4.1 INTRODUCTION

Human beings often undergo accelerative disturbances in many situations. For example, while landing and taking off air crafts, travelling in a tractor, operating a jack hammer and due to sudden movements of body during sports activities, the human body experiences external accelerations. Under normal conditions, the pumping action of the heart controls the flow of blood in human circulatory system. A pressure gradient owing to pumping action of heart is created throughout the arterial and venous network. This pressure gradient consists of two parts, one of which is constant and other pulsatile. It is observed that blood flow in the circulatory system is significantly affected by various factors. It is also found that the human body adopts for changes in velocity in blood to a certain level. But when the changes in the velocity are of very large magnitude and persist for a long period of time it is likely that such changes may cause serious physiological effects which may sometimes lead to fatality. It is observed that prolonged exposure to such acceleration resulted in several disorders like headache, loss of vision, increase in pulse rate, abdominal pain, venous pooling of blood in neck, lungs, brain etc. To protect the body from these ill effects several protective devices have been designed (Hiatt, et al. 1969). On the other hand, if accelerations are given to the body, by properly timing with respect to heart beat it is found that they have therapeutic effects on systemic circulation (Arntzenius et al. 1970). Experiments conducted on pigs showed that in the case of a cardiogenic shock, blood pressure and cardiac output increased when acceleration was given to the body in synchrony with heart beat. This method was suggested to be a salient feature for assisting circulation, particularly in the case of patients who are to be treated for
cardiogenic shock. It is also indicated that external disturbance due to acceleration input associated with large amplitudes may lead to serious cardiovascular disorders.

The study of dispersion of a soluble matter in a solvent flowing in channels or pipes has importance in many chemical and biological systems. The dispersion of a passive containment in a steady flow in a conduit is owing to the joint influence of the longitudinal diffusion and the interaction of advection and lateral diffusion. Taylor (1953, 1954a, 1954b) discussed the dispersion of a solute in a fluid flowing through straight circular pipe. Following Taylor, Aris (1956) and Lighthill (1966) studied the dispersion with different approaches. After these fundamental studies the subject attracted several researchers to further investigate both theoretically and experimentally.

The velocity profile in a steady laminar flow in a straight channel or a circular pipe is always parabolic, while in an oscillatory flow, it is significantly altered by a change in the frequency, the kinematic viscosity viz. with the Womersley frequency parameter. The mechanism which causes the dispersion in steady flow also causes dispersion in unsteady flow. But the overall results differ significantly. For example, the dispersion in an oscillatory flow can be much less than in steady flow, if the period of oscillation is large compared with transverse mixing time, (Smith 1982a).

Significant amount of work on dispersion in oscillatory flows has been done owing to the possible applications in many situations such as the injection of a chemical substance in blood flow (Caro, 1966), the discharge of outfalls in estuaries (Bowden, 1965) the mutual contamination length of two feed fluids when switching from one feed line to another as a part of manufacturing process and the mass
transport in capillaries etc. Bowden (1965) analysed the problem of horizontal mixing in the sea due to a shearing current. Besides the random processes described as horizontal turbulence, there are certain more regular processes which may contribute to horizontal mixing. It is observed that in a shearing current, where the vertical gradient of velocity combined with vertical turbulent mixing led to an effective diffusion in the horizontal direction. It was shown that this effect exists in an alternating flow, such as tidal current, as well as in a steady flow. The horizontal mixing by the shear flow in estuaries and coastal waters may be associated with tidal currents, density currents or wind – driven currents. In every case the effective coefficient of horizontal diffusion is inversely proportional to the coefficient of vertical eddy diffusion. It is also found that the presence of a stable density gradient enhanced the effective horizontal mixing. The observations of the experiments conducted in Mersey estuary and Iris sea are compared with the theoretical estimates.

It is observed that oscillatory flow effects the entire dispersion process when the amplitude of the pressure gradient is larger than the mean pressure gradient. Using the method of moments, Aris (1960) investigated the dispersion of a solute in an oscillatory flow. It is observed that the effective molecular diffusivity contained terms proportional to the sequence of the amplitude of the pressure pulse. Chatwin (1975) analysed the dispersion of a passive contaminant along the axis of a tube in which the flow is driven by a longitudinal pressure gradient varying harmonically with time. He showed that strong oscillatory effects dominate the character of contaminant cloud over time intervals of many periods. Smith (1982a, b), in his study showed that the sensitivity to time of release of contaminant during a cycle and the importance of the
location of a discharge source on the flow. Jaeger and Kurzweg (1983) obtained the
dispersion coefficient in an oscillatory flow and observed that it is proportional to the
sequence of the amplitude of the oscillation and the first power of frequency in the
Womersley number to vary from 3 to 15.

Jimenez and Sullivan (1984) investigated the dispersion of a contaminant in
both oscillatory and impulsively started flows, between two parallel plates. The initial
contaminant cloud was taken to be a thin sheet of contaminant over the flow cross –
section. It was observed that the optimal stream wise spread of the contaminant results
from a value of the molecular diffusivity which depends on kinematic viscosity and
frequency. Watson (1983) studied the exact analysis for the diffusion in an oscillatory
flow in a pipe of arbitrary cross section. The resultant flux of the diffusing substance
has been analytically evaluated for the cases of a circular pipe and channel. The
general behaviour of the flux for an arbitrary cross – section in the limiting cases of
slow and fast oscillations were discussed. Joshi et al (1983) validated the theoretical
predictions of Watson (1983) with their study on diffusion of Methane in air in a
circular pipe.

Mazumder and Das (1992), Jiang and Grotberg (1993) and Bandopadhyay and
Mazumder (1999) studied the dispersion of a solute in an oscillatory flow and found
that the frequency parameter played an important role on the transport of the mean
concentration of the solute. Jiang and Grotberg (1996) studied the dispersion
phenomenon in a curved tube in an oscillatory flow. It was observed that the axial
dispersion in a curved tube was greater than that in a straight tube. Sarkar and
Jayaraman (2000) analysed the dispersion of a solute in an annulus in the presence of
an oscillatory flow field. It is observed that the dispersion coefficient is a function of the frequency parameter, annular gap and the Schmidt number.

Due to the oscillatory nature of flow, there is a possibility that the flow might change direction before the dispersion process has had time to become fully effective to obtain accuracy, for small times of moments. Deshikachar (1987) studied the axial molecular diffusion of a solute in the laminar flow of an electrically conducting fluid oscillating with zero mean, between two parallel plates in the presence of a transverse magnetic field using perturbation analysis. The initial distribution of solute is considered as a step function. Their analysis is valid not only for all times but also for any initial distribution. Ramana and Sarojamma (2011) investigated the phenomenon of dispersion of a solute in blood vessels when the body is subjected to a periodic body acceleration using the generalized dispersion model. They have discussed results in large and small arteries. It is observed that the body acceleration enhances the dispersion coefficient in all the arteries.

In this chapter, a mathematical model is developed to understand the effects of magnetic field and external accelerations on the phenomenon of dispersion of a solute in blood flows. The mathematical formulation of the problem with appropriate initial and boundary conditions is given in section 4.2. Section 4.3 presents the solution of the mathematical model employing the generalized dispersion model. Results and discussion are given in section 4.4. The conclusions are presented in section 4.5.
4.2 MATHEMATICAL FORMULATION

Consider the flow of blood modelling in an artery assuming it to be a circular tube and blood is modeled as a Newtonian fluid. A uniform transverse magnetic field of strength $B_0$ is applied. For $t^* > 0$, the flow is subjected to a periodic body acceleration $G$ in the axial direction. Let $a_0$ be the amplitude, $f_b$ the frequency in Hz and $\phi$ the lead angle of $G$ with respect to the heart action, $\omega_b = 2\pi f_b$ is the circular frequency. The body acceleration $G$ is given by

$$G = a_0 \cos (\omega_b t^* + \phi)$$

(4.1)

We represent the pumping action of heart by the pressure gradient $\frac{\partial p^*}{\partial z^*}$ produced by it. In human beings $\frac{\partial p^*}{\partial z^*}$ takes the approximate form (Burton, 1966)

$$\frac{\partial p^*}{\partial z^*} = A_0 + A_1 \cos \omega_p t^*$$

(4.2)

where $A_0$ is the constant component of pressure gradient, $A_1$ is the amplitude of the fluctuating component, $\omega_p = 2\pi f_p$ and $f_p$ is the pulse frequency.

To simplify the analysis, we additionally make the following suppositions.

(i) the artery wall is rigid and its length is infinite.

(ii) the flow is laminar

(iii) there is rotational symmetry of the flow

(iv) variation of velocity along the tube length is small as compared to the rate of change of velocity with respect to time.
(v) frequency of body acceleration $f_b$ is so small that the wave effects can be neglected.

Under the above mentioned assumptions, the equation of motion for flow, following Sud and Sekhon (1985) and Sanyal et al. (2007) in cylindrical polar co-ordinates $(r, \theta, z)$ can be written in the dimensional form as

$$\rho \frac{\partial w^*}{\partial t^*} = \rho a_0 \cos (\omega_b t^* + \phi) + A_0 + \frac{1}{r} \frac{\partial w^*}{\partial r} \left( \frac{\partial^2 w^*}{\partial r^2} + \frac{1}{r^2} \frac{\partial w^*}{\partial \theta} \right) - \frac{\sigma B_0^2}{\mu} w^* \quad (4.3)$$

where $\rho$ is the density of blood, $\mu$ is the coefficient of viscosity of blood, $\sigma$ is the electrical conductivity of the blood.

**Non-Dimensionalisation**

We introduce the following non-dimensional variables

$$w^* = \frac{w}{w_0^*}, \quad r^* = \frac{r}{R}, \quad t^* = \frac{D_m t}{R^2}, \quad C^* = \frac{C}{C_0}, \quad z^* = \frac{D_m z}{R^2 w_0} \quad (4.4)$$

where $w$ represents the non-dimensional velocity, $w_0$ is velocity in plane Poiseuille flow given by $w_0 = -\frac{R^2}{4 \mu} \frac{dp}{dz}$; $R$ is the radius of the tube.

Using the above non-dimensional variables equation (4.3) reduces to

$$\frac{1}{Sc} \frac{\partial w}{\partial t} = D_1 \cos (\omega_1 t + \phi) + D_2 + D_3 \cos (\omega_2 t) + \left( \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial \theta} \right) - M^2 w \quad (4.5)$$

where

$$D_1 = \frac{\rho a_0 R^2}{\mu u_0}; \quad D_2 = \frac{A_0 R^2}{\mu u_0}; \quad D_3 = \frac{A_1 R^2}{\mu u_0}; \quad \omega_1 = \frac{\omega_b R^2}{D_m}; \quad \omega_2 = \frac{\omega_b R^2}{D_m}; \quad \omega_2 = \frac{\omega_b R^2}{D_m}; \quad A_0 = \frac{A_0}{A_0}; \quad F_r = \frac{\omega_l}{\omega_2}, \quad Sc = \frac{\nu}{D_m}$$

is the Schmidts number.
\[ M = B_0 \sqrt{\frac{\sigma}{\mu}} \] is the Hartmann number.

The initial and boundary conditions in the non dimensional form are given by

\[ w(r,0) = 2 \sum_{k=1}^{\infty} \frac{J_0(r \lambda_k)}{\lambda_k} \frac{D_1 + D_3 \cos \phi}{\lambda_k^2 + M^2} \]

(4.6a)

\[ w \text{ and } \nabla^2 w \text{ are all finite at } r = 0 \]

(4.6b)

\[ w = 0 \text{ and } \nabla^2 w = 0 \text{ at } r = 1 \]

(4.6c)

where \( \lambda_k \) are the roots of the equation \( J_0(\lambda_k) = 0 \)

Taking finite Hankel transform of equation (4.5)

\[ w^*(\lambda_k, t) = \sum_{k=1}^{\infty} \frac{J_1(\lambda_k)}{\lambda_k} \frac{D_2 + D_3 \cos \omega t + D_4 \cos(\omega t + \phi)}{M^2 + \lambda_k^2} \]

(4.7)

where \( w^*(\lambda_k, t) = \int_0^1 r w(r, t) J_0(r \lambda_k) dr \)

Initially at \( t = 0 \)

\[ w^*(\lambda_k, 0) = \sum_{k=1}^{\infty} \frac{J_1(\lambda_k)}{\lambda_k} \frac{D_2 + D_3 + D_4 \cos \phi}{M^2 + \lambda_k^2} \]

(4.8)

To solve the problem we employ Laplace transform in equation (4.3) with the help of (4.6b) and (4.6c)

\[ \frac{1}{Sc} \left( s \bar{w} - w(0, 0) \right) = \frac{D_2}{s} + \frac{D_3 s}{s^2 + \omega_1^2} + \frac{D_4 (s \cos \phi - \omega_1 \sin \phi)}{s^2 + \omega_1^2} + \left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \right) \bar{w} - M^2 \bar{w} \]

(4.9)

Now taking finite Hankel transform of (4.9) and using (4.8) we get

\[ \bar{w}^*(\lambda_k, s) = \frac{J_1(\lambda_k)}{\lambda_k} \left[ \frac{D_2}{\beta} \left( \frac{1}{s} \frac{1}{s + h} \right) + \frac{D_3 \beta}{\beta^2 + m^2} \left( \frac{-1}{s + h} + \frac{s}{s^2 + \omega_1^2} + \frac{m}{\beta \left( s^2 + \omega_1^2 \right)} \right) \right] \]
+ \frac{D_1 \beta \cos \phi}{\beta^2 + m^2 \omega_r^2} \left( -1 + \frac{s}{s^2 + \omega_r^2} + \frac{m \omega_r^2}{\beta^2 + \omega_r^2} \right) \right) - \frac{D_1 \omega_r m \sin \phi}{\beta^2 + m^2 \omega_r^2} \\
\times \left[ \frac{1}{s+h} - \frac{s}{s^2 + \omega_r^2} + \frac{m}{m \omega_r^2} \right] + m \sum_{k=1}^{\infty} \frac{\gamma}{\beta} \left( 1 + \frac{1}{s+h} \right) \right] \tag{4.10}

where \( m = \frac{1}{Sc} \), \( \beta = \lambda_k^2 + M^2 \), \( h = \frac{\beta}{m} \), \( \gamma = D_2 + D_3 + D_4 \cos \phi \)

Taking inverse Laplace transform of equation (4.10)

\[ w^*(\lambda_k, t) = \frac{J_1(\lambda_k)}{\lambda_k} \left[ \left\{ \frac{D_2}{\beta} + \frac{D_3 (\beta \cos \omega_r t + m \sin \omega_r t)}{\beta^2 + m^2} \right\} \\
+ \frac{D_1 (\beta \cos (\omega_r t + \phi) + \omega_r m \sin (\omega_r t + \phi))}{\beta^2 + m^2 \omega_r^2} \right] \\
- e^{-ht} \left[ \frac{D_2}{\beta} + \frac{D_3 \beta}{m^2 + \beta^2} + \frac{D_1 (\beta \cos \phi + \omega_r m \sin \phi)}{m^2 \omega_r^2 + \beta^2} \right] - \sum_{k=1}^{\infty} \frac{\gamma}{\beta} \right] \] \tag{4.11}

Taking finite Hankel inversion of equation (4.11) we obtain the required solution as

\[ w(r, t) = 2 \sum_{k=1}^{\infty} w^*(\lambda_k, t) \frac{J_0(\lambda_k r)}{J_1^2(\lambda_k)} \\
= 2 \sum_{k=1}^{\infty} \frac{J_0(\lambda_k r)}{\lambda_k J_1(\lambda_k)} \left\{ \frac{D_2}{\beta} + \frac{D_3 (\beta \cos (\omega_r t) + m \sin (\omega_r t))}{m^2 + \beta^2} \right\} \\
+ \frac{D_1 (\beta \cos (\omega_r t + \phi) + \omega_r m \sin (\omega_r t + \phi))}{\beta^2 + m^2 \omega_r^2} \right\} \\
- e^{-ht} \left[ \frac{D_2}{\beta} + \frac{D_3 \beta}{m^2 + \beta^2} + \frac{D_1 (\beta \cos \phi + \omega_r m \sin \phi)}{m^2 \omega_r^2 + \beta^2} \right] - \sum_{k=1}^{\infty} \frac{\gamma}{\beta} \right] \] \tag{4.12}

We consider the dispersion of a bolus of a solute which is initially of \( z_s \) units in length and of uniform concentration \( C_0 \). For a fully developed, laminar flow in a
tube, the unsteady convective diffusion equation which describes the local concentration \( C \) of the solute as a function of longitudinal (axial) coordinate \( z \), transverse (radial) co-ordinate \( r \) and time \( t \) can be written in non-dimensional form as

\[
\frac{\partial C}{\partial t} + w \frac{\partial C}{\partial z} = \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial C}{\partial r} \right) + \frac{1}{Pe^2} \frac{\partial^2 C}{\partial z^2} \right) C
\]  

(4.13)

where \( C = \frac{C^*}{C_o}, \quad z = \frac{D_m z^*}{R^2 w_0} \),

(4.14)

\( D_m \) is the molecular diffusivity, and \( Pe = \frac{R u_0}{D_m} \) is the Peclet number.

The initial and boundary conditions are

\[
C(0, r, z) = 1 \quad \text{if} \quad |z| \leq \frac{z_s}{2}
\]  

(4.15a)

\[
C(0, r, z) = 0 \quad \text{if} \quad |z| > \frac{z_s}{2}
\]  

(4.15b)

\[
C(t, r, \infty) = 0
\]  

(4.15c)

\[
\frac{\partial C}{\partial r}(t, 0, z) = \frac{\partial C}{\partial r}(t, l, z)
\]  

(4.15d)

### 4.3 Method of Solution

In order to solve the convective diffusion equation (4.13) along with the set of initial and boundary conditions (4.15a-d), we introduce the derivative expansion method developed by Gill and Sankarasubramanian (1970). Following their solution procedure we assume the concentration \( C(t, r, z) \) as a series expansion in \( \frac{\partial^n C_m}{\partial z^n} \) and express \( C(t, r, z) \) as
\[ C(t, r, z) = \sum_{n=0}^{\infty} f_n(t, r) \frac{\partial^n C_m}{\partial z^n} \]  

(4.16)

where \( C_m = 2 \int_0^1 C r \, d r \)  

(4.17)

is the mean concentration over a cross section.

Multiply equation (4.13) by \( 2 \, r \) and integrating with respect to \( r \) from 0 to 1, we get

\[
\frac{\partial C_m}{\partial t} = \frac{1}{Pe^2} \frac{\partial^2 C_m}{\partial z^2} - 2 \frac{\partial}{\partial z} \int_0^1 u(t, r) \, C(t, z, r) \, r \, dr
\]  

(4.18)

If we introduce (4.16) into (4.18), the following dispersion model for \( C_m \) is obtained as

\[
\frac{\partial C_m}{\partial t} = \sum_{i=1}^{\infty} K_i(t) \frac{\partial^i C_m}{\partial z^i}
\]  

(4.19)

where the \( K_i \)'s are given by

\[
K_i(t) = \frac{\delta_{i,2}}{Pe^2} - 2 \int_0^1 u(t, r) \, f_{i-1}(t, r) \, r \, dr, \quad i = 1, 2, 3\ldots
\]  

(4.20)

and \( \delta_{i,2} \) is the Kronecker delta. The first two terms on the right hand side of (4.19) describe the transport of \( C_m \) in the axial direction \( z \) through convection and diffusion respectively, and therefore the coefficients \( K_1 \) and \( K_2 \) are termed as the convection and diffusion coefficients for \( C_m \). For a steady flow in the absence of the magnetic field, \( K_1 = 0 \) and \( K_2 = 1/Pe^2 + 1/192 \) (Gill and Sankarasubramanian, 1970). But, both the dispersion coefficients \( K_1 \) and \( K_2 \) are harmonic functions of time when dispersion is considered in an oscillatory flow field (Jiang and Grotberg, 1993).
Substituting (4.16) in (4.13), using equation (4.19) and equating the coefficients of \( \frac{\partial^n C_m}{\partial z^n} \), gives the partial differential equation for \( f_n \) as

\[
\frac{\partial f_n}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f_n}{\partial r} \right) - u(t, r) f_{n-1} + \frac{1}{Pe^2} f_{n-2} - \sum_{i=1}^{n} K_i f_{n-i}, \quad n = 1, 2 \ldots (4.21)
\]

where \( f_{-1} = 0 \) and \( f_0 = 1 \). 

From equation (4.15), the initial and boundary conditions on \( f_n \) are obtained as

\[
f_n(t, 0) = 0 \quad \text{(4.22 a)}
\]

\[
\frac{\partial f_n}{\partial r}(t, 0) = 0 = \frac{\partial f_n}{\partial r}(t, 1), \quad n = 1, 2 \ldots \quad \text{(4.22 b)}
\]

Equations (4.20) and (4.21) lead to coupled system of partial equations with boundary and initial conditions described by equations (4.22a-b)

Thus, from (4.12) and using (4.20) and \( f_0 = 1 \), we get

\[
K_i(t) = -4 \sum_{k=1}^{\infty} \frac{1}{\lambda_k^2} \left[ \frac{D_2}{\beta} + D_3 \left[ \frac{\beta \cos \omega_2 t + m \sin \omega_2 t}{m^2 + \beta^2} \right] + D_4 \left[ \frac{\beta \cos(\omega_1 t + \phi) + \omega_1 m \sin(\omega_1 t + \phi)}{m^2 \omega_1^2 + \beta^2} \right] \right] 
- e^{-\beta t} \left[ \frac{D_2}{\beta} + D_3 \frac{\beta}{m^2 + \beta^2} + D_4 \left[ \frac{\beta \cos \phi + \omega_1 m \sin \phi}{m^2 \omega_1^2 + \beta^2} - \frac{\gamma}{\beta} \right] \right] \quad \text{(4.23)}
\]

Using equation (4.21) and \( f_0 = 1 \), the partial differential equation in \( f_n \) for \( n = 1 \) can be expressed as,

\[
\frac{\partial f_1}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f_1}{\partial r} \right) - w(t, r) - K_i(t) \quad \text{(4.24)}
\]

Using initial and boundary conditions from (4.22) as

\[
f_1(t, 0) = 0 \quad \text{(4.25a)}
\]
\[ \frac{\partial f_1}{\partial r}(t, 0) = 0 = \frac{\partial f_1}{\partial r}(t, 1) \]  

(4.25b)

Solving the non-homogeneous partial differential equation (4.24) satisfying the conditions (4.25a - b), the expression for \( f_1 \) is obtained as

\[
f_1(t, r) = \sqrt{2} \sum_{n=1}^{\infty} B_n(t) \frac{J_0(\mu_n r)}{J_0(\mu_n)}
\]  

(4.26)

\[
B_n(t) = -2 \sqrt{2} e^{-\mu_n^2 t} \sum_{m=1}^{n} E_1 \left( E_2 \left( e^{i\omega_2 t} - 1 \right) + E_3 \left( \mu_n^2 \cos \omega_2 t + \sin \omega_2 t \right) - \mu_n^2 \right) + \\
E_4 \left( \mu_n^2 \sin \omega_2 t - \cos \omega_2 t + 1 \right) + E_5 \left( \mu_n^2 \sin \omega_2 t + \mu_n^2 \cos \omega_2 t - \mu_n^2 \right) - \\
E_6 \left( \mu_n^2 \sin \omega_2 t - \omega_1 \cos \omega_2 t + \omega_1 \right) + E_7 \left( \mu_n^2 \sin \omega_2 t - \omega_1 \cos \omega_2 t + \omega_1 \right) + \\
E_8 \left( \mu_n^2 \sin \omega_2 t + \mu_n^2 \cos \omega_2 t - \mu_n^2 \right) - E_9 \left( \frac{e^{i\omega_2 - h}}{\mu_n^2 - h} - 1 \right) \]  

(4.27)

\[
E_1 = \frac{1}{\lambda_2^2 - \mu_n^2}, \quad E_2 = \frac{D_2}{\beta \mu_n^2}, \quad E_3 = \frac{D_3}{(m^2 + \beta^2)(\mu_n^2 + \omega_2^2)}, \quad E_4 = \frac{D_4}{P^2 + C_1^2}(\mu_n^2 + \omega_2^2), \\
E_5 = \frac{D_5}{\beta \cos \phi}, \quad E_6 = \frac{D_6}{(\beta^2 + m^2 \omega_2^2)(\mu_n^2 + \omega_1^2)}, \quad E_7 = \frac{D_7}{(\beta^2 + m^2 \omega_2^2)(\mu_n^2 + \omega_1^2)}\]  

where \( \mu_n \)'s are zeros of the Bessel's function \( J_1 \).

Using (4.26) and (4.20) we get the expression for \( K_2 \) as

\[
K_2(t) = \frac{1}{Pe^2} = 2\sqrt{2} \sum_{n=1}^{\infty} B_n(t) H_n(t) \]  

(4.28)
where \( H_n(t) = \sum_{k=1}^{\infty} \frac{1}{\lambda_k^2 - \mu_n^2} \left[ D_2 \frac{\beta \cos(\omega, t) + m \sin(\omega, t)}{m^2 + \beta^2} \right. \right.
\]
\[ + D_3 \left[ \frac{\beta \cos(\omega, t + \phi) + \omega m \sin(\omega, t + \phi)}{\beta^2 + m^2 \omega_i^2} \right] \]
\[ - e^{-t} \left( \frac{D_2}{\beta} + \frac{D_3}{m^2 + \beta^2} + \frac{D_1}{\beta^2 + \omega_i^2 m^2} \right) \left\{ \frac{\gamma}{\beta} \right\} \tag{4.29} \]

**Solution for Mean Concentration \( C_m \)**

Neglecting \( K_3(t) \) and higher order coefficients, the generalized dispersion model leads to

\[ \frac{\partial C_m}{\partial t} = K_1(t) \frac{\partial C_m}{\partial z} + K_2(t) \frac{\partial^2 C_m}{\partial z^2} \tag{4.30} \]

The initial and boundary conditions for \( C_m \) are given by

\[ C_m(0, z) = 1, \text{ if } |z| \leq z_s / 2 \tag{4.31a} \]

\[ C_m(0, z) = 0, \text{ if } |z| > z_s / 2 \tag{4.31b} \]

\[ C_m(t, \infty) = 0 \tag{4.31c} \]

The solution of the mean concentration for equation (4.30) with the help of the condition (4.31) is given by

\[ C_m = \frac{1}{2} \left[ \text{erf} \left( \frac{z_s - z}{2 \sqrt{\xi}} \right) + \text{erf} \left( \frac{z - z_s}{2 \sqrt{\xi}} \right) \right] \tag{4.32} \]

where \( z_i = z + \int_0^t K_1(\eta) d\eta \tag{4.33} \)

\[ \xi = \int_0^t K_2(\eta) d\eta \tag{4.34} \]
4.4 RESULTS AND DISCUSSION

The objective of the present investigation is to understand the combined effect of body acceleration and magnetic field on the dispersion of solutes in blood flows in human beings modelling blood as a Newtonian fluid. To obtain a quantitative idea of the effects of body acceleration on the phenomenon of dispersion of solutes in blood flow, the results are discussed in large and small arteries. The relevant data for various arteries is compiled from published literature (Batschelet, 1975; Mac Donald, 1974; Wolf et al. 1976; Milnor 1982) and is presented in Table 4.1

**Table 4. 1. Data for different arteries**

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Artery</th>
<th>Radius (×10⁻² m)</th>
<th>Average Velocity (× 10⁻² m s⁻¹)</th>
<th>A₀ (×10 Kg m⁻² s⁻¹)</th>
<th>A₁ (×10 Kg m⁻² s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Aorta</td>
<td>1.0</td>
<td>45.6</td>
<td>7.3</td>
<td>1.46</td>
</tr>
<tr>
<td>2</td>
<td>Femoral</td>
<td>0.5</td>
<td>50.0</td>
<td>32.0</td>
<td>6.4</td>
</tr>
<tr>
<td>3</td>
<td>Carotid</td>
<td>0.4</td>
<td>50.0</td>
<td>50.0</td>
<td>10.0</td>
</tr>
<tr>
<td>4</td>
<td>Coronary</td>
<td>0.15</td>
<td>98.25</td>
<td>698.65</td>
<td>139.74</td>
</tr>
</tbody>
</table>

The dispersion process in oscillatory flows varies from that of steady flow case. In an oscillator flow it is possible that the flow might have changed direction before the dispersion process had time to become fully effective. Due to fluctuations in velocity the dispersion coefficient assumes both positive and negative values. In a period of oscillation owing to the reversal flow the solute would be carried backward...
along with flow and thus negative values for dispersion are induced. Therefore, in a period of oscillation the dispersion of solutes contracts at each flow reversal. In addition to the fluctuating character of the dispersion of solute, the dispersion is further influenced by the body acceleration and magnetic field.

The results have been discussed in aorta, femoral, carotid and coronary arteries for different values of amplitudes of body acceleration, Hartmann number $M$, Schmidt’s number $Sc$ and the slug input length $z_s$.

Fig. 4.1 (a-d) describes the distribution of mean concentration along the axial direction at different times ($T_n = \frac{nT_r}{4}$ for $0 \leq n \leq 7$ and $T_r = \frac{2\pi}{\alpha^2 Sc}$), when the slug input length $z_s = 0.02$, $a_0 = 0.98$, $Sc = 100$, $M=1$ and the phase angle $\phi=\frac{\pi}{3}$. It is observed that in all the arteries the peak value of the mean concentration $C_m$ decreases as the dispersion time decreases and subsequently the profile becomes flattened. The peak value for $T_1$ is attained at origin in all arteries and the peak value drifts to the right of origin for the subsequent time values. The peak value in aorta is attained in the vicinity of origin. However, as the radius of the artery decreases the peak is drifted very much away from origin. In femoral and carotid the peak value of $C_m$ is attained in the intervals $(0, 0.5)$ while in coronary it is attained in $0 < z < 18$.

Fig 4.2 (a-d) depicts the variation of mean concentration $C_m$ versus axial distance for different values of the amplitude of the body acceleration when $Sc = 100$, $M=1$ and the phase angle $\phi=\frac{\pi}{3}$. It is seen that in aorta, when the amplitude of the body acceleration is 0.98 there is no much variation when compared to the corresponding case in the absence of body acceleration. But when $a_0 = 1.96$ there is a
significant decrease in the peak of the mean concentration. The peak value of the mean concentration is drifted towards the right of the origin. However, in femoral and carotid arteries the mean concentration is decreased significantly by the presence of body acceleration and further decreased with increase in amplitude of body acceleration. In femoral artery the peak value of mean concentration in the presence of body acceleration is reduced from 0.999 to 0.644 (\(a_0 = 0.98\)). When \(a_0 = 1.96\) it is half of the peak value of that in the absence of the body acceleration. It is observed that the effect of body acceleration in coronary on \(C_m\) is not appreciable.

Fig 4.3(a-d) presents the variation of mean concentration versus the axial distance when \(Sc = 100\) and \(a_0 = 0.98\) for different values of Hartmann number. It is observed that the presence of magnetic field increases the mean concentration. As the intensity of the magnetic field (i.e., Hartmann number) increases \(C_m\) is also increased. In aorta there is a two fold increase in the peak value of \(C_m\) when \(M = 2\) to that of the value in the absence of the magnetic field. The points of the peak values are drifted towards the origin as \(M\) increases. A similar behavior is noticed in femoral and carotid arteries. But in coronary artery the influence of magnetic field is relatively less. The peak value of mean concentration increases from 0.5062 to 0.7885 when \(M\) takes the value from 0 to 2 in coronary artery.

Fig 4.4 (a-d) shows the variation of dispersion coefficient \(K_2\) versus time when \(Sc = 1\), \(M = 7\) for different values of amplitude of body acceleration. Due to the oscillatory nature of the flow, \(K_2\) shows an oscillatory behavior. It assumes positive and negative values due to the forward and backward movement of the solute. It is also observed that it is harmonic. The presence of body acceleration increases the
magnitude of the dispersion coefficient. When \( M = 7 \) and \( a_0 = 1.96 \) the maximum value of \( K_2 \) in aorta is twice that of the corresponding case in the absence of body acceleration. In femoral and carotid arteries the impact of body acceleration is very meager and in coronary artery the effect of body acceleration is negligible. Fig 4.5(a-d) describes the effect of magnetic field on \( K_2 \) in one cycle of time. It is noticed that the presence of the magnetic field decreases the dispersion coefficient in all arteries. The negative values of \( K_2 \) are not symmetrical with reference to origin.

In aorta in the presence of magnetic field (\( M = 1 \)) the maximum value of \( K_2 \) is reduced to 3.025 from 4.2357 when \( M = 0 \). When \( M = 2 \), the maximum value of \( K_2 \) is three times less than that of the non-magnetic case. When \( M = 3, 4 \) the maximum and minimum values are symmetrical about the origin. For \( M = 4 \) the maximum value of \( K_2 \) is reduced by 12 times of that value corresponding to \( M = 1 \). When \( M = 7 \), \( K_2 \) is almost uniformly zero. A similar trend is noticed in the rest of the three arteries qualitatively. However, the magnitude of the dispersion coefficient increases with decrease in the size of the artery.

Fig 4.6 (a-d) describes the mean concentration \( C_m \) along the axial direction for different values of \( A_r \), the ratio of amplitudes of the body acceleration to that of the pressure gradient. When the amplitude of body acceleration is half of the amplitude of the pressure gradient, the peak value of the mean concentration in aorta occurs at \( z = 1 \) and when it is equal the point of peak value drifts to the right of \( z = 1 \) and it decreases from 0.0173 to 0.0154. With an increase in \( A_r \) the point of peak value of \( C_m \) drifts further and the magnitude is also decreased in aorta. In the remaining arteries also \( C_m \) shows a similar behavior qualitatively.
The variation of ratio of frequencies of body acceleration to that of pressure gradient (\(F_r\)) on dispersion coefficient is shown in Fig 4.7 (a-d). In aorta, when frequency of body acceleration is half of the pressure gradient, \(K_2\) assumes negative values attaining a minimum and starts increasing and assumes positive values again attaining a maximum value and shows a periodical behavior in the remaining cycle. When the frequencies are equal, \(K_2\) increases, assuming positive values and then decreases assuming negative values and shows the same behavior in the rest of the cycle as in the case \(F_r = 0.5\). When the frequency of body acceleration is twice that of the pressure gradient, it is qualitatively same as in the case \(F_r = 0.5\). In rest of all arteries a similar behavior is noticed.

Fig 4.8 (a,b) describes the variation of \(D_{eff}\) versus Schmidt’s number for different values of magnetic field. In all the arteries it is observed that the \(D_{eff}\) decreases with increase in Schmidt number. In aorta, in the absence of magnetic field, when \(a_0 = 9.8\), it is observed that the \(D_{eff}\) is very small as \(Sc\) approaches 5. The magnitude of \(D_{eff}\) decreases as the radius of the artery increases. The presence of magnetic field reduces \(D_{eff}\). When \(M = 2\), it is uniformly zero for all values of \(Sc\). In the remaining arteries, \(D_{eff}\) reduce with respect to magnetic field is same as that of the case in aorta. However, the values for \(D_{eff}\) when \(M = 2\) are different from zero and significant. In carotid for \(Sc = 1\) in the absence of magnetic field the value of \(D_{eff}\) is more than twice that of the corresponding value in femoral. In coronary artery, the corresponding values for \(M = 2\) are very high.

Fig 4.9 (a-d) illustrates the mean concentration for different values of Hartmann number in the four arteries when the observation point is inside the slug.
input. It is observed that the presence of magnetic field enhances the $C_m$ and it reduces with increase in time. A similar behavior is noticed in the remaining arteries also. Fig 4.10 (a-d) shows the distribution of mean concentration when the point of observation is outside the slug. In aorta it is noticed that there is a sudden rise in $C_m$ and attains its peak value and drops significantly in course of time. The presence of magnetic field and increase in magnetic field reduces the value of $C_m$ and the drop in its value after attaining its peak value is also controlled. In femoral the peak values are observed to be the same. In carotid qualitatively a similar behavior as in aorta is noticed. But the peak values are observed to be attained almost at the same time for all of M. In coronary the effect of magnetic field as $C_m$ versus time is negligible. Fig 4.11 (a-d) illustrates the variation of $C_m$ verses time when $M = 1$, $a_0 = 0.98$, $z = 0.5$ for different lengths of the solute. In all the arteries it is found that the solute disperses faster for shorter lengths of slug input lengths.

4.5 CONCLUSIONS

The dispersion of a solute in blood flow under the influence of periodic body acceleration and a uniform transverse magnetic field is studied. The governing equations of flow and dispersion are solved employing finite Hankel and Laplace transformation and generalized dispersion model. The study brings out the development of the mass transport due to the introduction of a solute in terms of the convection and dispersion coefficients. The results are discussed in large and small arteries. Due to the fluctuations in the velocity owing to the oscillatory flow the dispersion coefficient assumes positive and negative values. The effect of magnetic
field is found to decrease the dispersion coefficient and is significant in aorta. In aorta $K_2$ is reduced by 12 times of the corresponding value when $M = 1$. While in the other arteries the effect of magnetic field on $K_2$ is not as prominent as in aorta. The effect of body acceleration on the dispersion coefficient $K_2$ in aorta is found to be significant where as in femoral and carotid arteries its effect is meager. While in coronary artery it is negligible. The mean concentration $C_m$ is found to increase in aorta, femoral and carotid arteries and in coronary artery it is relatively less.
Fig 4.6. (a, b) of $C_m$ with $z$ for different values of $A_r$ when $\text{Sc} = 1$, $M=1$

in (a) Aorta (d) Femoral
Fig 4.6.(c, d) Variation of $C_m$ with $z$ for different values of $A_r$ when $Sc = 1$, $M=1$

in (c) Carotid (d) Coronary
Fig 4.1(a, b) Variation of $C_m$ with $z$ for different values of time ($T_n = \frac{nT_c}{4}$ for $0 \leq n \leq 7$) at $z_s = 0.02$, $Sc = 100$, $a_0 = 0.98$, $M=1$ in (a) Aorta (b) Femoral
Fig 4.1(c, d) Variation of $C_m$ with $z$ for different values of time ($T_n = \frac{nT_r}{4}$ for $0 \leq n \leq 7$) at $z_s = 0.02$, $Sc = 100$, $a_0 = 0.98$, $M = 1$ in (c) Carotid (d) Coronary
Fig 4.2. (a, b) Variation of $C_m$ with $z$ for different values of $a_0$ at $z_s = 0.02$, $Sc = 100$, $M=1$ in (a) Aorta (b) Femoral.
Fig 4.2. (c, d) Variation of $C_m$ with $z$ for different values of $a_0$ at $z_a = 0.02$, $Sc = 100$, $M=1$ in (c) Carotid (d) Coronary.
Fig 4.3.(a, b) Variation of $C_m$ with $z$ for different values of $M$ at $z_s = 0.02$, $Sc = 100$ in (a) Aorta (b) Femoral
Fig 4.3 (c, d) Variation of $C_m$ with $z$ for different values of $M$ at $z_s = 0.02$, $Sc = 100$, $M=1$ in (a) Carotid (b) Coronary
Fig 4.4(a, b) Variation of $K_2$ with $t$ for different values of $a_0$ at $Sc = 1$, $M = 7$ in

(c) Aorta (d) Femerol
Fig 4.4.(c, d) Variation of $K_2$ with $t$ for different values of $a_0$ at $Sc = 1$, $M = 7$ in

(c) Carotid (d) Coronary
Fig 4.5. (a, b) Variation of $K_2$ with $t$ for different values of $M$ at $Sc = 1$, $a_0 = 0.98$

in (a) Aorta (b) Femoral
Fig 4.5. (c, d) Variation of $K_2$ with $t$ for different values of $M$ at $Sc = 1$, $a_0 = 0.98$

in (c) Carotid (d) Coronary
Fig 4.8(a,b). Variation of $D_{\text{eff}}$ with Schmidt number for different values of $M$ when $a_0 = 9.8$ (a) Aorta, Femoral and Carotid (b) Coronary
Fig 4.9(a,b). Variation of $C_m$ with $t$ for different values of $M$ at $z_s = 0.02$, $Sc = 1$, $z = 0.005$ in (a) Aorta (b) Femerol
Fig 4.9(c,d). Variation of $C_m$ with $t$ for different values of $M$ at $z_s = 0.02$,

$Sc = 1, z = 0.005$ in (c) Carotid (d) Coronary
Fig 4.10(a,b). Variation of $C_m$ with $t$ for different values of $M$ at $z_s = 0.02,$

$Sc = 1, z = 0.05$ in (a) Aorta  (b) Femerol
Fig 4.10(c, d). Variation of $C_m$ with $t$ for different values of $M$ at $z_s = 0.02$.

$Sc = 1, z = 0.05$ in (c) Carotid (d) Coronary
Fig. 4.11 (a, b). Variation of $C_m$ with $t$ for different values of $z_s$ at $z=0.5$, $Sc = 1$, $M = 1$, $a_0 = 0.98$ in (a) Aorta (b) Femoral
Fig 4.11(c, d). Variation of $C_m$ with $t$ for different values of $z_s$ at $z=0.5$, $Sc = 1$, $M = 1$, $a_0 = 0.98$ in (c) Carotid (d) Coronary
Fig 4.7 (a, b) Variation of $K_2$ with $t$ for different values of $F_r$ when $Sc = 1$ in

(a) Aorta (b) Femoral
Fig 4.7 (c, d) Variation of $K_2$ with $t$ for different values of $F_r$ when $Sc = 1$ in

(c) Carotid (b) Coronary