\theta_2)^{k-1} + c_3(1 - g^*(\theta_3)) \sum_{k=1}^{\infty} \frac{F^*(s)^*}{s} g^k(\theta_3)^{k-1} + c_4(1 - g^*(\theta_4)) \sum_{k=1}^{\infty} \frac{F^*(s)^*}{s} g^k(\theta_4)^{k-1}\}
\]

\[\ldots (5.10)\]
\[ l^*(s) = 1 - (E + c_1 + c_2 + c_3 + c_4) + \{c_1 (1 - g^*(\theta_1)) \sum_{k=1}^{\infty} [F^*(s) g^*(\theta_1)]^{k-1} \]

\[ + c_2 (1 - g^*(\theta_2)) \sum_{k=1}^{\infty} [F^*(s) g^*(\theta_2)]^{k-1} + c_3 (1 - g^*(\theta_3)) \sum_{k=1}^{\infty} [F^*(s) g^*(\theta_3)]^{k-1} + c_4 (1 - g^*(\theta_4)) \sum_{k=1}^{\infty} [F^*(s) g^*(\theta_4)]^{k-1} \]
\[= 1 - (E + c_1 + c_2 + c_3 + c_4) + \left\{ c_1 \frac{1 - \left( \frac{\lambda}{\theta_1 + \lambda} \right)}{1 - \left( \frac{\alpha}{\theta_1 + \lambda} \right)} \frac{\alpha}{\alpha + s} + c_2 \frac{1 - \left( \frac{\lambda}{\theta_2 + \lambda} \right)}{1 - \left( \frac{\alpha}{\theta_2 + \lambda} \right)} \frac{\lambda}{\alpha + s} \right\} \]

\[+ c_3 \frac{1 - \left( \frac{\lambda}{\theta_3 + \lambda} \right)}{1 - \left( \frac{\alpha}{\theta_3 + \lambda} \right)} \frac{\alpha}{\alpha + s} + c_4 \frac{1 - \left( \frac{\lambda}{\theta_4 + \lambda} \right)}{1 - \left( \frac{\alpha}{\theta_4 + \lambda} \right)} \frac{\lambda}{\alpha + s} \}

\[= 1 - (E + c_1 + c_2 + c_3 + c_4) - \left\{ c_1 \frac{\theta_1}{\alpha \theta_1 + s(\theta_1 + \lambda)} + c_2 \frac{\theta_2}{\alpha \theta_2 + s(\theta_2 + \lambda)} \right\}

\[+ c_3 \frac{\theta_3}{\alpha \theta_3 + s(\theta_3 + \lambda)} + c_4 \frac{\theta_4}{\alpha \theta_4 + s(\theta_4 + \lambda)} \}

\[= -\left\{ \frac{\theta_1}{\alpha \theta_1 + s(\theta_1 + \lambda)} \frac{\theta_1}{\theta_1 + \lambda} + \frac{\theta_2}{\alpha \theta_2 + s(\theta_2 + \lambda)} \frac{\theta_2}{\theta_2 + \lambda} + \frac{\theta_3}{\alpha \theta_3 + s(\theta_3 + \lambda)} \frac{\theta_3}{\theta_3 + \lambda} \right\} \]

\[+ \left\{ \frac{\theta_4}{\alpha \theta_4 + s(\theta_4 + \lambda)} \frac{\theta_4}{\theta_4 + \lambda} \right\} \] \[s=0 \]

\[E(T) = \frac{c_1(\theta_1 + \lambda)}{\alpha \theta_1} + \frac{c_2(\theta_2 + \lambda)}{\alpha \theta_2} + \frac{c_3(\theta_3 + \lambda)}{\alpha \theta_3} + \frac{c_4(\theta_4 + \lambda)}{\alpha \theta_4} \]

\[\ldots \text{ (5.13)}\]

where

\[c_1 = \frac{\theta_2 \theta_3 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)} \]

\[c_2 = \frac{\theta_1 \theta_3 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} \]

\[c_3 = \frac{\theta_1 \theta_2 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_3)} \]

\[c_4 = \frac{\theta_1 \theta_2 \theta_3 (2\theta_4 - \theta_3 - \theta_2 - \theta_1)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)} \]
\[
\frac{d^2 l^*(s)}{ds^2} = -\left\{c_1 \frac{\theta_1 \alpha(\theta_1 + \lambda)}{a \theta_1 + s(\theta_1 + \lambda)} + c_2 \frac{\theta_2 \alpha(\theta_2 + \lambda)}{a \theta_2 + s(\theta_2 + \lambda)} + c_3 \frac{\theta_3 \alpha(\theta_3 + \lambda)}{a \theta_3 + s(\theta_3 + \lambda)} + c_4 \frac{\theta_4 \alpha(\theta_4 + \lambda)}{a \theta_4 + s(\theta_4 + \lambda)}\right\}
\]

\[
= \frac{2c_1 \theta_1 \alpha(\theta_1 + \lambda)^2}{(a \theta_1 + s(\theta_1 + \lambda))^2} + \frac{2c_2 \theta_2 \alpha(\theta_2 + \lambda)^2}{(a \theta_2 + s(\theta_2 + \lambda))^2} + \frac{2c_3 \theta_3 \alpha(\theta_3 + \lambda)^2}{(a \theta_3 + s(\theta_3 + \lambda))^2} + \frac{2c_4 \theta_4 \alpha(\theta_4 + \lambda)^2}{(a \theta_4 + s(\theta_4 + \lambda))^2} |_{s=0}
\]

\[
E(T)^2 = 2 \left\{\frac{c_1(\theta_1 + \lambda)^2}{(a \theta_1)^2} + \frac{c_2(\theta_2 + \lambda)^2}{(a \theta_2)^2} + \frac{c_3(\theta_3 + \lambda)^2}{(a \theta_3)^2} + \frac{c_4(\theta_4 + \lambda)^2}{(a \theta_4)^2}\right\}
\]

\[
V(T) = E(T)^2 - (E(T))^2
\]

\[
= 2\left(\frac{c_1(\theta_1 + \lambda)^2}{(a \theta_1)^2} + \frac{c_2(\theta_2 + \lambda)^2}{(a \theta_2)^2} + \frac{c_3(\theta_3 + \lambda)^2}{(a \theta_3)^2} + \frac{c_4(\theta_4 + \lambda)^2}{(a \theta_4)^2}\right) - \left(\frac{c_1(\theta_1 + \lambda)}{(a \theta_1)} + \frac{c_2(\theta_2 + \lambda)}{(a \theta_2)} + \frac{c_3(\theta_3 + \lambda)}{(a \theta_3)} + \frac{c_4(\theta_4 + \lambda)}{(a \theta_4)}\right)^2
\]

\[
\text{where}
\]

\[
c_1 = \frac{\theta_2 \theta_3 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)}
\]

\[
c_2 = \frac{\theta_1 \theta_3 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)}
\]

\[
c_3 = \frac{\theta_1 \theta_2 \theta_4}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)}
\]

\[
c_4 = \frac{\theta_1 \theta_2 \theta_3 (2 \theta_4 - \theta_3 - \theta_2 + \theta_1)}{(\theta_2 - \theta_1)(\theta_3 - \theta_1)(\theta_4 - \theta_1)(\theta_3 - \theta_2)(\theta_4 - \theta_2)(\theta_4 - \theta_3)}
\]
Numerical Illustration

The changes in $E(T)$ and $V(T)$ consequent to the variations in each of the parameters involved keeping others fixed is discussed below and the corresponding graphs are given.

<table>
<thead>
<tr>
<th>$\theta_1$</th>
<th>$E(T)$</th>
<th>$V(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>26.096</td>
<td>105.155</td>
</tr>
<tr>
<td>1.2</td>
<td>33.892</td>
<td>124.670</td>
</tr>
<tr>
<td>1.4</td>
<td>45.293</td>
<td>156.843</td>
</tr>
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<td>1.6</td>
<td>62.284</td>
<td>207.067</td>
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<td>1.8</td>
<td>88.556</td>
<td>286.355</td>
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<td>2.0</td>
<td>131.464</td>
<td>417.175</td>
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<td>2.2</td>
<td>207.262</td>
<td>649.594</td>
</tr>
<tr>
<td>2.4</td>
<td>357.890</td>
<td>1113.250</td>
</tr>
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<td>722.187</td>
<td>2238.390</td>
</tr>
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<td>2.8</td>
<td>2059.46</td>
<td>6382.790</td>
</tr>
</tbody>
</table>

Table 5.1. Variation in $E(T)$ and $V(T)$ for Changes in $\theta_1$ and $\theta_2 = 3, \theta_3 = 3.5, \theta_4 = 4, \lambda = 1.5, \alpha = 1$ are all fixed

![Figure 5.1a. Variation in $E(T)$ for Changes in $\theta_1$](image)
Figure 5.1b. Variation in V(T) for Changes in $\theta_1$

Table 5.2. Variation in $E(T)$ and $V(T)$ for Changes in $\theta_2$ and $\theta_1 = 0.2, \theta_3 = 0.5, \lambda = 3.5, \alpha = 0.2$ are all fixed

<table>
<thead>
<tr>
<th>$\theta_2$</th>
<th>$E(T)$</th>
<th>$V(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>69.971</td>
<td>9567.080</td>
</tr>
<tr>
<td>4.5</td>
<td>29.449</td>
<td>4104.800</td>
</tr>
<tr>
<td>5.0</td>
<td>16.793</td>
<td>2387.970</td>
</tr>
<tr>
<td>5.5</td>
<td>10.906</td>
<td>1583.470</td>
</tr>
<tr>
<td>6.0</td>
<td>7.627</td>
<td>1131.830</td>
</tr>
<tr>
<td>6.5</td>
<td>5.598</td>
<td>849.988</td>
</tr>
<tr>
<td>7.0</td>
<td>4.251</td>
<td>661.277</td>
</tr>
<tr>
<td>7.5</td>
<td>3.312</td>
<td>528.356</td>
</tr>
<tr>
<td>8.0</td>
<td>2.630</td>
<td>431.065</td>
</tr>
<tr>
<td>8.5</td>
<td>2.121</td>
<td>357.667</td>
</tr>
</tbody>
</table>

Figure 5.2a. Variation in $E(T)$ for Changes in $\theta_2$
Figure 5.2b. Variation in V(T) for Changes in $\theta_2$

Table 5.3. Variation in E(T) and V(T) for Changes in $\theta_3$ and $\theta_1 = 0.2$, $\theta_2 = 0.5$, $\theta_4 = 3.5$, $\lambda = 2.5$, $\alpha = 0.2$ are all fixed

<table>
<thead>
<tr>
<th>$\theta_3$</th>
<th>E(T)</th>
<th>V(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>128.360</td>
<td>14938.80</td>
</tr>
<tr>
<td>1.2</td>
<td>88.474</td>
<td>10276.30</td>
</tr>
<tr>
<td>1.4</td>
<td>67.695</td>
<td>7821.30</td>
</tr>
<tr>
<td>1.6</td>
<td>55.211</td>
<td>6326.85</td>
</tr>
<tr>
<td>1.8</td>
<td>47.072</td>
<td>5334.27</td>
</tr>
<tr>
<td>2.0</td>
<td>41.531</td>
<td>4638.38</td>
</tr>
<tr>
<td>2.2</td>
<td>37.736</td>
<td>4136.10</td>
</tr>
<tr>
<td>2.4</td>
<td>35.280</td>
<td>3773.28</td>
</tr>
<tr>
<td>2.6</td>
<td>34.052</td>
<td>3524.89</td>
</tr>
<tr>
<td>2.8</td>
<td>34.286</td>
<td>3392.10</td>
</tr>
</tbody>
</table>

Figure 5.3a. Variation in E(T) for Changes in $\theta_3$
Figure 5.3b. Variation in V(T) for Changes in $\theta_3$

Table 5.4. Variation in E(T) and V(T) for Changes in $\theta_4$ and $\theta_3 = 0.2$, $\theta_2 = 0.5$, $\theta_3 = 3.5$, $\lambda = 2.5$, $\alpha = 0.2$ are all fixed

<table>
<thead>
<tr>
<th>$\theta_4$</th>
<th>E(T)</th>
<th>V(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>100.289</td>
<td>13367.50</td>
</tr>
<tr>
<td>1.2</td>
<td>66.373</td>
<td>9043.250</td>
</tr>
<tr>
<td>1.4</td>
<td>48.482</td>
<td>6750.010</td>
</tr>
<tr>
<td>1.6</td>
<td>37.411</td>
<td>5333.580</td>
</tr>
<tr>
<td>1.8</td>
<td>29.787</td>
<td>4368.210</td>
</tr>
<tr>
<td>2.0</td>
<td>24.075</td>
<td>3660.830</td>
</tr>
<tr>
<td>2.2</td>
<td>19.451</td>
<td>3109.890</td>
</tr>
<tr>
<td>2.4</td>
<td>15.380</td>
<td>2654.010</td>
</tr>
<tr>
<td>2.6</td>
<td>11.404</td>
<td>2248.270</td>
</tr>
<tr>
<td>2.8</td>
<td>6.927</td>
<td>1846.710</td>
</tr>
</tbody>
</table>

Figure 5.4a. Variation in E(T) for Changes in $\theta_4$
Figure 5.4b. Variation in $V(T)$ for Changes in $\theta_4$

Table 5.5. Variation in $E(T)$ and $V(T)$ for Changes in $\alpha$ and $\theta_1 = 0.2$, $\theta_2 = 0.4$, $\theta_3 = 0.6$, $\theta_4 = 0.8$, $\lambda = 1$ are all fixed

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$E(T)$</th>
<th>$V(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>3100.690</td>
<td>138805</td>
</tr>
<tr>
<td>0.4</td>
<td>1550.350</td>
<td>34701.20</td>
</tr>
<tr>
<td>0.6</td>
<td>1033.560</td>
<td>15422.80</td>
</tr>
<tr>
<td>0.8</td>
<td>775.170</td>
<td>8675.31</td>
</tr>
<tr>
<td>1.0</td>
<td>620.139</td>
<td>5552.20</td>
</tr>
<tr>
<td>1.2</td>
<td>516.782</td>
<td>3855.69</td>
</tr>
<tr>
<td>1.4</td>
<td>442.956</td>
<td>2832.75</td>
</tr>
<tr>
<td>1.6</td>
<td>387.587</td>
<td>2168.83</td>
</tr>
<tr>
<td>1.8</td>
<td>344.522</td>
<td>1713.64</td>
</tr>
<tr>
<td>2</td>
<td>311.069</td>
<td>1388.05</td>
</tr>
</tbody>
</table>

Figure 5.5a. Variation in $E(T)$ for Changes in $\alpha$
Figure 5.5b. Variation in V(T) for Changes in α

Table 5.6. Variation in E(T) and V(T) for Changes in λ and θ₁ = 0.2, θ₂ = 0.4, θ₃ = 0.6, θ₄ = 0.8, α = 1 are all fixed

<table>
<thead>
<tr>
<th>λ</th>
<th>E(T)</th>
<th>V(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>620.139</td>
<td>5552.20</td>
</tr>
<tr>
<td>1.2</td>
<td>712.500</td>
<td>7412.50</td>
</tr>
<tr>
<td>1.4</td>
<td>804.861</td>
<td>9543.87</td>
</tr>
<tr>
<td>1.6</td>
<td>897.222</td>
<td>11946.30</td>
</tr>
<tr>
<td>1.8</td>
<td>989.583</td>
<td>14619.80</td>
</tr>
<tr>
<td>2</td>
<td>1081.940</td>
<td>17564.40</td>
</tr>
<tr>
<td>2.2</td>
<td>1174.310</td>
<td>20780.00</td>
</tr>
<tr>
<td>2.4</td>
<td>1266.670</td>
<td>24266.70</td>
</tr>
<tr>
<td>2.6</td>
<td>1359.030</td>
<td>28024.40</td>
</tr>
<tr>
<td>2.8</td>
<td>1451.390</td>
<td>32053.20</td>
</tr>
</tbody>
</table>

Figure 5.6a. Variation in E(T) for Changes in λ
5.5 Conclusions

(i) The parameter $\theta_1$ is allowed to vary with all other parameter being kept fixed. The changes in $E(T)$ and $V(T)$ are given in table 1, figure 5.1a and figure 5.1b. From the above table it can be seen that as $\theta_1$ increases the values of both expected time to recruitment and its variance decreases. This is due to the fact that since $\theta_1$ is the parameter of $Y \sim \exp (\theta_1)$, which is a part of the threshold $E(T)$ has $C_1$ in the numerator and as $\theta_1$ increases the value of $C_1$ increases, so $E(T)$ and $V(T)$ both shown are decreases.

(ii) If $\theta_2$ increases is allowed to vary keeping all the other parameters are fixed, it can be seen that, the expected time to recruitment and its variance show a decrease, as shown in table 5.2, figure 5.2a and figure 5.2b.

(iii) If $\theta_3$ increases then both expected time to recruitment and its variance show a decrease. This is by the fact that since $Y \sim \exp (\theta_3)$, the mean value of $y$ decreases as $\theta_3$ increases then the addition to the threshold due to $Y$ smaller and so the $E(T)$ and $V(T)$ show a decrease, as shown in table 5.3, figure 5.3a and figure 5.3b.
(iv) If $\theta_4$ increases then both expected time to recruitment and its variance show decreases. This by that fact that since $Y \sim \exp(\theta_4)$, the mean value of $y$ decreases as $\theta_4$ increases then the addition to the threshold due to $Y$ smaller and so the expected time to recruitment and its variance show a decrease, as shown in table 5.4, figure 5.4a and figure 5.4b.

(v) If the value of $\alpha$ which the parameter of the interarrival times between depletion increases then expected time to recruitment and its variance shows a decease, as shown in table 5.5, figure 5.5a and figure 5.5b. This is due to fact that, the interarrival times is distributed as $\exp(\alpha)$ and so $E(U) = 1/\alpha$. If $\alpha$ increases then $E(U)$ decreases and so more occasions of depletion take place. Hence $E(T)$ and $V(T)$ decreases.

(vi) If $\lambda$ which is the parameter of the distribution of $X$ which denotes the magnitude of depletion increases then $E(T)$ and $V(T)$ also increases. Since $g(.) \sim \exp(\lambda)$ as $\lambda$ increases then the average level of depletion $E(X)$ decreases. Hence it takes more time to cross the threshold. Hence expected time to recruitment and its variance is also increases, as shown in table 5.6, figure 5.6a and figure 5.6b.