Chapter 2

Braneworld Inflation on Hybrid Type Potentials

The most attractive model is the hybrid inflationary model. We have considered some variants of the hybrid model, namely inverted hybrid and mutated hybrid models on the RSII brane world. By applying slow roll approximation, we obtained constraints for the coupling constant of the inverted hybrid potential. Also we have derived the slow roll parameters and amplitudes of scalar perturbations using mutated hybrid inflation potential, in a form that can be used for constraining inflationary models.
2.1 Introduction

There are many types of inflation that are natural from the particle physics point of view. Despite many successes of inflation, there are still no realistic natural inflation models known in particle physics. Inflation explains many basic features of our universe [Guth, 1; Kolb; Gliner]. It is also thought to have generated the density perturbations needed to form galaxies and all the other large scale structure in the observable universe [Starobinsky]. Inflation generally requires small parameters in particle theory to provide a flat potential, needed for sufficient inflation and for the correct density fluctuations [Linde, 1; Randall, 2]. Probably the most attractive model of inflation are the hybrid inflation [Linde, 2; Linde, 3; Copeland; Garcia]. Given the success of hybrid inflation, it was subsequently suggested that the hybrid mechanism could be adapted to create an inverted hybrid model [Lyth, 1; King] in which the inflation field $\phi$ has a negative mass squared and rolls away from the origin, predicting a spectral index which can be significantly below 1 in contrast to virtually all other hybrid inflation models. The inverted hybrid inflation process appears in natural way in the supersymmetric theory given in [Tkach, 1; Obregon, 1; Tkach, 2]. It removes any necessity for fine-tuning and it reconnects inflation with particle physics. In this model the field $\phi$ is supposed to obtain a non-zero vacuum expectation value eventually. This model is not viable unless all its dimensionless couplings are extremely small. Recently Guzman et al., have studied inverted hybrid inflation scenario in the context of $n = 2$ supersymmetric quantum cosmology [Guzman]. In hybrid inflation the second field $\psi$ remain fixed. In the mutated hybrid inflation the field $\psi$ moves slowly during inflation as it adjusts to minimize the potential at a fixed value of the inflaton field $\phi$. Inflationary scenario on inverted and hybrid potentials have been reported earlier [Lyth, 1; Lyth, 3; King], but not on the brane world. We have done
this work in the RSII brane world [Randall,1]. The detailed discussion of RSII
brane world models is given in chapter 1 [Brax; Maartens; Langlois].

This chapter is organized as follows. In section 2.2 we give the hybrid
inflationary scenario, and its variants: inverted and mutated hybrid inflations. In
sections 2.3 we study the slow rol. inflation on the RS brane world using these
potentials.

2.2 Hybrid Inflation Models

Hybrid inflation starts with large field initial condition. In the scenario proposed by
Linde [Linde,2; Linde,3], the slowly rolling inflation field $\phi$ is not responsible for
most of the energy density. That role is played by another field $\psi$, which is held in
place by its interaction with the inflation field until the latter falls below a critical
value while rolling to its true vacuum, and inflation ends. Thus during hybrid
inflation, the inflaton field stabilizes the trigger field at the false vacuum. In this
respect, hybrid inflaton is similar to chaotic inflation. In the usual models of hybrid
inflation $\phi$ is rolling towards zero.

The simplest way of writing the inverted hybrid potential is [Lyth,1: Lyth,2; King]

$$V_H = \frac{1}{4} \lambda_\psi \left( \psi^2 - M^2 \right)^2 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} \lambda' \phi^2 \psi^2$$

$$= V_0 - \frac{1}{2} m_\psi^2 \psi^2 + \frac{1}{4} \lambda_\psi \psi^4 + \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} \lambda \phi^2 \psi^2$$ (2.1)

where $\phi$ and $\psi$ are real scalar fields, $V_0 = \frac{1}{4} \lambda_\psi M^4$, $m_\psi^2 = \lambda_\psi M^2$, $m_\phi$ is the mass
of $\phi$, $\lambda'$ is the coupling constant between two fields (to be distinguished from the
brane tension, $\lambda$), and the subscript H denotes ‘Hybrid’. In the hybrid models $\phi$
start out greater than some critical value, $\phi_c$, and falls below it for $\phi > \phi_c$. The field
\( \psi \) is held at origin \((\psi = 0)\) by its interaction with \(\phi\). In this regime, inflation occurs with the quadratic potential,

\[
V_H = V_0 + \frac{1}{2} m_\psi^2 \phi^2
\]

(2.3)

When \(\phi\) falls below some critical value \(\phi_c\), the \(\psi\) field rolls to its vacuum value so that \(V_0\) disappears and inflation ends. The critical value of the inflaton field, \(\phi_c\), defines the point where the "valley" in the \(\psi\) direction disappears. It is given by,

\[
\phi_c = \frac{m_\psi}{\sqrt{\lambda}}
\]

(2.4)

If, \(V_0\) dominates, inflation ends when \(\phi\) becomes less than \(\phi_c\), and the fields reach their true values, \(\langle \psi \rangle = M\) and \(\langle \phi \rangle = 0\).

The usual slow-roll parameters for GR case are [Riotto, 1; Liddle, 2],

\[
\epsilon = \frac{1}{2\kappa^2} \left( \frac{1}{V} \frac{\partial V}{\partial \phi} \right)^2
\]

(2.5)

\[
\eta = \frac{1}{\kappa^2} \frac{1}{V} \frac{\partial^2 V}{\partial \phi^2}
\]

(2.6)

where \(\kappa\) is related to Newton's constant.

In the case of RS II brane model, the above equations are [Maartens],

\[
\epsilon = -\frac{\dot{H}}{H^2} = \frac{1}{16\pi G} \left( \frac{V'}{V} \right)^2 \left( \frac{4\lambda (\lambda + \phi')}{(2\lambda + V)^2} \right)
\]

(2.7)

\[
\eta = \frac{V''}{3H^2} = \frac{1}{8\pi G} \left( \frac{V'}{V} \right)^2 \left( \frac{2\lambda}{2\lambda + V} \right).
\]

(2.8)

The number of e-foldings can be expressed as,

\[
A_z^2 \approx \left( \frac{512\pi}{75M_{pl}^4} \right) \frac{V^3}{V'^2} \left[ \frac{2\lambda + V}{2\lambda} \right]_\phi^{|k=at|}
\]

(2.9)
2.2.1 Inverted Hybrid Inflation

In the inverted hybrid inflation, the hybrid potential is used but the initial field (φ) expectation value is not large. In this scenario, the inflaton field, φ, slowly rolls away from the origin, and finally the trigger field, ψ, terminates inflation.

The simplest way of writing the inverted hybrid potential is [Lyth,1; King]

\[
V_{IH} = \frac{1}{4} \lambda_\psi \left( \psi^2 + M^2 \right) - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} \lambda \phi^2 \psi^2 + \frac{1}{4} \lambda_\phi \phi^4
\]  

(2.10)

\[
= V_0 + \frac{1}{2} m_\psi^2 \psi^2 + \frac{1}{4} \lambda_\psi \psi^4 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{1}{2} \lambda \phi^2 \psi^2 + \frac{1}{4} \lambda_\phi \phi^4
\]  

(2.11)

where $\phi^4$ term enables $\phi$ to possess a vacuum expectation value (VEV), and the subscript IH stands for Inverted Hybrid. This term, $\phi^4$, makes it different from the hybrid model. The inflation field $\phi$ has a negative mass squared. In the inverted case, $\phi$ is originally less than $\phi_c$, so that the inflation occurs and the false vacuum exists when $\phi < \phi_c$. While $\psi = 0$ during inflation, the potential becomes,

\[
V = V_0 - \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda_\phi \phi^4
\]  

(2.12)

The minimum value is at,

\[
\phi_m = \frac{m_\phi}{\sqrt{\lambda_\phi}}
\]  

(2.13)

For the inverted hybrid mechanism to work the $\phi_c < \phi_m$, otherwise $\phi$ would reach its minimum with $\psi$ still trapped in its vacuum state. If we impose stronger condition,

\[
\phi_c << \phi_m,
\]  

(2.14)

then we can neglect $\phi^4$ term in eq.(2.12) and achieve the inverted quadratic potential during inflation.
Defining a parameter $\mu$, where

$$\mu^2 = \frac{2V_0}{m_\phi^2}$$

we can express eq.(2.15) as,

$$V = V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^2 \right]$$

### 2.2.2 Mutated Hybrid Inflation

The potential is given by [Lyth, 1],

$$V = V_0 - \frac{1}{2} m_\phi^2 \phi^2$$

where $\psi$ is held at zero during inflation. In mutated hybrid inflations $\psi$ is held close to zero, but not at zero during inflation. In the model considered in this chapter, $\psi > 0$ and $\phi > 0$.

From eq. (2.18), by taking derivatives,

$$V_\phi = -\sigma \psi \phi^{-1} + \lambda \psi \phi^{-1} \psi'$$

$V_\phi = 0$ when $\psi = 0$ (for $p \geq 2$ and $q \geq 2$). This condition means, ($\phi$ is minimum at $\psi = \psi_*$)

$$\psi = \psi_* = \left( \frac{\sigma}{\lambda} \right)^{\frac{1}{q(p-1)}} \phi^{-\lambda(q-p)}$$

Assuming that $V_{\psi\psi} \big|_{\psi = \psi_*} \gg V_0$ so that $\psi$ is held fixed at $\psi = \psi_*$ during inflation one can derive the form of the effective potential during inflation as,
We will use this equation to study inflation on RS brane in the next section.

2.3 Studies of Inflation on RSII Braneworld

2.3.1 Inverted Hybrid Potential

We use the potential eq.(2.16), with the definition of eq. (2.15)

\[ V = V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^2 \right] \]  

(2.22)

Its derivative is,

\[ V' = -m_\phi^2 \phi \]  

(2.23)

\[ \frac{V'}{V} = -\frac{m_\phi^2 \phi}{V_0 (1 - \beta \phi^2)} = -\frac{m_\phi^2}{V_0} \phi \]  

(2.24)

The slow roll parameters, eq.(2.7) and eq.(2.8) become,

\[ \epsilon = \frac{m_{\rho'}^2 \lambda m_\phi^4 \phi^2}{4\pi V_0^3} = \frac{m_{\rho'}^2 \lambda m_\phi^4 \phi^2}{4\pi V_0^3} \]  

(2.25)

and,

\[ \eta = \frac{m_{\rho'}^2 \lambda V''}{4\pi V^2} = -\frac{m_{\rho'}^2 \lambda n_\phi^2}{4\pi V_0^2} \]  

(2.26)

The number of e-foldings,

\[ N = -\frac{4\pi}{m_{\rho'}^2 \lambda} \int \frac{(-) V_0^2}{m_\phi^2 \phi} = -\frac{4\pi}{m_{\rho'}^2 \lambda} \left[ \phi \frac{\phi_{\phi}}{\phi} \right] \]  

(2.27)

From eq. (2.27), the inflaton field at the end of inflation is,

\[ \phi_f = \phi \exp \left[ \frac{m_{\rho'}^2 \lambda m_\phi^2}{4\pi V_0^2} N \right] = \phi e^{-N} \]  

(2.28)

The ratio of parameters satisfies, for large \( V_0 \),
Thus using inverted hybrid potential we see that in the RS II model also the relative contribution of gravitational waves to CMB anisotropy is negligible, even though the slow roll parameters are different in brane world. This expression is different from that of FRW case [King] by a facto of \( \frac{1}{2} \). So we are concerned of scalar perturbations only.

The spectral index is effectively,

\[ n_s - 1 \approx -2|\eta| \]  \hspace{1cm} (2.30)

\[ 1 - \eta_s = 2|\eta| \]  \hspace{1cm} (2.31)

\[ \frac{m_\phi^2 \lambda m_\delta^2}{2\pi V_0^2} \]  \hspace{1cm} (2.32)

The scalar perturbation index on RS brane model [Maartens] is,

\[ A_s^2 = \frac{512\pi}{75 m_{\phi'}^2} \frac{V^3}{V''^2} \left( 1 + \frac{V}{2\lambda} \right)^3 \]  \hspace{1cm} (2.33)

\[ = \frac{512\pi}{600 m_{\phi'}^2} \frac{V''^6}{V''^2 \lambda^3} \]  \hspace{1cm} (2.34)

Substituting in the above eq.(2.28),

\[ \phi_\tau = \sqrt{\frac{64\pi}{75} \frac{V_0^2}{m_{\phi'}^2 \lambda^2}} \frac{e^{-\eta N}}{A_s} \]  \hspace{1cm} (2.35)

\[ = \sqrt{\frac{64\pi}{75} \frac{V_0^2}{m_{\phi'}^2 \lambda^2}} \frac{4\pi V_0^2}{m_{\phi'}^2 \lambda m_\delta^2 \lambda} \frac{1}{\lambda A_s} \]  \hspace{1cm} (2.36)
where $A = \frac{4}{75\pi \, m_{\phi} \sqrt{\lambda^*} \, A^*}$

(2.37)

The minimum value of $\phi$, found from the eq. (2.39) is

$|\eta| = \frac{1}{N}$

(2.41)

Substituting in eq.(2.39) we get,

$\phi = A \, e \, N$

(2.42)

The eq.(2.4) is

$\phi = \frac{m_\psi}{\sqrt{\lambda'}} \cdot \text{where} \, m_\psi = \sqrt{\lambda'_\psi} \, M$

(2.43)

$\therefore \phi = \sqrt{\frac{\lambda'_\psi}{\lambda'}} \, M$

(2.44)

The inflaton field at the end of inflation satisfies eq.(2.14). Multiplying it by $\lambda_*$ and substituting from eq. (2.13), we get,

$\lambda_* \ll \left( \frac{m_{\phi}}{\phi_i} \right)^2$

(2.45)

$m_{\phi}/\phi_i = \frac{m_{\phi}}{A \, e^{-\eta N}} = \frac{m_{\phi} |\eta|}{A \, e^{-\eta N}}$

(2.46)

$= \frac{m_{\phi} |\eta| \, e^{\eta N}}{\sqrt{4 \, V_0} \, \sqrt{75\pi \, m_{\phi} \sqrt{\lambda^*} \, A^*}}$

(2.47)
\[ \frac{d}{d|\eta|} \left( \frac{m_{\phi}}{\phi_f} \right)_{\text{max}} = \frac{3}{2} B |\eta|^{1/2} e^{-|\eta|N} - B |\eta|^{3/2} N e^{-|\eta|N} \]

\[ = B |\eta|^{1/2} e^{-|\eta|N} \left[ \frac{3}{2} - |\eta|N \right] = 0 \]

We get, \[ |\eta| = \frac{3}{2} N \]

\[ \left( \frac{m_{\phi}}{\phi_f} \right)_{\text{max}} = \pi \sqrt{75} A_s \left( \frac{3}{2N} \right)^{3/2} e^{-\gamma/2} \]

\[ = \pi \sqrt{75} e^{-\gamma/2} \left( \frac{3}{2} \right)^{3/2} N^{-\gamma/2} A_s \]

The COBE normalization gives \( A_s = 4 \times 10^{-10} \), thus we get a constraint for the parameter, for \( N = 50 \),

\[ \lambda_f \ll \left[ \left( \frac{m_{\phi}}{\phi_f} \right)_{\text{max}} \right]^{\gamma/2} \leq 3 \times 10^{-13} \]  

Thus we see that the value of \( \lambda_f \) must be very small. The corresponding value is of the same order in the FRW case [Kin3]. Thus we see no improvement in the braneworld scenario. This unusually small value of the coupling parameter makes it not viable. This is the drawback of the inverted hybrid inflation. The inflationary scenario is possible only if the dimensionless couplings are very small as \( \leq 3 \times 10^{-13} \).
2.3.2 Mutated Hybrid Inflation

As explained in the section 2.2.2, the effective potential during inflation is of the form,

$$V = V_0 \left( 1 - \mu \phi^{-\alpha} \right)$$

(2.58)

The index $\alpha$ characterizes the potential

Derivatives can be written as,

$$V' = V_0 \mu \alpha \phi^{-\alpha-1}$$

(2.59)

$$V'' = -V_0 \mu \alpha (\alpha + 1) \phi^{-\alpha-2}$$

(2.60)

The slow roll parameters are,

$$\epsilon = \frac{m_{pl}^2}{16\pi} \left( \frac{V'}{V} \right)^2 \frac{1 + V/\lambda}{1 + V/(2\lambda)}$$

(2.61)

$$= \frac{m_{pl}^2 \lambda \mu^2 \alpha^2}{4\pi V_0} \phi^{-\alpha(\alpha+1)}$$

(2.62)

and,

$$\eta = \frac{m_{pl}^2 \nu^*}{8\pi} \frac{1}{\nu \left[ 1 + V/(2\lambda) \right]}$$

$$= -\frac{m_{pl}^2 \lambda \alpha (\alpha + 1) \mu}{4\pi V_0} \phi^{-\alpha-2}$$

(2.63)

(2.64)

The scalar spectral index of scalar perturbation is,

$$n_s = 1 - 6\epsilon + 2\eta$$

$$= 1 - \frac{1}{2\pi} \frac{m_{pl}^2 \lambda \alpha \mu}{v_0} \left[ 3 \mu \alpha \phi^{-2\alpha-2} + (\alpha + 1) \phi^{-\alpha-2} \right]$$

(2.65)

The number of e-folds can be expressed as,

$$N = -\frac{8\pi}{m_{pl}^2} \int_{\phi}^{\infty} \frac{\nu'}{\nu'} \left( 1 + \frac{\nu}{2\lambda} \right) d\phi = \frac{-4\pi}{m_{pl}^2 \lambda} \int_{\phi}^{\infty} \frac{\nu^2}{\nu'} d\phi$$

(2.66)
From the above step the inflaton field can be written as,

\[ \phi^{a+2} = \phi^{a+2} + \frac{m_{pl}^2 \lambda \mu \alpha (\alpha + 2)}{4 \pi \nu_0} N - \frac{m_{pl}^2 \lambda (\alpha + 1) \alpha \mu}{4 \pi \nu_0} \phi^{-a-2} \]  \hspace{1cm} (2.68)

\[ = \phi_f^{a+2} + \frac{\alpha + 2}{2} \chi N \]  \hspace{1cm} (2.69)

where \( \chi = \frac{1}{2 \pi} \frac{m_{pl}^2 \lambda \alpha \mu}{\nu_0} \)  \hspace{1cm} (2.70)

The tensor to scalar ratio of perturbations is

\[ R = 6 \frac{m_{pl}^2}{\pi} \left( \frac{\nu'}{\nu} \right)^2 \frac{\lambda}{\nu} \]  \hspace{1cm} (2.71)

\[ = 6 \frac{m_{pl}^2}{\pi} \mu^2 \alpha^2 \lambda \phi^{-2a-2} \]

\[ = i 2 \chi \mu \alpha \left[ \phi_f^{a+2} + \frac{(\alpha + 2) \chi N}{2} \right]^{-2a-1} \]  \hspace{1cm} (2.72)

The eq. (2.53) becomes,

\[ n_s = 1 - \frac{R}{4} - \frac{(\alpha + 1) \chi}{\phi_f^{a+2} + \frac{(\alpha + 2) \chi N}{2}} \]  \hspace{1cm} (2.73)

We can use this relation to get a theoretical plot, in order to compare with observational values.

Also the eq. (2.55) for e-fold number becomes,

\[ N = \frac{4 \pi \nu_0}{m_{pl}^2 \lambda \mu \alpha (\alpha + 2)} \] \[ \frac{m_{pl}^2 \lambda (\alpha + 1) \alpha \mu}{4 \pi \nu_0} \left[ \frac{1}{\eta_{(\phi_i)}} - \frac{1}{\eta_{(\phi)}} \right] \]  \hspace{1cm} (2.74)
We have expressed the number of e-foldings in terms of slow roll parameter and the index of potential.

From eq.(2.72) $R$ is maximum at

$$x = \frac{1}{n\alpha} \left( \phi_f \right)^{\alpha+1}$$

$$\therefore \quad R_{\text{max}} = \frac{12 \mu}{N} \phi_f^{-\alpha} \left[ \frac{3\alpha + 2}{2\alpha} \right]^{-2(n+1)}$$

The scalar perturbation amplitude is,

$$A_s^2 = \frac{512\pi}{m_{pl}^6} \left( 1 + \frac{\nu}{2\lambda} \right)^3$$

$$= \frac{512\pi}{8 \times 75} \frac{\nu_0^4}{m_{pl}^6 \lambda^3 \mu^2 \alpha^2} \phi^{2\alpha+2}$$

$R$ can be written in terms of $A_s^2$ independent of $\alpha$, 

$$R = \frac{6}{\pi} \frac{m_{pl}^2}{\nu_0} \eta^2 \alpha \lambda \phi^{-2\alpha-2}$$

$$= \frac{128\nu_0^3}{25\lambda^2 m_{pl}^4 A_s^2} \frac{1}{A_s^2}$$

Using observational values we can obtain constraint for $\nu_0/\lambda$.

### 2.4 Conclusions

We have discussed brane world inflation in the RSII model with inverted hybrid and mutated hybrid potentials.

We find using the inverted hybrid potential that the relative contribution of gravitational waves to CMB anisotropy is negligible in the case of RSII model.
Chapter 2

Applying slow roll conditions we have derived the slow roll parameters, amplitudes of scalar perturbations, and number of e-folds to the end of inflation in a form that can be used for constraining inflationary models. By using COBE normalized value we obtained the constraint for the coupling constant and found that it should be fine tuned as in the FRW inflation, in order to be consistent with structure formation.

In the case of mutated hybrid potential, using observational value of amplitude of tensor perturbation, we have obtained the upper bound for the energy scale of inflation and the model parameters, the constant of potential function and brane tension.

The comparison with observational values, and hence the constraints can be obtained using the computer program COSMOMC [Lewis], which is a Markov-Chain Monte-Carlo engine for exploring cosmological parameter space. Also this method can be used to constrain all the models of inflation. We have not discussed the observational constraints, as the computational work for obtaining observational plot is not complete which is being carried out presently.

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