Chapter 3

Charging of Dust Grains in Plasma

3.1 Limitations of existing theories:

Dust particles emerged in plasma acquire electric charges collecting electrons and ions, by photo-emissions, secondary electron emission, thermoionic emission etc. The study of the charge fluctuation problem of dusty plasma is important for the interpretation of various astrophysical and laboratory phenomena. Charge fluctuation plays an important role in dusty plasma. It leads to damping of the dust-acoustic mode, unstable high frequency plasma modes and negative energy mode for streaming ions in the background of fixed dust. Interesting phenomena are observed when the charge to mass ratio of a dust grain is large due to the influence of electric and magnetic forces on the grain. Hill and Mendis[1982c] have suggested that the F-ring, composed of micron-sized grains, could carry a current of $10^8$ A which would significantly change the planetary magnetic field. Goertz and Morfill[1983] have shown how the currents in a dusty plasma cloud could polarize the cloud and cause its radial motion, which they relate to the rapid growth of spokes of Saturn’s ring. Ran-
dom fluctuation of charge on dust grains may grow in magnitude and duration. It may promote agglomeration by lowering the repulsive barrier for particle-particle collisions and enhance particle transport by inducing diffusive motion across magnetic field lines, thus producing behavior that is not explained by assuming the charge to be constant.

Most of the dusty plasma charging theories are based on theories of electrostatic probes in plasmas. The currents are termed as “orbit-limited” when the condition $r_d \ll \lambda_D \ll \lambda_{mf}$ applies where $r_d$ is the grain radius, $\lambda_D$ is Debye length and $\lambda_{mf}$ is a collisional mean free path between neutral gas atoms and either electrons or ions.

The standard continuous charging model neglects the fact that the electron and ion currents collected by the particle actually consist of individual electrons and ions. The charge on the particle is an integer multiple of the electron charge. Electrons and ions arrive at the particle’s surface at random time and charge fluctuates in discrete steps and at random time about the steady state value.

Effect of ion trapping has been ignored in deriving the above charging theories. Negative charges on $\alpha$ particle create a Debye sheath, which is an attractive potential well for positive ions. A passing ion can be trapped in this well when it suffers a collision within the particle’s Debye sphere, loses energy and suffers a change in orbital angular momentum. It remains trapped there, in an orbit bound to the particle, until it is detrapped by another collision. These trapped ions shield the charged dust grain from external electric field. Electromagnetic forces on the grain are thereby reduced, and other forces, such as gravity and gas drag, become more significant.

It is already said that orbit limited theory of charging of dust grains is valid only when collisional mean free path between neutrals and ion-electrons exceeds Debye-shielding length of plasma i.e. for $\lambda_{mf} \gg \lambda_D \gg r_d$. In this case, currents are calculated by assuming that the electrons and ions
are collected if their collisionless orbits intersects the probe surface. The effec-
t of collisions has been completely neglected. The OML theory fails if
\[ \lambda_{mfp} < r_d \left( -\frac{e\phi}{kT_i} \right)^{1/2} \] [Allen 1992]. This provides a limitation to the use of
OML theory in cases where collisional effect is not negligible. No plasma is
entirely collisionfree nor infinite in extent.

In section 3.2, we are giving a theory of random charging of dust grains in
plasma. It is assumed that electrons and ions in plasma perform random walk
before hitting the grain surface. Considering this Brownian type of motion and
Chandrasekhar's idea of absorption of particles having random motion with
an absorbing boundary condition, charging currents for electrons and ions are
built up. It is hoped that theory will be valid even in the weakly ionized
plasma.
3.2 Theory of random charging of dust particles in dusty plasma

When dust particles are emerged in plasma, they get charged due to random collection of electrons and ions. In the present theory, it is assumed that charging of the grains is basically a Brownian process in which electrons and ions are performing random walk before hitting the grains surface. Electrons and ions keep sticking to the grains until a substantial field builds up which repels the electrons and thus an ambipolar diffusion sets near the dust grain. Chandrasekhar's idea of absorbing boundary condition for particles having random motion is used to describe the process of charging of dust grains. The diffusion co-efficient of electrons and ions are calculated by using the strong field approximation of Chapman and Enskog, which is valid for the region \( d < \frac{e^2}{T} < \lambda_D \) where \( d \) and \( \lambda_D \) are inter-particle distance and Debye-shielding length respectively. Our theory may describe charging equation of dust grains in weakly ionized plasma.

3.2.1 Introduction

Dust is an almost ubiquitous component of the interstellar, interplanetary, circumplanetary environments. These dust grains are immersed in ambient plasma and radiative environments. They therefore, get electrically charged and consequently coupled to the plasma through electric and magnetic fields, with the coupling becoming stronger as the grain charges decreases. The dust particles in plasmas are unusual charge carriers. They are many orders of magnitude larger time-dependent charges. The basic difference between these charges carrying dust particles in plasma and multiply charged ionic plasma is
this fluctuation of charge. Electrons and ions are randomly moving and they hit the dust surfaces at random intervals giving rise to charge fluctuation. The charge on a small dust particle is of fundamental interest, because it allows the coupling between the fields and particles environment to the dynamics of the dust grains. Hence the study of charging mechanism of dust grains is important for proper understanding of instabilities, wave modes and overall collective behavior of plasma. In the last decade, the collective modes in dusty plasma are mainly investigated and little attention has been paid on the formulation of kinetic equation for the charging of the dust grains, transport properties of dusty plasma etc. The existing standard model of charging is valid only for collisionless plasma. In the theory that we have developed, the collisional effect is considered through the introduction of diffusion coefficient in the charging equation. The diffusion coefficient is evaluated by using strong field approximation of Chapman and Enskog which is valid for weakly ionized plasma. The charging equation derived in this way may be appropriate for charging of aerosols in the atmosphere.

The nanometer sized smoke and dust particles with concentration of several thousand per cubic centimeter resulting from meteoroid ablation has been reported to occur at heights of about 80 kms above the upper atmosphere [Turco et al.1982]. Recent work has shown that the particles become electrically charged by collecting electrons and ions from the ionosphere. The upper mesosphere also comprises the D region of the ionosphere, and as a result the meteoric dust particles and the growing ice particles exist in a weakly ionized plasma[Reid 1997]. Electrical charging plays a major role in the growth process[Jensen 1991]. In the upper atmosphere and lower ionosphere, diffusion of ions and electrons cannot be ignored if undisturbed conditions are not considered[Klostermeyer 1994]. In the above situation, collisional mean free path is comparable or smaller than the Debye length. Therefore, orbital limit assumption for the description of the charging of the currents cannot be used[Allen
The upper atmosphere, at heights above 80 kms, is the region of the atmosphere, where most ablation of incoming meteoric material takes place, and is thought to contain tiny nanometer size smoke and dust particles with concentrations of several thousand per cubic centimeter. Particles become electrically charged by collecting free electrons and positive ions from the background ionosphere. The upper mesosphere also comprises the D region of the ionosphere, and as a consequence the meteoric dust particles and the growing ice particles exist in a weakly ionized plasma. Electrical charging is likely to play a major role in the growth process. The theory of random charging may be applicable in this situation.

Electrons and ions are performing random walk in plasma and strike the surface of the dust grains at random intervals. The charging of the dust grains, thus can be considered as a Brownian process. It is a unidirectional process in the sense that electrons and ions keep sticking to the grains until a substantial field builds up which repels the electrons and thus an ambipolar diffusion sets near the dust grain. Chandrasekhar describes the process of random walk of a particle with an absorbing boundary condition. Charging of the grains is a similar process described by Chandrasekhar and thus we hope that formulation of kinetic equation for the charging of the grains is possible. In deriving our theory, it is assumed that charged particles are deflected by single collisions with the dust grains, rather than by multiple scattering by other charged particles.

At high latitudes in the summertime, a particularly striking feature is observed in the ionization near the mesopause. Measurements of the electron density in this region using Langmuir probes and RF capacitance probes show a pronounced depletion. This sharp depletion in electron density must be due to either a sudden decrease in the electron production rate or a sudden increase in the electron loss rate. Since the large water cluster ions exist only at
or very near the mesopause, it is possible that the electron depletion could be due to considerably enhanced recombination co-efficient associated with these ions. Havnes et al suggested that the apparent reduction in electron density observed with Langmuir probes could be caused by the inability of the electrostatic probes to deflect massive, positively charged dust particles. A proper theory of aerosol charging in weakly ionized plasma is important for the study of effect of net negative charge present in mesospheric aerosols on coagulation. Coagulation is the only known growth mechanism for meteoric smoke particles in the mesosphere. These particles act as nucleation sites for the crystal formation.

3.2.2 Formulation

The charging of the dust grains immersed in plasma is basically a Brownian process, in which electrons and ions hit the grain surface randomly. Here Chandrasekhar's idea of random walk of particles with absorbing barrier is considered [Chandrasekhar 1943].

Let there be an absorbing wall at $m_1$. Fraction of a large number of particles initially at $x = 0$ and which are deposited on the absorbing wall at $x = x_1$ per unit time is

$$W(x, t) = \frac{x_1}{t} \frac{1}{2\sqrt{\pi Dt}} \exp \left( -\frac{x_1^2}{4Dt} \right)$$

where $x_1 = m_1$, $l$ is the net displacement of particles, $t$ is the step length and $D$ is the diffusion rate. This probability is used for calculating the probability that plasma particles performing Brownian motion get attached to the dust particles and subsequently for calculating the plasma current. The effect of collision is taken into consideration by the introduction of the term "diffusion co-efficient" which is here evaluated using Chapman-Enskog method.

We consider the motion of a group of electrons whose number density is
small compared with the number density of the dust grain, so that collision between these particles can be ignored. The diffusion flux is

$$ j = -D \nabla N $$

(3.2)

where $D$ is the diffusion coefficient and $N$ is the number density of the test particles (electrons or ions). The kinetic equation for the distribution function of the electrons taking into account their diffusion is [Smirnov 1981]

$$ v_e f_{e0} \frac{\nabla N}{N} = \int (f'_{e'} f'_d - f_e f_d) \ (\vec{v}_e - \vec{v}_d) \ d\sigma dv_d $$

(3.3)

where $f_{e0}$ is the Maxwellian distribution function for the electrons, $f_e$ is the distribution function of the electrons with diffusion and $f_d$ is the Maxwellian distribution function of the dust particles.

Assuming that the density gradient is along x-axis and multiplying equation (3.3) by momentum $m_e v_{ex}$ of the electrons in the x direction and integrating over the velocities we get the left hand side of the equation as $T(\partial N/\partial x)$ and finally the equation (3.3) becomes

$$ T \frac{\partial N}{\partial x} = \mu \int f_e f_d g \sigma*(g) dv_e dv_d $$

(3.4)

where $\mu = m_e m_d/(m_e + m_d)$ is the reduced mass of the electrons and the dust particle, $g = |\vec{v}_e - \vec{v}_d|$ is the relative velocity of collision and $\sigma*(g) = \int (1 - \cos\theta) d\sigma$ is the diffusion cross-section of collision between the electron and the dust particles.

The distribution function for the test particles in the Chapman-Enskog approximation can be written as

$$ f(v_e) = f_0(v_e) \left[ 1 - v_{ex} \frac{\partial \ln N}{\partial x} h(v_e) \right] $$

(3.5)

Replacing this equation into the right hand side of equation (3.4) and assuming that $h(v_e)$ is constant, we find
\[
\frac{T \frac{\partial N}{\partial x}}{N} = \frac{1}{N} \frac{\partial N}{\partial x} \frac{\mu m_d}{(m_e + m_d)} < g^2 g \sigma^*(g) > h \quad (3.6)
\]

Here we have used \( v_e = v_c + m_d g / (m_e + m_d) \), where \( v_c \) is the velocity of centre of inertia and \( m_e \) and \( m_d \) are the masses of the electrons and the dust particle. The angle brackets mean averaging over the Maxwellian distribution. Thus we get

\[
h = \frac{3T m_e}{\mu^2 N_1 < g^3 \sigma^* >} \quad (3.7)
\]

and the flow of electrons in this case is

\[
j_x = \int v z f(v_e) dv_e = -\frac{hT}{m_e} \frac{\partial N}{\partial x} \quad (3.8)
\]

Putting the value of \( h \) in equation (3.7) and comparing with equation (3.2) we find the diffusion co-efficient as

\[
D_{ed} = \frac{3T^2}{\mu^2 N < g^3 \sigma^*(g) >} \quad (3.9)
\]

Now, collision cross-section for plasma particles and dust particles can be obtained as

\[
\sigma^*(g) = \pi r_d^2 \left( 1 - \frac{2e \alpha g}{m_\alpha v^2} \right) \quad (3.10)
\]

where \( e_\alpha \) is the electron (ion) charge, \( g \) is the charge of the dust particle, \( m_\alpha \) is the mass of the electron or ion and \( r_d \) is the radius of the dust particle. Therefore,

\[
<g^3 \sigma^*(g) >= \int f_\alpha f_d |\vec{v}_e - \vec{v}_d|^3 \sigma^*(g) dv_e dv_d \quad (3.11)
\]

Writing \( \beta = T_e/T_d, v = (m_e v_e + \beta m_d v_d)/(m_e + \beta m_d), M = m_e + \beta m_d, m = (\beta m_e m_d)/M \) and \( \omega = v_e - v_d \), we obtain
\[ \langle g^3 \sigma^*(g) \rangle = \int \left( \frac{nm}{4\pi K^2 T^2} \right)^{3/2} \exp \left[ -\frac{(mv^2 + Mv^2)}{2KT} \right] \, w^3 \sigma^* \, dw \]  

(3.12)

After performing the integration we get

\[ \langle g^3 \sigma^*(g) \rangle = r^2 \sqrt{\frac{8M^2 KTe}{\pi^2 m^3}} \left[ 1 + \frac{eqm}{2m_e KTe} \right] \]  

(3.13)

Substitution of the expression for \( \langle g^3 \sigma^*(g) \rangle \) in equation (3.9) gives the diffusion co-efficient of electron in the form

\[ D_e = \frac{3T_e^2 (m_e + \beta m_d)}{4\mu^2 N_e r_d^2} \sqrt{\frac{(m_e m_d)^3}{2KT_e^2 (m_e + \beta m_d)^5}} \left( 1 + \frac{eqm_d}{2K(m_e T_e + m_d T_e)} \right)^{-1} \]  

(3.14)

Similarly the diffusion coefficient of ions will be

\[ D_i = \frac{3T_i^2 (m_i + \beta m_d)}{4\mu^2 N_i r_d^2} \sqrt{\frac{(m_i m_d)^3}{2KT_i^2 (m_i + \beta m_d)^5}} \left( 1 - \frac{eqm_d}{2K(m_i T_i + m_d T_i)} \right)^{-1} \]  

(3.15)

where \( \mu' = m_i m_d / (m_i + m_d) \) is the reduced mass of the ions and the dust particles. \( N = N_e, N_i \) for electron and ion density respectively. The ambipolar diffusion near the dust grain can be written as

\[ D_{amb} = D_i \left( 1 + \frac{T_e}{T_i} \right) \]  

(3.16)

3.2.3 Charging Equation:

To determine the ion current on a dust particle, we consider a dust particle at position \( (x_1, t) \) with Debye length \( \lambda_D \). Equation (3.1) gives the fraction of large number of particles undergoing random motion initially at \( x = 0 \), that are deposited on the absorbing wall at \( x = x_1 \) per unit time. The Debye sphere surrounding a dust particle can be considered to be such an absorbing
boundary, where plasma particles viz., electrons and ions having Brownian motion get canceled with the probability that can be obtained by modifying equation (3.1). Thus, the probability that particles starting from \( x = 0 \) are deposited on the dust particle is

\[
W(x_1, t) = \frac{x_1}{t} \frac{1}{2 \sqrt{\pi D t}} \exp \left[ -\frac{x_1^2}{4Dt} \right] \tag{3.17}
\]

where \( \alpha = i.e. \) Upto the Debye sphere surrounding the dust particle, the velocity of the ions will be their random velocity \( v \); but when the ions cross the boundary of the Debye sphere, their velocity becomes \( (v + v_\lambda) \), where velocity \( v_\lambda \) inside the Debye sphere due to the attracting field is obtained from the relation

\[
\frac{1}{2} m_i v_\lambda^2 = q(\phi - \phi_f) \tag{3.18}
\]

or

\[
v_\lambda = \sqrt{\frac{2q(\phi - \phi_f)}{m_i}} \tag{3.19}
\]

Hence the probability that ions having velocity between \( v \) and \( v + dv \) will hit the surface of the dust particle is

\[
W(v) = (v + b) \frac{1}{2 \sqrt{\pi D t}} \exp[-(v + b)^2a] \tag{3.20}
\]

where \( b = v_\lambda = \sqrt{2q(\phi - \phi_f)/m_i} \) and \( a = t/4D_i \).

Now we can write the relation of ion current through the dust particle surface as

\[
I_i = \pi r_d^2 \int_0^\infty c N_i v W(v) dv \tag{3.21}
\]

After straightforward calculations we get
\[ I_i = 4N_e \pi r_d^2 eD_i \left[ \frac{1}{t^2} - \frac{1}{\pi D_i t^{3/2}} \sqrt{\frac{2q(\phi - \phi_f)}{m_i}} \right] \] (3.22)

Similarly up to the Debye sphere electrons will be moving with random velocity \( v \). When they cross the Debye sphere surrounding the dust particle, (i.e. when they enter sheath region of the dust particle) they will be under a retarding potential and will experience a negative acceleration. Hence velocity inside the Debye sphere is \( v - v_\lambda \) where \( v_\lambda = \sqrt{2q(\phi - \phi_f)/m_e} \). Hence the probability that electrons starting from \( x = 0 \) are deposited on the surface of the dust particle is

\[ W(x_1, t) = \frac{x_1}{t} \frac{1}{2\sqrt{\pi D_e t}} \exp \left[ -\frac{x_1^2}{4D_e t} \right] \] (3.23)

and in terms of velocity of the particles we can write the electron current as

\[ W(v) = (v - v') \frac{1}{2\sqrt{\pi D_e t}} \exp[-(v - v')^2a'] \] (3.24)

where \( v' = v_\lambda = \sqrt{2q(\phi - \phi_f)/m_e} \) and \( a' = t/4D_e i \).

Electron current through the dust particle surface as

\[ I_e = -\pi r_d^2 \int_{v_{\text{min}}=v'}^{\infty} eN_e v W(v) dv \] (3.25)

Performing the integration, finally we have,

\[ I_e = -\frac{N_e \pi r_d^2 eD_e}{4} \left[ \frac{1}{t^2} - \frac{1}{\pi D_e t^{3/2}} \exp \left( -\frac{t}{4D_e} \frac{q(\phi - \phi_f)}{m_e} \right) \right] \] (3.26)

Equations (3.22) and (3.26) give the ion and electron currents respectively flowing to a dust grain at any time \( t \).

3.2.4 Discussion and Conclusion:

In section 3.2.2, a theory of random charging of dust grains in plasma is discussed. Time dependent current equations are obtained for charging of dust
particles by electrons and ions. It is mentioned earlier that the standard OML theory of charging is applicable only under the condition $\lambda_{mf} >> \lambda_D >> r_d$ i.e. for plasma, where effect of collision is negligible. In the theory developed in this chapter, effect of collision is considered through the introduction of diffusion co-efficient in the expressions for electron and ion currents. On the other hand, diffusion co-efficient has been derived using Chapman-Enskog approximation, which is mainly suitable for partially ionized plasma. It is assumed that density of electrons (or ions) is much small compared to the density of dust grains, so that collision between these particles can be neglected. The flux of ions and electrons to the grain constitute a loss mechanism for the plasma particles. Low energy ions are neutralized at the grain surface and leave as neutral particles, whereas the incident electrons are absorbed into the grain material. Consequently, the plasma concentration in the region where dust grains are present, can be drastically reduced [Whipple et al. 1985]. Under this condition, above assumption is correct. Thus, in this theory, effect of collision of plasma particles with the dust grains has been considered in a situation appropriate for weakly ionized plasma. In OML theory, the plasma outside the sheath is assumed to be perfectly neutral.

In deriving the current equations, it is assumed that the Debye sphere surrounding the dust grain behaves like an absorbing wall where electron (or ions) performing random walk get absorbed at random time intervals. The probability of being absorbed has been calculated using Chandrasekhar's idea of absorbing boundary wall. It is hoped that the theory discussed here may give a proper equation for charging of dust grains in weakly ionized plasma when effect of collision cannot be neglected.

In the upper atmosphere which also comprises the D region of the ionosphere, the meteoric dust particles and the growing ice particles exist in weakly ionized plasma. In the upper mesosphere and lower thermosphere, transport and in particular, diffusion of ions and electrons can be ignored if only undis-
turbed conditions are considered. VHF radar echoes are produced by refractive index fluctuations with scales on the order of 1m. At such scales, neutral gas fluctuations clearly fall into the viscous subrange of the energy spectrum, whereas the corresponding electron density fluctuations may belong to the viscous convective or viscous-diffusive subrange, so that diffusion of ions, electrons and charged particles in general cannot be neglected. The theory of random charging of dust grains presented here might find proper applications in such areas.

The present investigation offers a possible mechanism for charging of dust grains which gets charged via the random walk of the plasma particles. The above formulation of kinetic equation for charging of grains may be suitable for explaining the transport properties of particles and charging process in the polar mesosphere.