Introduction

Proximity spaces and contiguity spaces, and more recently nearness spaces, have been studied not just because they provide various approaches to uniform structure. Possibly of greater importance is that they can be used as a means of introducing compactifications and more general extensions of the topological spaces on which they are defined. Riesz [29] was probably the first to recognize this connection. Since then the idea was used by Freudenthal [16], Alexandroff [1], Smirnov [30], Leader [23] and Ivanov and Ivanova [21,22] among others. Nearness structures have been introduced by Herrlich [19]. He used them extensively to study $T_1$-extensions (particularly $T_1$-compactifications) of $T_1$-topological spaces. Nearness structures have also been investigated and used to study extension problem by Bentley [5,6], Bentley and Herrlich [7] and Herrlich [19].

Reed [27] using work of Bentley [5,7] and Herrlich [19,20] studied the 1-1 correspondence between the class $\mathcal{X}(X)$ of all cluster generated LO-nearness spaces and the class $\mathcal{G}(X)$
of all principal $T_1$-extensions of a given $T_1$-space. She succeeded in showing that the mapping induces a 1-1 correspondence between the contiguous nearness spaces in $\mathcal{C}(X)$ and the compactifications in $\mathcal{E}(X)$. The proximal nearness spaces are mapped onto the linkage compactifications (she called them clan complete extensions). The Efremović proximal nearness spaces correspond to the $T_2$-compactifications.

It may be mentioned that Chattopadhyay, Njåstad and Thron [11] studied the 1-1 correspondence between principal $T_1$-extensions of a non-topological $T_1$-closure space and the cluster generated Riesz nearnesses on the space. For their purpose they introduced a new class of nearnesses which they called weakly contiguous nearnesses. Chattopadhyay [13] have shown that most of the results of [11] can be reestablished for $R_1$-closure spaces without requiring the spaces to be $T_1$.

Recently Reed [28] introduced the concepts of Wallman nearnesses and Wallman-type (W-T) extensions of $T_1$-topological spaces. The concepts grew out of an attempt to determine what $T_2$-compactifications, the (classical) Wallman compactifications and the one-point compactifications of $T_1$-spaces have in common. She succeeded in proving that all the aforesaid well known compactifications share the common platform of W-T extensions. Being inspired by this result one might ask: whether all 'nice' extensions do belong to the class of W-T extensions.
Though the commonness of the above-mentioned compactifications primarily inspired her work, Reed [28] finally became interested in the conditions which would insure the compactness of W-T extensions. She established some sufficient conditions on the underlying proximity which would guarantee the compactness of a W-T extension. Reed [28] concluded her paper with an open question in which she expressed that it would be of interest to obtain other conditions on a LO-proximity \( \pi \) which would guarantee the compactness of the associated W-T extension corresponding to the proximity \( \pi \).

Our work in [14] and in the thesis has been in part motivated by the above question of Reed. The work in [14] and a part of the thesis has been devoted to characterize completely those W-T extensions which are compact or linkage compact. Both have been done in terms of the trace system of the extension as well as in terms of nearness generating the extension. These characterizations led us to establish a necessary and sufficient condition on a LO-proximity \( \pi \) which would guarantee the compactness of the associated W-T extension corresponding to \( \pi \). This necessary and sufficient condition might be thought to be a possible answer to the question raised by Reed in [28].

So far either our work or the work of Reed [28] have suggested little or nothing for the existence of a noncompact W-T extension. If it happens that all the W-T extensions are
compact then the conditions for the compactness of a W-T extension investigated by Reed and by us would completely be meaningless. Thus a meaningful question that one could ask in this context would be: whether all Wallman-type extensions are compact.

Reed [28] defined a nearness to be a Wallman nearness if it can be generated by the set of Wallman \( \pi \)-clans in the usual way (see Gagrat and Thron [18]), where \( \pi \) is a LO-proximity. It is important to note that if for every Lodato proximity \( \pi \), the set of \( \pi \)-clans and the set of Wallman \( \pi \)-clans do generate the same nearness then the class of Wallman nearnesses and the class of proximal nearnesses would be identical or equivalently the class of W-T extensions and the class of principal \( T_1 \) linkage compact extensions would be identical and hence one achieves nothing new by introducing a new concept like 'Wallman nearnesses or W-T extensions'. Thus to show that the concept 'Wallman nearness' as introduced by Reed is really a new one a negative answer to the following question: whether the set of \( \pi \)-clans and the set of Wallman \( \pi \)-clans do generate the same nearness where \( \pi \) is a Lodato proximity - is desirable.

It is well known that every linkage compact space is compact but the converse is not true in general (see [11]). Since these concepts are identical in the class of Hausdorff species,
T$_2$-compactifications and the linkage compact T$_2$-extensions are identical. However in the class of principal T$_1$-extensions, compactifications and linkage compactifications are distinct concepts. In this context one may ask: whether the above distinctness of compactifications and linkage compactifications is maintained in the class of W-T extensions.

Before giving a chapterwise/sectionwise classification of our work in the thesis we feel to say a few more words. When Reed [28] introduced Wallman-type extensions the name 'Wallman-type compactification' had already been used for another approach to generalizing Wallman compactifications. Recall that Wallman [34] defined T$_1$-compactifications for all T$_1$-spaces by using as points the set of all ultraclosed filters. Thus there is but one Wallman compactification for each T$_1$-space. During last 40 years Wallman's method has been generalised and applied to study extension problem by many authors. A natural generalization is to consider all $\mathcal{B}$-ultrafilters, where $\mathcal{B}$ is a base for the closed sets of the space. This can be done for all T$_1$-spaces with minimum assumptions on the base $\mathcal{B}$. In order to conclude that the construction leads to a compactification of the space it suffices to assume that $\mathcal{B}$ is closed under finite unions and intersections. However, to be equivalent to a Wallman-type extension in the sense of Reed additional condition to be called for which will be discussed in the thesis.
The idea of using a base $\mathcal{B}$ for the closed sets of the space in the construction of an extension was used by Frink [17]. Since he was interested only in $T_2$-compactifications, he had to impose a number of conditions on the base he studied. His construction led to a great deal of further work on the part of others who all used the term 'Wallman-type compactification'. Since his extensions are $T_2$-compactifications, they are Wallman-type extensions in the sense of Reed (see[28]).

In what follows we give a chapter (section) wise classification of our work in the thesis. The whole thesis has been divided into four chapters.

In Chapter I of the thesis we have collected some definitions and results on grills, filters and also on proximities and nearnesses. Among the results collected here some are known, some are known but modified suitably for our purpose and a few are new. In Section 1.3, we provide a number of equivalent characterizations of ultraclosed filters and also some other properties of ultraclosed filters which are important for the following chapters. In Section 1.4 we collect some known results on clusters of a nearness structure and prove some unknown but simple results which are found to be useful to investigate extensions of topological spaces via nearness structures. We conclude this section by showing that the two definitions of contiguval nearness, one introduced
by Reed [27] and the other by Chattopadhyay, Njåstad and Thron [11], are equivalent.

In Chapter II we mainly characterize those W-T extensions which are compact or linkage compact. For this purpose we reinvestigate in Section 2.1, the theory of principal $T_1$-extensions of a $T_1$-topological space. A proof in [11] has been suitably modified to establish the result (Theorem 2.1.21) concerning when a principal $T_0$-extension is compact (linkage compact). We have also restated a few more results (viz. Theorem 2.1.23, Corollary 2.1.24) from [11] after a suitable modification for our purpose. A few new results have also been established concerning the separation axioms and trace systems of extensions which are found to be useful in our work. The Section 2.1 is concluded with an example which might be of some interest, to show that the correspondence between principal $T_1$-extensions and LO-nearnesses need not be 1-1 in general unless we are confined to the cluster generated nearnesses.

In Section 2.2, a few results concerning Wallman $\pi$-clans have been established. We also restate after a little modification and prove some results concerning Wallman nearnesses and covered proximities from Reed [28] for our purpose in the next section.
In Section 2.3, we characterize Wallman-type extensions in the class of principal $T_1$-extensions in terms of the trace systems and also characterize Wallman nearnesses in the class of cluster generated LO-nearnesses and then we use the results to characterize completely those $W$-$T$ extensions which are compact or linkage compact. Both have been done in terms of trace systems as well as in terms of the nearnesses generating the extensions. These characterizations led us to establish a necessary and sufficient condition on a covered proximity $\pi$ which insures the compactness (linkage compactness) of $W$-$T$ extensions and thereby an answer to a question of Reed that she raised in [28] is obtained.

In Section 2.4 we have also constructed $T_1$-compactifications of $T_1$-spaces by using $\mathfrak{B}$-ultrafilters as points where $\mathfrak{B}$ is a base for the closed sets of the respective spaces. The technique of our construction, to some extent, similar to the one used by Wallman [34]. We are able to show by imposing a 'disjunction' condition on the base $\mathfrak{B}$ that such compactifications are all Wallman-type extensions in the sense of Reed. Recall that the (classical) Wallman compactifications are Wallman-type extensions — is one of the results proved by Reed in [28]. We conclude this section by showing that the result follows quite trivially from our results of this section, because the set of all closed sets satisfies the 'disjunction' condition.
Recall that we have raised a number of reasonable questions which could be asked in the context of investigation in the area of Wallman nearnesses or W-T extensions. These questions including some others are settled by supplying sufficiently many examples in the last chapter.

Chapter III has been divided into two sections. In the last section of this chapter we proved that the set of all Wallman nearnesses compatible with a $T_1$-space is a complete lattice with respect to the ordering as the set inclusion. The smallest and the largest element of the lattice are explicitly described. It has further been shown that this lattice need not be a sublattice of the lattice of all LO-nearnesses compatible with a given $T_1$-space.

In the first section of Chapter III we prove a number of results on Wallman nearnesses which will mainly be used in the last chapter to construct the examples. Some of the results of this section are apparently technical in nature but they are found to be very much useful in the last chapter. Even though all the results proved in this section have been used in the construction of examples in the last chapter, a few of them may be thought to be interesting by themselves.

As it has already been pointed out, we supply sufficiently many examples in Chapter IV to settle a number of questions including those which have already been raised. In Chapter IV
we state all examples to be given and we exhibit their relative positions in a diagram before going to discuss them in detail.

Finally to conclude the introduction it may be pointed out that we have included in the references not only those books and articles which we have referred to in the thesis but also the books and articles in which related topics are found. In this connection it is worth mentioning that Chandler's book [8] and the book of Alo and Shapiro [3] have a good bibliography in the related area.