Chapter 6

Integrated model for Line Balancing with Workstation Inventory Management

6.1 Introduction

Integrated line balancing involves a network of interconnected activities for the ultimate provision of products and service packages required by the end customers. The domain of integrated line balancing covers all movements and storage of raw materials, inventory of work-in-progress
and finished goods from point-of-origin to point-of-consumption. Basically, integrated line balancing is the planning, organizing and controlling of sourcing, procurement, conversion and logistic activities. It is used to characterize all the inter-related components and processes required to ensure that the right amount of product is in the right locations at the right time at the lowest possible cost. In the recent past, integrated line balancing has become a very important part of any business activity. This importance is going to increase further due to growing uncertainty in the global business environment and cost minimization has again become the bull’s eye.

However, the form of integrated line balancing depends on the type of industry under study. Industries can be broadly divided into two types on the basis of their final offers. One is the manufacturing sector and other one is the service sector. So far as manufacturing sector is concerned, there exists a very close relationship between the increase in productivity and increase in resultant profit. Since integrated line balancing is a management technique through which the better quality products are delivered to the customers at lower costs by making a balance between the inbound and the outbound logistics, this increases productivity, improves quality and adds to profit. For most of the manufacturing sector,
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A line of production is located between inbound and outbound logistics. Therefore, this line of production plays an important role in the overall integrated line balancing process. The details may differ, but the basic flow line principles remain the same. Items are processed as they pass through a series of workstations along a production line, i.e., different tasks are performed in different workstations. One or more tasks can be performed in a workstation but the activity time for each workstation should be near about equal so that the line will be balanced. In a nutshell, the problem of line balancing deals with the distribution of activities among the workstations so that there will be maximum utilization of human resources and facilities without disturbing the work sequence. But this problem should not be addressed in isolation. This is because in each workstation supply of materials and planning of inventory play important roles for ensuring smooth functioning of the assembly line. So, the excellence in integrated line balancing can only be achieved by undertaking a system approach and by simultaneously minimizing the total inventory cost and balancing the line of production. Better coordination of different links that connect source with the destination can create competitive advantage for a company. In fact, along with the company, its suppliers, its channel members and its customers can all
benefit from better coordination and usage of such linkages. For example, if we consider the case of modern rice milling facilities and if we examine the conversion area, we can classify the activities involved with the conversion process into four different groups such as steeping, parboil, dryer and hulling. Though, each group of activity may consist of one or more activities, those four groups are performed in four separate areas. For each separate area, we may consider sub-operative areas forming a network of workstations and a network of stock chambers. Therefore, to maximize the efficiency of the entire chain of operation, balancing of the assembly line is needed. And for each workstation, configuration of separate stock chamber is also needed to feed the respective workstation. For these stock chambers, the need for inventory planning, which is one of the major decision areas in integrated line balancing can be highly felt. Since we have to perform all these sub-activities under an overall system activity, the approach should be an integrated system approach where the inventory decision of integrated line balancing and balancing decision of assembly line will be considered jointly as one set of decisions.

To study the integrated problem of supply chain, there is a need to link line balancing and customers' rate of demand with the entire supply chain. This linking has an important role towards achieving excellence in
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the complete task of moving from source to destination. Our objective in this current work is to design the integrated model for line balancing with workstation inventory where, given the rate of customers’ demand, the total of inventory cost and the cost of balancing loss of the assembly line will be jointly minimized so that the entire optimization approach can be a holistic one.

6.2 Notation

K  number of jobs
N  number of workstations
t_i  task time or assembly time of i^{th} job
W_j  j^{th} workstation
a(i,j)  assignment variable taking value 1 if task i is assigned to workstation j and taking value 0, otherwise
L_j  idle time of j^{th} work station
C  cycle time
C_t  trial cycle time
6.3 Methodology and Mathematical Formulation

To formulate the integrated problem of cost minimization, we like to split the same into three interrelated parts, viz. determination of customers' rate of demand, deciding about the stock of materials for consumption during production activity and balancing of the workstations. At the end, we propose to rejoin their part-wise measures on a common scale and undertake the joint optimization task.
For us, on one hand the assembly line will be balanced, when the idle time in each work station is in minimum level and in turn, the cost of the production will be minimized when the balancing loss is minimized. The measure of balancing loss of an assembly line is defined as the loss resulting from allocation of work elements to workstations and is given by (see Ray Wild, 2004)

$$B = \left\{ \frac{(NC - \sum_{i=1}^{K} t_i)}{NC} \right\} \times 100\%,$$

where the numerator of which indicates the idle time per $C$ unit time of work. The corresponding cost per unit time can be expressed as,

$$\frac{(NC - \sum_{i=1}^{K} t_i)}{C} M$$

(1)

Inventory cost that mainly deals with ordering cost, holding cost and the shortage cost is minimized under the plan of optimum ordering quantity. If we do not permit any shortage that badly affects the total assembly line, the total cost of the supply chain for our purpose will be the sum total of ordering cost, average holding cost and the cost of balancing loss to be observed per unit time. To link the entire system with the rate of customers’ demand we consider the inverse of the same to arrive at the
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cycle time of operation. Due to variations in the market demand it is preferable to express the same in terms of lower and upper bounds, $R_L$ and $R_H$ respectively. Then the cycle time must lie in the interval $[1/R_H, 1/R_L]$. Now, given a cycle time $C$, demand, $D_j$, for the materials may be expressed as $D_j = \sum_{i=1}^{K} a(i,j)/C$ under the assumption that each work element has one unit of consumption. Holding cost for a particular workstation $j$ will be $W_H_j = \sum_{i=1}^{K} (a(i,j) \cdot C_H)$, under a similar assumption and argument. With the help of our proposed notation, we can express the ordering cost for the $j^{th}$ workstation as,

$$\sum_{i=1}^{K} \frac{a(i,j)}{Q_j} \cdot \frac{C_o}{C},$$

Therefore, the total ordering cost for the $N$ stock chambers of the $N$-workstation line balancing system is

$$\sum_{j=1}^{N} \left[ \sum_{i=1}^{K} \frac{a(i,j)}{Q_j} \cdot \frac{C_o}{C} \right] (2)$$
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Average holding cost for the \( j \)th workstation will be

\[ Q_j \sum_{i=1}^{K} (a(i,j) \cdot CH_i) / 2. \]

Hence, the total average holding cost for the system is as follows,

\[
\sum_{j=1}^{N} \left[ \frac{Q_j \sum_{i=1}^{K} (a(i,j) \cdot CH_i)}{2} \right]
\]

(3)

To this end we add the cost of balancing loss which is \( NC - \sum_{i=1}^{K} t_i \) \( \cdot M / C \).

The cycle time \( C \), being linked with the average rate of demand of the customers, may be determined in terms of an interval corresponding to interval estimator of average rate of demand of the customers. Thus, \( C \) varies between two points \( C_{\text{min}} = 1/R_H \) and \( C_{\text{max}} = 1/R_L \) and the optimum \( C \) along with optimum \( Q_j \) values will be determined from the total cost of the plan. Thus, our objective is to minimize total cost of the supply chain including \( N \) stock chambers and \( N \) workstations where the integrated objective function, as obtained from (1), (2) and (3), is

\[
Z = \sum_{j=1}^{N} \left[ \frac{\sum_{i=1}^{K} a(i,j)}{Q_j} \cdot C_o / C \right] + \sum_{j=1}^{N} \left[ \frac{1}{2} Q_j \sum_{i=1}^{K} (a(i,j) \cdot CH_i) \right] + \left[ NC - \sum_{i=1}^{K} t_i \right] \cdot M / C
\]
and our objective is to minimize $Z$, subject to (i) precedence constraints as given by the technology and (ii) cycle time constraints as determined from the market demand.

Under the condition that the $i^{th}$ task can be assigned to only one workstation, we must have,

$$\sum_{j=1}^{N} a(i,j) = 1 \quad i = 1, 2, \ldots, K.$$ 

Also, according to precedence constraints if task $i'$ is to be assigned before assigning task $i$, that is $i' < i$, then

$$a(i,j) \leq \sum_{r=1}^{i'} a(i', r). \quad \forall \quad i' < i.$$ 

Further, since each workstation can at the most be assigned $C$ unit of time we have,

$$\sum_{i=1}^{K} a(i,j) t_i \leq C \quad \forall \quad j=1, 2, \ldots, N$$

Thus, a mathematical programming formulation of the integrated optimization problem can be written as,
minimize

\[ Z = \sum_{j=1}^{N} \left[ \sum_{i=1}^{K} a(i, j) \frac{C_o}{Q_j} \right] + \sum_{j=1}^{N} \left[ \frac{1}{2} Q_j \sum_{i=1}^{K} (a(i, j) \cdot C H_j) \right] + \left[ NC - \sum_{i=1}^{K} t_i \right] \cdot M / C \]

subject to,

\[ \sum_{i=1}^{N} a(i, j) = 1 \quad i = 1, 2, \ldots, K \]

\[ a(i, j) \leq \sum_{i=1}^{K} a(i', r) \quad \forall \quad i' < i \quad \text{and} \quad i', i = 1, 2, \ldots, K \]

\[ \sum_{i=1}^{K} a(i, j) t_i \leq C \quad \forall \quad j = 1, 2, \ldots, N \]

\[ C_{\min} \leq C \leq C_{\max} \]

\[ a(i, j) = 0, 1 \quad \forall \quad i, j \quad i = 1, 2, \ldots, K. \quad j = 1, 2, \ldots, N. \]

Obviously, this is a mixed nonlinear programming problem. To arrive at the optimum solution we may take the course of iterative algorithm. For this, we formulate holding cost, ordering cost and cost of balancing loss and add them up to get the total cost of the supply chain and denote it by \( Z \). Iteratively we minimize \( Z \) by optimization procedure to get the minimum cost of the chain. Inputs of the system include the values of
ordering cost, holding cost and average cost of one man hour. Highest and lowest rate of market demand is also fed into the program. After successful iterations we get the desired cycle time, assignments of work elements to workstations and order quantity for each stock chamber associated with each workstation. The next section includes an worked out example to demonstrate the functioning of the proposed method.

6.4 The Algorithm

1. Formulate the objective function.

2. Complete the model formulation by restricting the objective function using precedence constraints, keeping in mind the zoning constrains and nonnegative constraints.

3. Set cycle time $C$, determine the minimum number of workstations $N_{\text{min}}$ and calculate the $C_{\text{min}}$ value.

4. Set the trial cycle time $C_i$ at $C_{\text{min}}$.

5. Solve the formulated problem for the particular $C_i$ value.

6. After getting the complete distribution of tasks to workstations, the value of objective function is calculated.
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7. For each $C_i$ value, the new value of the objective function is compared with the previous value of the objective function. If the new value of the objective function is less than the previous least value of the objective function, the new solution is stored as the basis for next comparison. Otherwise keep the previous one as the basis for comparison.

8. Increase the trial cycle time $C_i$ by one unit until it crosses $C$ value. If $C$ value is crossed, go to step 11.

9. Repeat step 5 to 8.

10. Check whether all the work elements have been assigned to specified number of workstations. If not, increase the value of $N_{min}$ by 1 and go to step 4.

11. Print the best solution in terms of overall minimum value of objective function.

6.5 Worked Out Example

To explain how the proposed method works, we consider in Figure 6.1 a conversion process where figure within a circle represents task number and that close to a circle represents corresponding task time. Precedence constraints are represented by the arrows.
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Figure 6.1: Precedence diagram of workstations along with the task times.

Work elements and precedence constraints as obtained from this conversion process are fed into the program developed for this purpose. In addition, we have given the values of ordering cost ($C_0$), holding cost for each job ($CH_i$), task time or assembly time of each job ($t_i$), trial cycle time ($C_t$) and minimum cycle time ($C_{\text{min}}$) and average cost of one manhour ($M$) as input data.

Cycle time ($C$) and ordered quantity for each work station ($Q_j$) are the decisions variables. We will determine the optimum values of those variables from the iterative run of our program. The conversion process can be summarized in a tabular form in terms of the binary variables.
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a(i,j)s and is given in Table 6.1 along with the choices of the cost parameters.

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<th>Immediate Predecessor</th>
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Table 6.1: Precedence relation, task times of work elements and cost parameters

The optimum policy and configuration is presented in Table 6.2.
Table 6.2: Final Optimum Configuration and policy

Thus there will be 5 workstations with 5 stock chambers. Balancing loss of the system works out as 7.412%. The total cost of operation per unit time is 4667.7 unit.

6.6 Conclusion

We have presented a mathematical programming approach for solving an integrated model of line balancing with workstation inventory management problem with the objective of integrated cost minimization, resulting in optimum provision of materials in each stock chamber and determination of ideal cycle time so that the demand of the market can be met in time.
Till date, determination of cycle time was overlooked. Techniques were there for isolated minimization of balancing loss given a cycle time. Following our approach, we can easily calculate the ideal cycle time for our system according to the demand in the market. It will be possible to adjust the cycle time as and when needed. Our model will also minimize raw materials inventory as well as finished goods inventory and the total cost of the chain. We know that the cost of the supply chain is mainly the running cost of the chain. Our objective is to minimize this operational cost. More we can reduce the operational cost greater will be our flexibility in adding value to the chain. As a result of this point wise optimization and chronological improvement in the system, the quality of the supply chain will get increased providing distinctive competence to the company.