Chapter 2

Transport Processes in Dusty Plasma

2.1 A review on transport processes in plasma

When macroscopic plasma parameters like temperature, velocity of directed motion, density of a given species vary spatially, parameters are levelled off after a sufficiently long time owing to migration of the gas particles. The equilibrium is established by energy transfer from the regions with higher temperature to the regions with lower temperature, by transfer of momentum if there are gradients of velocity and by transfer of a given species if its density varies in the gas.

A plasma with spatial inhomogeneity caused by plasma density variation may be considered for example. On account of their random thermal motions, the electrons have collisions with the other particles. Even though individually the the electrons move at random, the collisional interactions create a tendency for the electrons to drift from the high density to the low density regions. Similarly if there is a variation of the plasma temperature in space, the electrons tend to drift from the high temperature to the low temperature
regions and this drift is promoted by the collisional interactions of the electrons with the other particles. The drifting of the particles in a spatially inhomogeneous plasma caused by the collisional interactions is called diffusion. The drifting of electrons can occur even in a spatially homogeneous plasma if an external force is present. When an external electric field is present, electrons individually move at random because of their finite temperature but on the average, the electrons have a velocity in the direction opposite to that of the electric field. This process is called mobility. In a spatially inhomogeneous plasma acted on by an external force, particle current is due to both diffusion and the mobility.

We define some characteristic lengths which play a significant role in the discussion of transport processes:

\[ b \Rightarrow \text{Impact parameter} \]

\[ b_0 \Rightarrow \text{Impact parameter corresponding to a deflection of } \alpha = \pi/2 \]

If a collision takes place between an ion of charge \( e_i \) and an electron charge \( e_e \), the well known dispersion formula of Rutherford yields

\[ b_0 = \frac{|e_i e_e|}{m_e v_r^2} \]

where \( m_e \) is the electron mass, \( v_r \) is relative velocity of particles.

\[ d_0 = (\frac{1}{N})^{1/3} \Rightarrow \text{Mean distance between the particles (} N \text{ is the number density of particles) } \]

\[ \lambda_D = (\frac{K T}{N_e})^{1/2} \Rightarrow \text{Debye shielding distance which indicates approximately the distance over which the negative charge can deviate significantly from the positive charge.} \]

The term transport phenomena is customarily used in plasma physics to identify plasma properties associated with collisional effects. The diffusion of electrons in a tenuous, low-temperature, partially ionized plasma is governed by electron-neutral collisions. There are various processes that occur as a result of ionizing collisions and electron-ion recombination in the plasma.
Depending upon the relative values of the above defined characteristic lengths, the collision process can be divided into the categories:

Interval 1: $b < b_0$: Most collisions satisfying this inequality involve only two particles. This follows from the fact that the smallest distance between particles $b$ is much smaller than the mean distance $d_0$ between the particles.

Interval 2: $d_0 \geq b \geq b_0$: Many particles collisions often occur in this interval. This is an intermediate group between the well defined interval 1 interval 3.

Interval 3: $\lambda_D \geq b > d_0$: In this interval, collisions cannot be considered as independent since several particles will affect each other simultaneously.

Interval 4: $b > \lambda_D$: Particles with this parameter will participate in an organized oscillating motion. These plasma oscillations propagate in the plasma with wavelengths greater than $\lambda_D$. Hence the concept of collisions loses its meaning and the mutual influence of the particles results in density variations which can be included in the left side of Boltzmann equation.

From the discussion of above classification, collision term can be divided as,

$$\left(\frac{\partial f}{\partial t}\right)_{coll} = \left(\frac{\partial f}{\partial t}\right)_d + \left(\frac{\partial f}{\partial t}\right)_c$$

The first term on the right represents the effect of close collisions (defined by $b \leq b_0$) and the second term, the effect of other influences (defined by $\lambda_D \geq b > b_0$).

The close collision term $\left(\frac{\partial f}{\partial t}\right)_c$ is relatively easy to evaluate for particles which are both spherically symmetric and elastic. Chapman-Enskog's method of strong field approximation is used in this case for the calculation of transport co-efficients. The method is more appropriate for transport calculations in a weakly ionized gas. The model assumes that the charged particles are deflected by single collisions with neutral atoms rather than by multiple scattering by other charged particles.

The distant collision term $\left(\frac{\partial f}{\partial t}\right)_d$ is evaluated by using Fokker-Planck model, which is appropriate for calculating transport behavior in a fully ionized gas.
The model is based on the notion that in a fully ionized plasma, a large angle deflection of a particle by collisions is produced most rapidly by a succession of small-angle scattering with distant particles. The basic idea is to consider a particle, a so-called test particle, moving through the plasma subjected to many small deflections caused by those particles in the interval $\lambda_D > b > b_0$.

2.2 Importance of the study of transport phenomena in dusty plasma

Dust impurities can considerably influence the electromagnetic and kinetic properties of plasma. Although many theoretical and experimental studies on electromagnetic and kinetic processes in dusty plasma have been done, little stress has been given on the study of transport properties of dusty plasma.

In the case of grain charging by plasma currents, it is not possible to describe plasma relaxation disregarding electron and ion absorption by grains. The important case of plasma relaxation is the expanding plasma. The expansion of a semi-infinite plasma with dust-particles was shown to differ significantly from the process without dust[Lonngren 1990; Luo et al.1992; Yu et al.1992]. But in these studies, changes in the dust-particle charge during the plasma expansion have been disregarded.

The calculation of transport co-efficients of dusty plasma is important for the understanding of plasma processing, plasma etching, synthesis of submicron particles etc. Thermal plasma are being used to process submicron sized powder of high quality refractory materials. Such powders have a wide range of uses in surface casting, high density ceramics, dispersion strengthening of metals [Sayce 1971]. A wide variety of compounds and alloys can be synthesized in submicron powder form by plasma processing.

RF and DC glow discharge plasmas are extensively utilized in the manufac-
turing of semiconductor integrated circuits. Dust particles of size from tenths of micron to microns have been observed in these plasmas. These particles are detected on the wall surfaces and electrodes using laser light scattering at several optical frequencies [Barnes et al. 1992]. However, no complete theoretical explanation is available that can fully explain the observed particle transport phenomena. Barnes et al. [62] have presented a theory describing the transport of dust particles in electro-positive glow-discharge plasmas. They have considered an argon discharge with $10^{10} \text{ cm}^{-3}$ plasma density; ion and electron temperatures of 2.0 and 0.05 eV respectively and neutral argon pressure of 100 mT. They have shown that the typical forces like electrostatic, ion drag, gravitational on the particles not only depend upon the ionic velocity but also they have different power law dependency on the particle radius suggesting that the dominant force changes as the particle grows larger. The electrostatic force is found to be proportional to the particle radius, whereas momentum transfer forces are proportional to particle area and hence the radius squared. The ion drag force only has the dependency over certain limited ranges of ion velocities. Eddy and diffusion ion currents parallel to electrode surfaces have significant effects on particles suspended above wall surfaces due to the efficiency of ion drag force at low ionic velocities. It has recently been shown that the effective Debye length is reduced for small dust particles and low ion velocities in quiescent plasma [Dougherty]. The ion drag force is also reduced since it is dependent on the Debye length. At higher concentrations, the dust particles coacervate at plasma-sheath boundaries. The charge on each particle is reduced and it has been postulated that they exist in a condensed phase. [Ikezi 1986]. A great deal of numerical simulation and sophisticated theories are necessary to express the dust-particle transport in a proper manner.

Particulates have severe effects on the quality and reliability of microelectronic devices. Hence, particle contamination is a major concern in modern very large scale integration (VLSI) and packaging fabrication lines. Plasmas
are widely used during device fabrication for etching, deposition and sputtering. Controlling particles in plasma tools is an essential aspect of tool contamination reduction. To meet this goal, it is required to understand particle dynamics and transport processes. Particles in a plasma rapidly acquire negative charge by attachment of electrons and get suspended. The suspended particles are primarily influenced by electrostatic and gas drag forces and can be transported throughout the plasma. The importance of particle trapping is key in the process of particle control. For the proper development of plasma contamination control techniques, transport theory in dusty plasma is to be developed extensively [Selwyn 1991].

Physical processing e.g. heat transfer, current flow etc. in presence of charged grains are much more complex than in an ordinary plasma because of the presence of large electronic charges on the grains. Inside the Debye sphere of the grains, one encounters a very strong electric field which considerably alters the transport properties. Knowledge of the transport co-efficients will help to understand these phenomena properly.

In section 2.3, we present our theory on transport properties of dusty plasma. In this section, our work is mainly confined in dealing with transport processes outside the Debye sphere i.e. in the region $e\phi/T < n^{-1/3} < \lambda_D$. We find that even in this region, there is modification to the usual transport co-efficient. For evaluating diffusion-co-efficients of electrons and ions in dusty plasma, different terms of Fokker-Planck equation have been calculated taking into consideration the effect of dust grains. We have also developed Fokker-Planck equation in the enlarged phase space i.e. $(\vec{x}, \vec{v}) \rightarrow (\vec{x}, \vec{v}, q)$ and hence we have evaluated the diffusion co-efficient of dust grains in charge space. The charging time and slowing-down time are also calculated.

On the other hand, in the region $n^{-1/3} < e\phi/T < \lambda_D$, i.e. inside the Debye sphere, the electric field becomes very strong. For calculating diffusion co-efficients in this region, we have followed strong field approximation of
Chapman and Enskog. The theory is presented in chapter 3 in the context of “Charging of dust grains in plasma”.
2.3 Theory of Transport Properties of Dusty Plasma

Presence of dust grains in plasma modifies its transport properties. Study of transport co-efficients and relaxation time of dusty plasma is important for plasma processing, plasma etching, synthesis of submicron particles etc. Fokker Planck equation is developed in the enlarged phase space $q,v,t$. Different terms of Fokker-Planck equation in charge space give slowing down time for charging process and diffusion like phenomena in the same way as Fokker-Planck equation in velocity space. The charging time and slowing-down time obtained from our theoretical model can throw some light on the formation of spokes in Saturn's ring, time-scale of formation of rings etc.

2.3.1 Introduction:

Dusty plasma contains charged grains whose motion is influenced not only by gravity and radiation pressure but also by plasma drag and electromagnetic forces. Electromagnetic forces are responsible for many interesting effects such as resonant orbit perturbations, modification of density wave dispersion characteristics and angular momentum transport in planetary rings [Goertz 1989]. The fluctuation of grain charge can cause the angular momentum of a grain in planetary magnetosphere to change and it can lead to radial transport which may have significant effects on the evolution of planetary rings. Dust particles present in the spoke of Saturn may be responsible for the mass transport away from the synchronous orbit leading to depth minimum observed there [Northrop 1992]. Here study has been made of the implication of Fokker-Planck description of charge dynamics on the formation of spokes of Saturn's rings.

Here we have developed Fokker-Planck equation for the charging of dust
grains. we have modified the phase space from velocity space to charge space, i.e. \((r, v) \rightarrow (r, v, q)\). Dust charge fluctuation can be treated to be small compared with the charge present on the dust grain and the distribution function can be expanded in terms of \(\Delta q\) and \(\Delta v\). The diffusion coefficient and slowing-down time of electrons and ions of plasma have been calculated in presence of dust grains. The diffusion coefficient of dust grains in charge space and typical time scales of charging of the grains are also calculated. Finally numerical values of these parameters are found out for Saturn’s environments and the effect of dust grains in plasma on these transport co-efficients are investigated. Dust charge fluctuation is ignored in our model. In the region \(a \ll d < \lambda_D\), where \(a, d, \lambda_D\) represent grain radius, inter-grain distance and Debye length of the plasma respectively, charged dust particles can be considered as massive point particles similar to multiply charged negative (or positive) ions [Mendis 1994]. This condition is satisfied in planetary rings, where we have applied our model to find out various physical quantities.

In section 2.3.2, Fokker-Planck equation is written for the particles in presence of dust grains and the diffusion co-efficient of these particles are calculated. Slowing down time is calculated for the plasma particles in section 2.3.3. On the other hand, diffusion co-efficient in charge space for dust grains and their charging time are investigated by developing Fokker-Planck equation in charge space in section 2.3.4. All these parameters are numerically evaluated in section 2.3.5. Results are discussed in section 2.3.6.

2.3.2 Diffusion coefficients of plasma particles in presence of dust grains

In ordinary phase space, Boltzmann equation is:

\[
\frac{\partial f}{\partial t} + \bar{v} \cdot \nabla f + \frac{q}{m} (\bar{E} + \bar{v} \times \bar{B}) \frac{\partial f}{\partial v} = \left( \frac{\partial f}{\partial t} \right)_{coll}
\]  

(2.1)
where $f$ is the distribution function. The electric and magnetic fields acting on each particle consists of fields external to plasma and fields induced by all other plasma particles. The latter fields are actually particle interaction forces. Interaction forces may be divided into two classes. The first is associated with the collective motion of relatively large particle volumes, that may result from charge separation and the passage of current which induce macroscopic electric and magnetic fields. The space scale of variation in these fields greatly exceed the average distance between the particles. On the other hand, the second type of interaction, viz. the close interaction of particles can be reduced to collisions. The effect of collision is to change the magnitude and direction of particles rather sharply on a very short time scale. There can be both close and distant collisions. The close collision term can be evaluated using Chapman-Enskog’s method of strong field approximation, whereas, the distant collision term is evaluated by using Fokker-Planck’s equation. The definitions of close and distant collisions are discussed already in section 2.1. It is mainly focussed on the distant collision term in this chapter and diffusion coefficients are evaluated using Fokker-Planck’s equation.

The term \((\frac{\partial f}{\partial t})_{\text{coll}} = (\frac{\partial f}{\partial t})_d + (\frac{\partial f}{\partial t})_c\) of rhs of (2.1) gives the abrupt change in $f$ in a given incremental volume in phase space, of which \((\frac{\partial f}{\partial t})_d\) is evaluated as follows. \((\frac{\partial f}{\partial t})_d\) is written as \((\frac{\partial f}{\partial t})\). Fokker-Planck equation for ion and electrons is given by:

$$\frac{\partial f}{\partial t} = - \frac{\partial}{\partial v} \left( f \langle \frac{\Delta v}{\Delta t} \rangle \right) + \frac{1}{2} \frac{\partial^2}{\partial v \partial v} \left( f \langle \frac{(\Delta v \Delta v)}{\Delta t} \rangle \right)$$

(2.2)

This equation represents the net change in $f$, the single particle distribution function over a collision time $\Delta t$. The first term in right hand side of (2.2) is the expression for dynamical friction. This process tends to slow down or speed up particles to drag them towards the average velocity. The second term is the velocity space diffusion and it results in a spreading of the velocities of the particles, in opposition to the dynamical friction. Hence, diffusion coefficients
of plasma particles in velocity space is determined from this term. For this, we examine the elastic collision of dust particles and electrons or ions considered as target and test particles respectively, in velocity space [Shohet 1971].

Relative velocities of the particles before and after collision are,

\[ \vec{g}_{ad} = \vec{v}_a - \vec{v}_d; \quad \vec{g}'_{ad} = \vec{v}'_a - \vec{v}'_d \]  

(2.3)

where \( \vec{v}_a, \vec{v}_d \) are velocities of electrons (or ions) and dust grains before collision whereas \( \vec{v}'_a, \vec{v}'_d \) are respective velocities after collision. We find averages \( \Delta v \) and \( \Delta v \Delta v \) first over all possible deflection angles and then over the complete range of allowed relative velocities.

The change in relative velocity due to collision is

\[ \Delta g_{ad} = 2g_{ad} \sin\left(\frac{\chi}{2}\right) \]  

(2.4)

Probability of deflection through angle \( \chi \) for Coulomb collision is

\[ \frac{d\sigma}{d\Omega} = \frac{n_d k^2}{4g_{ad}^4 \sin^2(\frac{\chi}{2})} \]  

(2.5)

where \( k = \frac{g_{ad} e_a}{4\pi e m_r} \), and for collision between plasma particles and dust grains,

\[ m_r = \frac{m_e m_a}{m_d + m_a} \quad ; \quad \alpha = e, i \]

In (2.5), \( n_d \) is the density of scattering particles (here dust grains) per unit volume of position space. \( z \)-component of \( \Delta g_{ad} \) is given by

\[ (\Delta g_{ad})_z = -2g_{ad} \sin^2(\chi/2) \]

Then average deflection of the particles in \( z \)-direction is obtained as

\[ \Delta g_z = \int_{0}^{2\pi} \int_{X_{\min}}^{X_{\max}} (\Delta g)_z \frac{d\sigma}{d\Omega} g^2 \sin\chi d\chi d\phi f(\vec{v}_d) \]  

(2.6)

where \( f(\vec{v}_d) \) is the velocity distribution of the target particles (dust grains). Here we have dropped the subscript of \( g_{ad} \). Using (2.4) and (2.5) in (2.6) we get

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\[
\Delta g_z = \left( \frac{q_a e_a}{2\varepsilon_0 m_r g^2} \right)^2 f(v_d) g^2 \int_{\chi_{\min}}^{\chi_{\max}} \frac{\cos(x/2)d(x/2)}{\sin(x/2)} n_d \]  \quad (2.7)

The angular averages for \(\Delta g_z\) and \(\Delta g_\psi\) are found to be zero. The minimum value of \(\chi\) occurs when impact parameter 'b' becomes equal to \(\lambda_D\), Debye shielding distance, and is given by,

\[
\tan \left( \frac{\chi_{\min}}{2} \right) = \frac{q_a e_a}{4\pi\varepsilon_0 g^2 \lambda_D m_r} \]  \quad (2.8)

Hence, evaluating \(\chi_{\min}\) from (2.8) and taking \(\chi_{\max}\) to be \(\pi\) and after calculating the integration, equation (2.6) becomes

\[
\Delta g_z = \left( \frac{q_a e_a}{2\varepsilon_0 m_r g^2} \right)^2 n_d f(v_d) g^2 \ln \Lambda \]  \quad (2.9)

where \(\Lambda = \sin(\frac{\chi_{\min}}{2})\) We can write (2.9) in vector form as,

\[
\Delta \vec{g} = \frac{1}{\pi} \left( \frac{q_a e_a}{2\varepsilon_0 m_r g^2} \right)^2 \ln \Lambda n_d \left| \frac{\vec{g}_{\odot}}{\vec{g}_{\odot}^2} \right|^3 f(v_d) \]  \quad (2.10)

Velocities of dust grain (target particles) and electron (test particles) in the centre of mass frame can be written as

\[
\vec{v}_d = \vec{G} + M_\odot \vec{v}_{\odot} \]

\[
\vec{v}_\alpha = \vec{G} + M_\odot \vec{v}_{\odot} \]

where \(\vec{G}\) is the velocity of centre of mass and

\[
M_\odot = \frac{m_\odot}{m_\alpha + m_\odot}; \quad M_\alpha = \frac{m_\alpha}{m_\alpha + m_\odot},
\]

so that

\[
\Delta \vec{v}_\alpha = M_\odot \Delta \vec{g} = \frac{n_d}{\pi} \left( \frac{q_a e_a}{2\varepsilon_0 m_r g} \right)^2 M_\odot \ln \Lambda \left| \frac{\vec{g}_{\odot}}{\vec{g}_{\odot}^2} \right|^3 f(v_d). \]  \quad (2.11)
where $\Delta \tilde{v}_\alpha$ is the change in velocity of the test particles i.e. electrons or ions. After simplification of the Fokker-Planck term using (2.11), we get

$$< \Delta v_\alpha > = \Gamma \frac{\partial H_{ad}}{\partial v_\alpha}, \quad (2.12)$$

$$< \Delta v_\alpha \Delta v_\alpha > = \Gamma \frac{\partial^2 F_{ad}}{\partial v_\alpha \partial v_\alpha}, \quad (2.13)$$

where

$$\Gamma = \frac{4\pi q_d^2 e^2}{m_a^2} \ln \Lambda \quad (2.14)$$

$$H_{ad} = n_d m_a \int \frac{f(\tilde{v}_d)}{\tilde{v}_d - \tilde{v}_\alpha} dv_{dz} dv_{dy} dv_{dx}, \quad (2.15)$$

$$F_{ad} = n_d \int f(\tilde{v}_d) | \tilde{v}_d - \tilde{v}_\alpha | dv_{dz} dv_{dy} dv_{dx}. \quad (2.16)$$

Assuming the velocity distribution of dust grains to be Maxwellian

$$f(\tilde{v}_d) = \left( \frac{m_d}{2\pi K T_d} \right)^{3/2} \exp \left( -\frac{m_d v_{d, z}^2}{2 K T_d} \right)$$

we finally get diffusion co-efficient for electrons and ions in presence of dust grains as

$$D_{ad} = < \Delta v_\alpha \Delta v_\alpha >$$

$$= \Gamma \frac{\partial^2 F_{ad}}{\partial v_\alpha \partial v_\alpha}$$

$$= \frac{8\pi q_d^2 e \alpha}{m_a^3} \ln \Lambda \left[ -\frac{m_a}{m_d} n_d \left( \frac{m_d}{2\pi K T_d} \right)^{1/2} \exp \left( -\frac{m_d v_{d, z}^2}{2 K T_d} \right) \right.$$  

$$- \left( \frac{\pi K T_d}{2m_d} \right)^{1/2} \text{erf} \left( \frac{m_d v_{d, z}^2}{2 K T_d} \right)^{1/2} \left( \frac{m_d v_{d, z}^2}{2 K T_d} \right) \left. \right] \quad (2.17)$$

Here $m_d$, $T_d$, $v_d$, $q_d$ are the mass, temperature, velocity and charge, respectively, of the target particle (dust grain) and $m_\alpha$, $T_\alpha$, $v_\alpha$, $e_\alpha$ are the corresponding quantities of test particles (electrons or ions). Equation (2.17) gives diffusion co-efficient of plasma particles in velocity space, modified by the presence of dust grains.
2.3.3 Slowing-down time of plasma particles in presence of dust grains:

Slowing-down time of plasma particles in presence of dust grains can be defined as the time scale in which the momentum of the particles relaxes to the mean momentum due to encounters with dust grains and is given by (Sturrock, 1994)

$$\tau_s = \frac{v_R}{|\frac{dv_R}{dt}|}$$

where $v_R(t) = \frac{1}{n_f} \int f_T vd\nu (f_T$ is defined in equation (2.19)).

We consider a plasma consisting of electrons, ions and dust grains described by the distribution functions

$$f_e = n_{e0} g_e = n_{e0} \left( \frac{m_e}{2\pi K T_e} \right)^{3/2} \exp\left(-\alpha_e^2 v^2 \right)$$

$$f_i = n_{i0} g_i = n_{i0} \left( \frac{m_i}{2\pi K T_i} \right)^{3/2} \exp\left(-\alpha_i^2 v^2 \right)$$

$$f_d = f_d(q, \nu, t)$$

$$= n_{d0} g_d \delta(q - q_0)$$

$$= n_{d0} \left( \frac{m_d}{2\pi K T_d} \right)^{3/2} \exp\left(-\alpha_d^2 v_d^2 \right) \delta(q - q_0) \quad (2.18)$$

where $\alpha_j^2 = m_j/2K T_j$; $j = e, i, d$. We also assume that a beam of test particles of density $n_T$, passing through plasma, is distributed at time $t = 0$ by (Krall and Trivelpiece, 1973)

$$f_T(\nu, t) = n_T g_T(\nu, t) = n_T \delta(\nu - \nu_0) \quad (2.19)$$

Using (2.12) - (2.19), Fokker-Planck equation can be written as

$$\frac{\partial f}{\partial t} = -\Gamma \frac{\partial}{\partial \nu} \left[ f \frac{\partial H_{ad}}{\partial \nu_\alpha} \right] + \frac{1}{2} \Gamma \frac{\partial^2}{\partial \nu \partial \nu_\alpha} \left[ f \frac{\partial^2 F_{ad}}{\partial \nu_\alpha \partial \nu_\alpha} \right] .$$

or
\[
\frac{1}{\Gamma_T} \frac{\partial g_T}{\partial t} = -\frac{\partial}{\partial v} \left[ \frac{\partial}{\partial v} \left( \frac{e^2 m_T + m_e}{g_T^2} \int d^3 v_d \frac{n_{e0} g_e}{g_d} \right) \right. \\
+ \frac{e^2}{g_T^2} \frac{m_T + m_i}{m_i} \int d^3 v_d \frac{g_i n_{i0}}{g_d} \left. \right] \\
+ \frac{1}{2} \frac{\partial^2}{\partial v \partial \vartheta} \left[ \frac{\partial}{\partial v} \left( \frac{e^2}{g_T^2} \int d^3 v_d \frac{(n_{e0} g_e + n_{i0} g_i) g_d}{g_d} \right) \right. \\
+ \left. \int \frac{d^3 v_d}{g_T^2} d^3 v_d n_{d0} g_d \delta(q - q_0) g_d \right],
\]
(2.20)

where

\[
\Gamma_T = \frac{4\pi q_T^2 q^2}{m_T^2} \ln \Lambda
\]

Defining \( v_R(t) = \int g_T v d v \), we get from (2.20)

\[
\frac{dv_r}{dt} = -\Gamma_T \left[ \left( \frac{n_{e0} e^2}{q_T^2} \frac{m_T + m_e}{m_e} \right) \frac{\partial}{\partial v_r} \left( \frac{1}{v_r} \text{erf}(a_e v_r) \right) \left( \frac{n_{i0} e^2 m_T + m_i}{q_T^2} \right) \right. \\
+ \left. \frac{\partial}{\partial v_r} \left( \frac{1}{v_r} \text{erf}(a_i v_r) \right) + \left( \frac{n_{d0} q_T^2 m_d + m_T}{q_T^2} \right) \frac{\partial}{\partial v_r} \left( \frac{1}{v_r} \text{erf}(a_d v_r) \delta(q - q_0) \right) \right]
\]
(2.21)

After simplification of (2.21) and using the definition \( \tau_s = \frac{v_n}{|d v_r / d t|} \), we get following expressions for slowing-down time of electrons and ions respectively:

\[
\tau_{s,e} = \frac{m_e^2 v_r^3}{4\pi e^4 \ln \Lambda \left[ (n_{e0} + n_{i0} + n_{d0} Z_d^2) + \left( \frac{n_{e0} m_e}{m_e} + n_{e0} + \frac{n_{i0} m_i}{m_i} \right) \right]}
\]
(2.22)

\[
\tau_{s,i} = \frac{m_i^2 v_r^3}{4\pi e^4 \ln \Lambda \left[ (n_{e0} + n_{i0} + n_{d0} Z_d^2) + \left( \frac{n_{i0} m_i}{m_i} + n_{i0} + \frac{n_{d0} m_d}{m_d} \right) \right]}
\]
(2.23)

Expressions (2.22) and (2.23) reduce to standard results of electron-ion plasma (Sturrock 1994) if one replaces the dust by electrons or ions as target particles. The slowing down times for plasma particles in presence of dust grains are numerically evaluated in section 2.3.5 and the effect of dust grains is highlighted.
2.3.4 Diffusion co-efficient of dust grains in charge space:

When dust particles are emerged in plasma, they get (negatively) charged due to the collection of ions and electrons. Unlike electrons or ions, the charge on the dust grains fluctuated due to random hitting of the plasma particles. The fluctuation can be treated as small compared with the charge present on the dust grains and thus distribution function of dust particles can be expanded in terms of $\Delta q$. Considering this effect, we develop Fokker-Planck equation for dust grains in the enlarged phase space i.e. in $(q, v)$ space. Different terms of Fokker-Planck equation describe diffusion like phenomena and slowing down time for dust grains.

Fokker-Planck equation for dust grain in $(q, v)$ space can be written as:

$$
\frac{\partial f_d}{\partial t} = -\frac{\partial}{\partial v} \left[ f \left( \frac{\Delta v}{\Delta t} \right) \right] - \frac{\partial}{\partial q} \left[ f \left( \frac{\Delta q}{\Delta t} \right) \right] + \frac{1}{2} \frac{\partial^2}{\partial v^2} \left[ f \left( \frac{\Delta v}{\Delta t} \right) \right] + \frac{1}{2} \frac{\partial^2}{\partial q^2} \left[ f \left( \frac{\Delta q}{\Delta t} \right) \right] \tag{2.24}
$$

where

$$
\begin{align*}
\langle \frac{\Delta v}{\Delta t} \rangle &= \int \Delta v \text{Prob}(v - \Delta v, \Delta v)(\Delta v) \\
\langle \frac{\Delta q}{\Delta t} \rangle &= \int \Delta q \text{Prob}(q - \Delta q, \Delta q)(\Delta q) \\
\langle \frac{\Delta v \Delta v}{\Delta t} \rangle &= \int \Delta v \Delta v \text{Prob}(v - \Delta v, \Delta v)(\Delta v) \\
\langle \frac{\Delta q \Delta q}{\Delta t} \rangle &= \int \Delta q \Delta q \text{Prob}(q - \Delta q, \Delta q)(\Delta q) \tag{2.25}
\end{align*}
$$

Here $\text{Prob}(q - \Delta q, \Delta q)$ being the probability that a particle with charge $q$ attains charge $q + \Delta q$ after time $\Delta t$ has elapsed and $\text{Prob}(v - \Delta v, \Delta v)$ being the probability that a particle with velocity $v$ attains velocity $v + \Delta v$ after time $\Delta t$ has elapsed.

First term in (2.25) viz. $\langle \Delta v/\Delta t \rangle$ has the dimension of force per unit mass.
It tends to slow down or speed up particles until they reach the average velocity \(\langle \Delta v / \Delta t \rangle\). The process is called dynamical friction.

Second term in (2.25) can be written as

\[
\left\langle \frac{\Delta q}{\Delta t} \right\rangle = \frac{1}{\tau_{fi}} \int (q - q_0) \text{Prob}(q - \Delta q, \Delta q) d(\Delta q)
\]  

(2.26)

It gives the average change in charge over the time interval \(\tau_{fi}\), time scale of charge fluctuation. To obtain an expression for \(\tau_{fi}\), we consider the charge of a dust particle 'q' as a continuous variable and linearize the current in the vicinity of the steady state charge \(q_0\) defined by the condition \(I_i(q_0) + I_e(q_0) = 0\).

The charging equation for dust is,

\[
\frac{dq}{dt} = -\frac{q}{\tau_{fi}} = I_i + I_e
\]

which gives,

\[
\tau_{fi} = \frac{1}{-I_i'(q_0) - I_e'(q_0)}
\]

(2.27)

where

\[
I_a' = \frac{\partial I_a}{\partial q}
\]

Physically, fluctuation time \(\tau_{fi}\) gives the time interval between the charged states of a dust grain.

Third term in (2.24), viz. \(\langle \Delta u \Delta v \rangle\) as usual represents diffusion in velocity space. In equilibrium, diffusion is exactly balanced by dynamical friction. In analogy with this, fourth term in (2.24), viz. \(\langle \Delta q \Delta q \rangle\) represents diffusion of dust grains in charge space. It is well known, any density or velocity gradient is always followed by diffusion. Particles have natural tendency to diffuse from a more denser region to a less denser one. Similarly, when there are excess of dust grains with a particular charge, there will be diffusion in charge space. This term can be simplified to

\[
\frac{1}{2} \left\langle \frac{\Delta q \Delta q}{\Delta t} \right\rangle = \frac{1}{2\tau_{fi}} \int (q - q_0)^2 \text{Prob}(q - \Delta q, \Delta q) d(\Delta q)
\]
from which one can obtain the diffusion co-efficient in charge space as

\[ D_q = \frac{e^2}{2} \left[ I_e(q_0) - I_i(q_0) \right] \]  

(2.28)

In deriving (2.28), we take the probability per unit time for absorbing an electron or ion as \( I_a/e_a \) (Cui et al.1994), where \( \alpha = e, i \).

### 2.3.5 Numerical Results

Using the above model, we now calculate the charging time, diffusion coefficients and slowing-down times for Saturn's spokes and rings (Table 1). For Saturn's spokes we use the following parameters (Mendiset et al.1994): \( n_e \sim n_i = 0.1 - 10^2 \text{ cm}^{-3} \), \( n_d \sim 1 \), \( T_e \approx T_i = 2 \times 10^4 \text{K} \), \( a = 10^{-4} \text{ cm} \), \( m_d = 10^{-6} \text{ gm} \), and surface potential of the grain \( \phi_s = -4 \text{V} \). For Saturn's E (F) ring we take (Mendis and Rosenberg, 1994) \( n_e \sim n_i = 10(10 - 10^2) \text{ cm}^{-3} \), \( n_d = 10^{-7} - 10^{-8}(< 30) \text{ cm}^{-3} \) with \( T_e \sim T_i = 10^5 - 10^6 \text{ K} \), \( a = 10^{-4} \text{ cm} \) and \( m_d = 10^{-6} \text{ gm} \).

<table>
<thead>
<tr>
<th></th>
<th>( \tau_{\text{ch}} ) (sec)</th>
<th>( D_q ) (Coul² sec⁻¹)</th>
<th>( D_{ed} ) (cm² sec⁻¹)</th>
<th>( D_{id} ) (cm² sec⁻¹)</th>
<th>( \tau_{s,e} ) (sec)</th>
<th>( \tau_{s,i} ) (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spokes</td>
<td>( 7.57 \times 10^2 )</td>
<td>( 3.170 \times 10^{-17} )</td>
<td>0.155</td>
<td>( 3.724 \times 10^{-3} )</td>
<td>( 9.644 \times 10^{-55} )</td>
<td>( 3.247 \times 10^{-48} )</td>
</tr>
<tr>
<td>E ring</td>
<td>( 1.045 \times 10^3 )</td>
<td>( 2.295 \times 10^{-17} )</td>
<td>( 2.620 \times 10^{-10} )</td>
<td>( 6.114 \times 10^{-12} )</td>
<td>( 2.342 \times 10^{-61} )</td>
<td>( 1.697 \times 10^{-31} )</td>
</tr>
<tr>
<td>F ring</td>
<td>( 1.045 \times 10^2 )</td>
<td>( 2.296 \times 10^{-16} )</td>
<td>0.358</td>
<td>( 4.185 \times 10^{-3} )</td>
<td>( 5.746 \times 10^{-58} )</td>
<td>( 2.021 \times 10^{-49} )</td>
</tr>
</tbody>
</table>

Several conclusions can be drawn from these results. Concentration of dust particle density has an interesting effect on the diffusion co-efficients of electrons and ions. Table 1 shows that the diffusion co-efficients and slowing-down time for F ring are much higher than that for E ring, which can be related to the higher dust density of F ring. The charging time of dust particles due
to absorption of electrons and ions is found to be on the order of $10^2 - 10^3$ sec, which is consistent with the estimate of Goertz (Goertz 1989).

2.3.6 Discussion

It is shown that transport properties of plasma get modified due to presence of dust grains. Diffusion co-efficient and slowing down time have been derived for different species in plasma. Higher dust density is found to result in higher diffusion co-efficient in velocity space and slowing-down time in F-ring of Saturn. Transport phenomena play a major role in planetary rings and magnetosphere. Charged grains are subject to azimuthal and electromagnetic forces in planetary magnetospheres, so their angular momentum is not constant. This produces a radial transport and distribute grains throughout the magnetosphere. Moreover, fluctuations of the grain charge can lead to a radial diffusion of grains.

Voyager observations have revealed the existence of radially elongated thin structures called spokes which appear dark in backward scattered but bright in forward scattered light, which suggests that they contain small micron-sized or submicron grains. Although many theories are coming up, there is no universally accepted explanation of these structures till now. The radial displacement of the dust grains contained in the spokes causes a significant transport of angular momentum which can lead to exponential growth of perturbations in the ring surface mass density. Our theoretical model of transport properties of dusty plasma may be quite relevant in explaining these observations.

Micron-submicron sized grains have also been identified in the dense A and B rings of Saturn. They also dominate the population in outer F,G,E rings and spokes. Dust-magnetosphere interaction may be important in determining the structure of Saturn's E,G and F rings. For the E ring, transport processes like diffusion, plasma drag and sputtering may cause the broad extent of dust
distribution within Saturn's magnetosphere. An active source for E ring particles is required, since dust particles in E region are of short life time due to particle transport.

The theoretical model discussed here may be useful for the collisional dynamics of ring particles when one considers the tidal interaction between Saturn's inner satellites and the rings. It may also help in studying the density variation in rings which may be caused by diffusion instability.

The diffusion of dust grains in charge space discussed here is rather a new, but at the same time, quite natural concept. Density gradient or velocity gradient are always accompanied by diffusion of particles. In the same way, when there is gradient of charge on different dust grains, there may be diffusion in charge. The fluctuation of dust charge may be related to the phenomena of diffusion in charge space. Further study of this type of diffusion process in a magnetized plasma is required.

Knowledge of transport coefficients may be useful in various laboratory plasma also. The subject of dusty plasma was initiated by the semiconductor industry. With the arrival of VLSI and ULSI (Very/ Ultra Large Scale Integration) plasma etching and plasma deposition techniques have become indispensible. With the decreasing feature size on the device the tolerance for dust particles become smaller. Most of the research effort in this field is aimed at eliminating the dust. Hence the study of trapping and transport of dust particles in processing plasma has become important. There are two approaches for plasma contamination control: to avoid particle formation and secondly, to influence the movement and transport of particles. It is unlikely that particle formation can be completely stopped since they result from polymer deposition, wall flaking, sputtering etc, i.e. the techniques used in the processing. Hence the best approach to limit contamination is the transport of particles, such that particles formed do not contaminate the product. Elaborate study on transport properties of dusty plasma is therefore needed.