A powerful promotional tool that attracts new customers is the trade credit period offered by the supplier to the retailer and encourages retailer to buy more, is the permissible delay period. Permissible delay in the payment can be understood as if the most credit card issuers (or banks) frequently offer customers a teaser interest rate (say $I_{c_1}$), which is significantly lower than the regular interest rate of $I_{c_2}$ (with $I_{c_2} > I_{c_1}$) for only 6 months or a year (say, $M_2$) to lure new customers from their competitors. Consequently, the customer faces a progressive interest charged from the bank. If the customer pays the outstanding balance by the grace period (say $M_1$, which is generally 25 days), then the bank does not charge any interest. If the outstanding amount is paid after $M_1$, but by $N_1$ (with $N_1 > M_1$), then the bank charges the customer the teaser interest rate of $I_{c_1}$ on the unpaid balance. If the customer pays the outstanding amount after $N_1$, then the bank charges the regular interest rate of $I_{c_2}$. In the classical EOQ model, it is assumed that the retailer must pay to the supplier for the items as soon as he receives them. However, in practice supplier is willing to offer the retailer a fixed credit period without interest during the permissible delay period. Before the end of the credit period, retailer can sell the goods and accumulate revenue. A number of researchers over the years have already dealt with the inventory model under the situation.
The model incorporating the facility of permissible delay in payment was first discussed by Haley and Higgins (1973). Goyal (1985) explored a single item EOQ model under permissible delay in payments. In real life situations, there are products like volatile liquids, medicines, and materials, etc. in which the rate of deterioration is very large. Therefore, the loss due to deterioration should not be ignored. Aggarwal and Jaggi (1995) extended Goyal’s (1985) model to allow for deteriorating items. Shortages are of great importance especially in a model that considers a delay in payment due to the fact that shortages can affect the quantity ordered to benefit from the delay in payment. Jamal et al. (1997) generalized the model of Aggarwal and Jaggi (1995) to allow for shortages and make it more applicable in real the world. In the above cited references, all the models were developed for a single warehouse and assumed that the available warehouse had unlimited capacity. However, this assumption is debatable in real life situations. A common practical situation is that of a limited storage space, whereas the extra storage capacity can be acquired in the form of rented warehouse if the retailer’s existing storage capacity is insufficient to store the ordered quantity. Motivated by the trade credit policy given by supplier, first time Shah and Shah (1992) developed a deterministic inventory model under the permissible delay in payment with two storage facilities. In that model, the items were considered non-deteriorating without shortage and the time horizon was infinite. Chung and Huang (2004) extended Goyal’s (1985) model by considering limited storage capacity of the retailer in which the items were considered non-deteriorating. In that model, shortage was not allowed and the time horizon was infinite with constant demand rate. Ouyang et al. (2006) developed an inventory model for deteriorating items with permissible delay in payments. The purpose of that study was to find an optimal replenishment policy for minimizing the total relevant inventory cost. Chung and Huang (2007) presented a two-warehouse inventory model for deteriorating items under trade credit financing. In that model, shortages were not allowed and time horizon was infinite. The rate of deterioration in both warehouses was considered the same. Singh S.R and Singh S. (2007) considered a lot size model for items having linear demand under the effect of permissible delay and inflation. Singh S.R and Singh S. (2008) considered a production model for items under the effect of inflation and permissible delay in payments. Singh et al. (2008) presented a two-
warehouse inventory model for deteriorating items with constant demand rate where shortages were allowed and partially backlogged. Singh and Jain (2009) proposed a deterministic inventory model with time varying deterioration rate and a linear trend in demand over a finite planning horizon. Geetha and Uthayakumar (2010) developed an Economic Order Quantity (EOQ) model for deteriorating items with permissible delay in payments and single storage facility. In that model, shortages were allowed and partially backlogged.

This chapter deals an inventory model for decaying items with time & stock dependent demand rate. To make the study more practical, the effect of permissible delay in payment and inflation has been taken into account. In real life situation, the storage cost varies with time. The storage time can be classified into different ranges, each with its distinctive unit holding cost. Thus, two times dependent holding cost step functions are considered that is retroactive and incremental holding cost. Shortages are allowed in inventory with partial backlogging. We provided the solution procedure for finding the total cost. Further we use a numerical example to illustrate the model and sensitivity analysis.
II. ASSUMPTIONS AND NOTATIONS

The inventory model is developed on the basis of the following assumptions and notations.

A. Assumptions:

- Demand rate is depending on stock & time.
- Constant deteriorating rate of the on hand inventory in considered.
- The holding cost is varying as an increasing step function of time in storage.
- Shortages are allowed with partial backlogging.
- The effect of inflation is considered.
- Permissible delay on payment is taken into account.

B. Notations:

- $I_b(t)$: Inventory level.
- $I_{mb}$: Maximum inventory
- $P_b$: The unit price cost
- $F_{b,j}$: Holding cost of the item in period $j$.
- $F_b(t)$: Holding cost of the item at time $t$, $F_b(t)=F_{b,j}$ if $t_{j-1} < t < t_j$.
- $C_{sb}$: The set up cost per order.
- $C_2$: Shortage cost per item for backlogged items.
The unit cost of lost sales.

Denote the fraction where \( t \) is the waiting time up to the next replenishment. We take \( B(t) = \frac{1}{1+\delta t} \), where the backlogging parameter \( \delta \) is a positive constant.

Demand rate where \( a, b \) and \( c \) are positive constants and \( I(t) \) is the inventory level at time \( t \).

Time length of each cycle

Deterioration rate

Number of distinct time periods with different holding cost rates.

Inflation rate

The integrate cost of vendor and all buyer per unit item.

The period of permissible delay in setting account

The time at which shortage occur.

III. MATHEMATICAL MODEL AND SOLUTION

Inventory level starts with the maximum inventory \( I_{mb} \) at \( t = 0 \) and this inventory gradually depletes to zero at time \( t_1' \) due to the simultaneous effect of demand and deterioration, shown by the Fig. 2.

\[
I_{bl}(t) + \theta I_{bl}(t) = [a + bI_{bl}(t) + ct], \quad 0 \leq t \leq t_1' \quad \ldots (1)
\]

After time \( t_1' \) partial backlogging occurs and the change in the inventory is directed by the following differential equation.
\[ I_{b_2}'(t) = -\frac{[a + ct]}{1 + \delta\left(\frac{T}{n} - t\right)} \quad t_{1} - t \leq T / n \quad \ldots (2) \]

Boundary condition is \( I_{b_1}(t_{1}') = 0 = I_{b_2}(t_{1}') \)

\[ I(t) \]

\[ I_{mb} \]

\[ t_{1}' \quad \text{Lost Sale} \]

**Fig 1. Inventory system for buyer when shortage is allowed**

Now the solutions of the above differential equations are

\[ I_{b_1}(t) = \left(\frac{a}{\theta + b} - \frac{c}{(\theta + b)^2}\right)\left(e^{(\theta+b)(t_{1} - t)} - 1\right) + \frac{c}{(\theta + b)}\left(t_{1}' e^{(\theta+b)(t_{1}' - t)} - t\right) \quad 0 \leq t \leq t_{1}' \quad \ldots (3) \]
\[
I_{b_2}(t) = -\frac{1}{\delta} \left[ a + \frac{c}{\delta} \left( 1 + \frac{T}{n} \right) \right] \log \left( \frac{1+ \delta \left( \frac{T}{n} - t_i' \right) }{1+ \delta \left( \frac{T}{n} - t_i \right) } \right) + \frac{c}{\delta} (t - t_i') \quad t_i' \leq t \leq T/n \quad \ldots(4)
\]

By using the boundary condition \( I_{mv} = I_{v_2}(0) \) and \( I_{mb} = I_{b_1}(0) \), we have

\[
I_{mb} = t_i' (a + c t_i') + \frac{t_i'^2}{2} \left[ a(\theta + b) - c + c t_i' (\theta + b) \right] \quad \ldots(5)
\]

**Case I: Retroactive holding cost increase**

The yearly holding cost:

\[
HC_b = \frac{F_{b_2} n}{T} \int_0^{t_i'} I_{b_1}(t)e^{-rt} \, dt
\]

\[
= \frac{F_{b_2} n}{T} \left[ \frac{(a + c t_i') t_i'^2}{2} - \frac{t_i'^2}{2} \left( (\theta + b)(a + c t_i') - c \right) \right] \quad \ldots(6)
\]
Case II: Incremental holding cost increase:

\[ HC_b = \frac{n}{T} \left[ F_{b_1} \int_{0}^{t} I_{b_1}(t)e^{-rt} dt + F_{b_2} \int_{t_i}^{t_2} I_{b_1}(t)e^{-rt} dt + \ldots + F_{b_c} \int_{t_{i-1}}^{t_{i+1}} I_{b_1}(t)e^{-rt} dt \right] \]

\[ = \frac{n}{T} \sum_{j=1}^{c} F_{b_j} \int_{t_{j-1}}^{t_j} I_{b_1}(t)e^{-rt} dt \]

\[ = \frac{n}{T} \sum_{j=1}^{c} F_{b_j} \left[ \frac{(a + ct_1')}{2} \left[ -(t_1' - t_j)e^{-rt_j} + \frac{e^{-rt_j}}{r} + (t_1' - t_{j-1})e^{-rt_{j-1}} - \frac{e^{-rt_{j-1}}}{r^2} \right] + \left\{ \frac{a(\theta + b) - c + ct_1' (\theta + b)}{2} \right\} \right] \]

\[ \left\{ -(t_1' - t_j)^2 \frac{e^{-rt_j}}{r} + 2(t_1' - t_j) - 2 \frac{e^{-rt_j}}{r^2} + (t_1' - t_{j-1})^2 \frac{e^{-rt_{j-1}}}{r} - \frac{2(t_1' - t_{j-1})e^{-rt_{j-1}}}{r^2} + 2e^{-rt_{j-1}} \right\} \]

\[ \ldots \ldots (7) \]

(i) \textbf{When } t_1' > M

Interest payable per cycle per unit time is

\[ IP = \frac{n_P I_P}{T} \int_{M}^{t_1} I_{b_1}(t)e^{-rt} dt \]

\[ = \frac{n_P I_P}{T} \int_{M}^{t_1} \left[ (t_1' - t)(a + ct_1') + \frac{(t_1' - t)^2}{2} \left\{ a(\theta + b) - c + ct_1' (\theta + b) \right\} \right] e^{-rt} dt \]
\[ \frac{nP_aI_e}{T} \left[ (a + ct_1')(t_1' - M)(t_1' - \frac{M}{2} + \frac{rM^2}{2}) \right. \\
+ \left\{ \frac{(\theta + b)(a + ct_1') - c}{2} \right\} (t_1' - M) \left\{ -Mt_1' + \frac{M^2}{2} (t_1' - M) \right\} \right] \]  

\[ \text{Interest earned per cycle per unit time is} \]

\[ IE = \frac{nP_aI_e}{T} \int_0^{t_e} e^{-\eta}(a + bI_{bt} + ct)dt \]

\[ = \frac{nP_aI_e}{T} \left[ (a + ct_1')(t_1' - \frac{rt_{t_1^2}}{2} + bt_{t_1^2}) + b \left\{ \frac{(a + ct_1')(\theta + b) - c}{2} \right\} \left\{ \frac{1}{2} - \frac{2}{r} \right\} t_{t_1^2} \right] \]

\[ \text{Interest earned during } (M - t_1') \text{ is} \]

\[ P_{b}I_e \left[ (a + ct_1')(t_1' - \frac{rt_{t_1^2}}{2} + bt_{t_1^2}) + b \left\{ \frac{(a + ct_1')(\theta + b) - c}{2} \right\} \left\{ \frac{1}{2} - \frac{2}{r} \right\} t_{t_1^2} \right]^{M} e^{-\eta} dt \]

(ii) When \( t_1' \leq M \)

Interest earned up to time \( t_1' \) is

\[ IE = P_{b}I_e \int_0^{t_e} e^{-\eta}(a + bI_{bt} + ct)dt \]

\[ = P_{b}I_e \left[ (a + ct_1')(t_1' - \frac{rt_{t_1^2}}{2} + bt_{t_1^2}) + b \left\{ \frac{(a + ct_1')(\theta + b) - c}{2} \right\} \left\{ \frac{1}{2} - \frac{2}{r} \right\} t_{t_1^2} \right] \]
\[ P_b I_c \left[ (a + ct_1')(t_1' - \frac{rt_1'^2}{2} + bt_1'^2) + b \left( \frac{(a + ct_1')(\theta + b) - c}{2} \right) \left( 1 - \frac{2}{r} \right) t_1'^2 \right] \left( \frac{e^{-\eta_i}}{r} - \frac{e^{-\eta_M}}{r} \right) \]

Total Interest earned per unit time is

\[ IE = \frac{nP_b I_c}{T} \left[ (a + ct_1')(t_1' - \frac{rt_1'^2}{2} + bt_1'^2) 
+ b \left( \frac{(a + ct_1')(\theta + b) - c}{2} \right) \left( 1 - \frac{2}{r} \right) t_1'^2 \right] \left( 1 + \frac{e^{-\eta_i}}{r} - \frac{e^{-\eta_M}}{r} \right) \]  

......(10)

In this case customer pays no interest.

The annual deteriorated cost is:

\[ DC_b = \frac{nP_b}{T} \left[ I_{Mb} - \int_{0}^{t_1'} (a + bI_{b_1} + ct)e^{-rt} \, dt \right] \]

\[ = \frac{nP_b}{T} t_1'^2 \left[ \left( \frac{a(\theta + b) - c}{2} \right) + \frac{ar}{2} - \frac{ab}{2} + \left( \frac{1}{r} - \frac{1}{4} \right) b(a(\theta + b) - c) \right] \]  

......(11)

The set up cost per year is:

\[ SC_b = \frac{nC_{ab}}{T} \]  

......(12)
Shortage cost per cycle

\[ \text{AS}_b = \frac{nC_2}{T} \left[ \int_{t_i}^{T} -I_{b2}(t)e^{-rt} \, dt \right] \]

\[
= \frac{nC_2}{\delta T} \left[ \frac{c}{\delta r} \left( \frac{T}{n} - t_i \right) \left( 1 - e^{-\frac{T}{n}} \right) - \left\{ a + \frac{c}{\delta} \left( 1 + \frac{\delta T}{n} \right) \right\} \left( \frac{T}{n} - t_i \right) \right. \\
\left. - \left\{ \frac{r}{4} \left( \frac{T}{n} + t_i \right) - \left( 1 - \frac{r}{2\delta} \left( 1 + \frac{\delta T}{n} \right) \right) \right\} \right] 
\]

.....(13)

Opportunity cost per cycle due to lost sales

\[ \text{OC} = \frac{nC_3}{T} \left[ \frac{T}{r^2} \left[ 1 - \frac{1}{1 + \frac{\delta}{\delta r}} \left( 1 + \frac{\delta T}{n} \right) \right] (a + ct) e^{-rt} \, dt \right] \]

\[
= \frac{nC_3}{T} \left[ \left( e^{-\frac{T}{n}} - e^{-t_i} \right) \left\{ \frac{c}{r^3} - \frac{c}{r^2} - \frac{2\delta c}{r^3} - \frac{\delta a}{r^3} \right\} - \frac{\delta c T}{r^3} e^{-\frac{T}{n}} \right] \\
+ \frac{\delta c t_i}{r^3} e^{-t_i} - \frac{c \delta}{r^3} \left( \frac{T}{n} - t_i \right) e^{-t_i} \right] 
\]

.....(14)
Case I: Retroactive holding cost increase:

(i) When \( t'_1 > M \)

Total Cost ( \( TC_{1a} \) ) = \( HC_b + DC_b + SC_b + AS_b + \) Interest payable - Interest earned

Where \( HC_b, DC_b, SC_b, AS_b \), Interest payable and Interest earned are (6), (11), (12), (13), (8) and (9) respectively.

The necessary and sufficient conditions for the total relevant cost per unit time to be minimize are

\[
\frac{\partial TC_{1a}}{\partial t'_1} = 0
\]

\[
\frac{\partial^2 TC_{1a}}{\partial t'_1^2} > 0
\]

and \( \frac{\partial^2 TC_{1a}}{\partial t'_1^2} > 0 \)

(ii) When \( t'_1 \leq M \)

Total Cost ( \( TC_{1b} \) ) = \( HC_b + DC_b + SC_b + AS_b - \) Interest earned

Where \( HC_b, DC_b, SC_b, AS_b, OC, HC_v, DC_v, SC_v \) and Interest earned are (6), (11), (12), (13), and (10) respectively.
The necessary and sufficient conditions for the total relevant cost per unit time to be minimize are

\[ \frac{\partial TC_{lb}}{\partial t_1} = 0 \]

\[ \frac{\partial^2 TC_{lb}}{\partial^2 t_1} > 0 \]

and \[ \frac{\partial^2 t_1}{\partial^2 t_1} \]

Case II: Incremental holding cost increase:

(ii) When \( t_1' > M \)

Total Cost (\( TC_{2a} \)) = \( HC_b + DC_b + SC_b + AS_b \) + Interest payable - Interest earned

Where \( HC_b, DC_b, SC_b, AS_b \), Interest payable and Interest earned are (7), (11), (12), (13), (8) and (9) respectively.

The necessary and sufficient conditions for the total relevant cost per unit time to be minimize are

\[ \frac{\partial TC_{2a}}{\partial t_1'} = 0 \]

\[ \frac{\partial^2 TC_{2a}}{\partial^2 t_1'} > 0 \]

and \[ \frac{\partial^2 t_1}{\partial^2 t_1} \]
(ii) When $t'_1 \leq M$

Total Cost \((TC_{2b}) = HC_{b} + DC_{b} + SC_{b} + AS_{b}\) - Interest earned

Where $HC_{b}$, $DC_{b}$, $SC_{b}$, $AS_{b}$ and Interest earned are (7), (11), (12), (13), and (10) respectively.

The necessary and sufficient conditions for the total relevant cost per unit time to be minimize are

$$\frac{\partial TC_{2b}}{\partial t'_1} = 0$$

$$\frac{\partial^2 TC_{2b}}{\partial^2 t'_1} > 0$$

and

These equations are solved by the mathematical software Mathematica 5.2.
IV. NUMERICAL EXAMPLE

A numerical example is considered to illustrate the model as shown in Table 1 and Table 2. The following values of parameters are used in the example.

\[ P = 240 \text{ unit}, \ n = 2, \ a = 130 \text{ unit}, \ P_b = 5, \ C_{ab} = 24, \ \theta = 0.03, \ b = 0.03, \ I_p = 0.08, \ I_c = 0.06, \ M = 25 \text{ days}, \]

\[ C_2 = 0.4, \ r = 0.02, \ c = 0.03, \ \delta = 0.05, \ C_3 = 0.4, \text{ and } T = 60 \text{ days}. \]

Table 5.1

<table>
<thead>
<tr>
<th>Retroactive holding cost (( F_{t M} = 0.70 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>when ( t_1' &gt; M )</td>
</tr>
<tr>
<td>( t_1' )</td>
</tr>
<tr>
<td>Total cost</td>
</tr>
</tbody>
</table>
V. SENSITIVITY ANALYSIS

The results of the sensitivity of the optimal solution to changes in the different parameters are as follows:

(i) All of $TC_{1a}, TC_{1b}$ are very sensitive to changes in the value of the inflation rate $r$.

(ii) $TC_{1a}$ is moderately sensitive to changes in the value of the parameter $P, M, \theta$ and $r$. Moreover, $TC_{1a}$ is increases (decreases) with increase (decrease) in $P, M, \theta$ and $r$.

(iii) $TC_{1b}$ is low sensitive to changes in the value of the parameter $\theta, M$ while it is sensitive to changes in the value of the parameter $P, r$. Moreover, $TC_{1b}$ is increases (decreases) with increase (decrease) in $P, M, \theta$ but it decreases (increases) with increase (decrease) in $r$.

Table 5.2:

<table>
<thead>
<tr>
<th>Variation parameter</th>
<th>Total Cost</th>
<th>Percentage variation in parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>-10%</td>
</tr>
<tr>
<td>P</td>
<td>$TC_{1a}$</td>
<td>1389.36</td>
</tr>
<tr>
<td></td>
<td>$TC_{1b}$</td>
<td>1531.95</td>
</tr>
<tr>
<td></td>
<td>$TC_{1a}$</td>
<td>$TC_{1b}$</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>M</td>
<td>1359.22</td>
<td>1585.23</td>
</tr>
<tr>
<td></td>
<td>1541.26</td>
<td>1606.7</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>1437.78</td>
<td>1595.13</td>
</tr>
<tr>
<td></td>
<td>1465.92</td>
<td>1601.09</td>
</tr>
<tr>
<td>r</td>
<td>1384.01</td>
<td>1947.77</td>
</tr>
<tr>
<td></td>
<td>1473.37</td>
<td>1461.13</td>
</tr>
</tbody>
</table>
Fig 1: Variation in $TC_{1a}$ and $TC_{1b}$ w.r.t. 'P'

Fig 3: Variation in $TC_{1a}$ and $TC_{1b}$ w.r.t. 'M'
Fig 5: Variation in $TC_{1a}$ and $TC_{1b}$ w.r.t. 'θ'

Fig 7: Variation in $TC_{1a}$ and $TC_{1b}$ w.r.t. 'r'
VI. CONCLUSION

An inventory model deals for decaying items with stock and time dependent demand under permissible delay in payments. Shortages are allowed with partial backlogging. Backlogging rate is waiting time for the next replenishment. The environment of whole study has been taken as inflationary. In this study, variable holding cost i.e. Retroactive holding cost and Incremental holding cost has been considered. Cost minimization technique is used in this study. Numerical assessment is given to illustrate the theoretical results. The solution obtained has also been analyzed through sensitivity, and a conclusion is made that the model is very practical and suitable to the realistic situations. All these facts together make this study very unique and matter-of-fact.