CHAPTER 3
TIME DOMAIN WINDOWING METHOD

3.1 INTRODUCTION

ICI compression using a correlative coding schemes can suppress the ICI component without reducing the spectral efficiency by a factor of two. However, this scheme usually cannot provide satisfactory performance in terms of the carrier-to-interference ratio (CIR) as it has a drawback that the detection errors generated at the receiver would be propagated through the OFDM symbols. An improved version of this method namely time domain windowing technique using correlative coding where the data samples of the tail subcarriers in an OFDM symbol are encoded with the data samples of the head ones in the subsequent OFDM symbol. For low complexity, the modified correlative coding scheme is realized using windowing function parameters.

3.2 FREQUENCY CORRELATIVE CODING

Frequency domain equalization is used to remove the fading distortion in the OFDM signal for a frequency non-selective, time varying channel. Once the coefficients of the equalizer is found, linear or decision feedback equalizers are used in frequency domain. Since ICI is different for each OFDM symbol, the pattern of the ICI for each OFDM symbol needs to be calculated. ICI is estimated through the insertion of frequency domain pilot symbols in each symbol. This method is shown in Figure 3.1.
Correlative coding is another method used to suppress the ICI. This method does not reduce the bandwidth efficiency. In this coding, new symbols are determined from old symbols using the correlation polynomial $F(D) = (1 - D)$. Without any loss in the bandwidth, 3.5dB improvement in CIR level is gained in this method using BPSK. A nonlinear adaptive filter is also used to reduce ICI. This method converges slowly as it uses higher order statistics.

### 3.2.1 Realization of Correlative Coding

Frequency domain correlative coding is a simple solution to ICI problems, and makes OFDM systems less sensitive to frequency errors, thus it reduces the system complexity and increasing bandwidth efficiency. The correlative coding between signals modulated on subsequent subcarriers are used to compress ICI in OFDM system and the ICI is measured using subcarrier frequency offset response.
The (1-D) type of correlative coding is chosen and the subcarrier frequency offset response is introduced in terms of Doppler shifts in the channel. For better ICI suppression performance, a higher order correlation polynomial can be used. However, the error propagation will come out in the decoding process which may degrade the BER performance.

A simplified block diagram of the proposed OFDM system with correlative coding is shown in Figure 3.2. The structure of an OFDM system with correlation coding can be derived from conventional single carrier systems. By using BPSK modulator, the serial modulated signal, $a_k$ is coded using correlative coding, where $k$ is the subcarrier index with $k=0, 1 \ldots N-1$ and $N$ is the total number of subcarriers. Denoting ‘D’ as the unit delay of the subcarrier index $k$, the proposed coding with correlation polynomial $F(D) = (1 - D)$ is performed as

$$b_k = a_k - a_{k-1} \quad (3.1)$$
Where $a_k$ is the signal sequence before correlative coding and $b_k$ is the coded symbols.

Then the coded symbol $b_k$, are modulated on $N$ subcarriers. The symbol $b_k$ takes three possible values (-2, 0, 2). Equation (3.1) introduces the correlation between the adjacent symbols ($b_k$, $b_{k-1}$). To avoid the error propagation in the decoding procedure due to correlative coding, precoding is performed before the BPSK modulation. In OFDM systems, the ICI signal on each subcarrier is a function of the channel frequency offset and the signal values modulated on all subcarriers. For the OFDM, the main sources affecting the BER performances are the AWGN and the ICI. When the frequency error exists, then without considering AWGN, the received signal on each subcarrier can be recognized as a sum of the expected signal and the interference signal. For an OFDM system with $N$ subcarriers, if the channel frequency offset normalized to the subcarrier separation is denoted by $\varepsilon$, then the received signal on subcarrier $k$ denoted as $r_k$ is given by

$$r_k = \frac{1}{N} \sum_{n=0}^{N-1} \sum_{l=0}^{N-1} b_l \exp \left( j2\pi n(l - k + \varepsilon) \right)$$

(3.2)

$$r_k = \sum_{l=0}^{N-1} b_l S(l - k)$$

(3.3)

Where,

$$S(l - k) = \frac{\sin (\pi \varepsilon)}{N \sin \left( \frac{\pi (\varepsilon + l - k)}{N} \right)} \exp \left( j\pi(N - 1)\varepsilon - (l - k) \right)$$

(3.4)
The received signal $r_k$ can be expressed as a sum of the desired signal $C_k$ and the undesired ICI signal $I_k$

$$r_k = C_k + I_k \quad (3.5)$$

where

$$C_k = b_k S(0) \quad (3.6)$$

$$I_k = b_l S(l-k) \quad (3.7)$$

The desired signal value $C_k$ depends only on the signal transmitted on subcarrier $k$ and $I_k$ depends on the signals transmitted on all the other subcarriers. The carrier to interference ratio (CIR) of the OFDM system with (1-D) type correlative coding can be obtained from the following equation as suggested by Zhao.Y et al (1998).

$$CIR = \frac{\sin^2 (\pi \epsilon)}{(\pi \epsilon)^2} \frac{1}{\sum_{l=0}^{N-1} |S(l)|^2 - \frac{1}{2} \sum_{l=2}^{N-1} |S(l)S^*(l-1) + S(l-1)S^*(l)|} \quad (3.8)$$

To analyze ICI level with respect to the frequency error, it is necessary to have a corresponding basic ICI function with respect to the system frequency error. At the transmission side of OFDM systems, signals $a_k$ when $k = 0, \ldots, N-1$ are modulated onto $N$ subcarriers. It can be done by performing IFFT to the signal sequence $a_k$. If the frequency error is sufficiently large, it is possible that $I_k > C_k$ occurs. In such a case, a data decision error can be made even in the absence of the AWGN. The BER of the OFDM systems increases rapidly when frequency error increases. The
condition $\varepsilon < 0.05$ is necessary to maintain acceptable system performance, explained by Zhao.Y et al (1998).

### 3.3 TIME DOMAIN WINDOWING

Time domain windowing is used to reduce the sensitivity to linear distortions and frequency errors (ICI). Window may be realized with a raised cosine or other kind of function that fulfills the Nyquist criterion. Raised cosine window is used in order to reduce the ICI effects. However, this intuitive window is shown to be sub-optimum and a closed solution for optimum window coefficients. A condition for orthogonality of windowing schemes in terms of the FFT of the windowing function is derived. The FFT can be considered as a filter bank with N filters where N is the FFT size.

The frequency response of the $n^{th}$ filter $H_n(F)$ is

$$|H_n(F)| = \left| \frac{\sin(\pi(F - n))}{\sin(\pi(F - n)/N)} \right|$$

(3.9)

where $F = N. f/f_s$ and $f_s$ is the sampling rate at the receiver.

This filter has the shape of a periodic sinc function. The FFT operation in the receiver performs transform in blocks of only N samples. This is equivalent to using a square window of length $T_s$ in time domain corresponding to a sinc function in frequency domain. The filter bank consisting of N filters having sinc shape is shown in Figure 3.3(a). Carriers are represented by ideal Dirac distributions placed on the filter maxima. The maximum of one filter coincides with the zero crossing of all others, allowing the carriers to separate without suffering any ICI.
As explained unwindowed OFDM system has rectangular symbol shapes and hence, in the frequency domain the individual sub-channels will have the shape of sinc functions. The use of a window on N samples (in time domain) before the FFT reduces the side lobe amplitude of this sinc function but also leads to an orthogonality loss between carriers. A window which reduces the side lobes and preserves the orthogonality is called Nyquist window. This window will reduce the amplitude of the filter side lobes depending on the roll-off factor. The side lobe magnitudes of the frequency response of a raised cosine window for different roll-off factors (\(\alpha\)) are given in Figure 3.4.

![Filter bank for rectangular windowing.](image)

(a) Filter bank for rectangular windowing.  
(b) Filter bank for a 2N rectangular window.

**Figure 3.3 Position of carriers in the FFT filter bank**

The Nyquist window uses the part of the guard interval that is not disturbed by multipath reception. If the estimate of maximum echo delay is known then the length of undisturbed guard period can be calculated. One can choose the roll-off factor of the window accordingly. Therefore, the length of the window adapts to the transmission conditions. To reduce the sensitivity to frequency errors, useful part of the signal and unused part of the guard period is shaped with the Nyquist window function.
After Nyquist windowing the sub-carriers has lost their orthogonality. Thus a symmetrical zero padding is performed in order to complete a total of \(2N\) samples. Therefore, \(2N\) filters will be used in the FFT process. The advantage of having twice as many filters (\(2N\)) on the filter bank shown in Figure 3.3(b) is that the area under the filter curve is one half of that of the \(N\)-filter case for the same maxima value.

Thus the odd or even filters integrate the same carrier power but only one half of the white noise power, leading to an improvement in carrier to noise ratio. In the receiver the outputs of the FFT with even-numbered subscripts are then used as estimates of the transmitted data and the odd-numbered ones are discarded. Since not all of the received power is being used in generating data estimates, windowing reduces overall Signal-to-noise Ratio (SNR) compared with OFDM without windowing.

![Figure 3.4 Frequency response of a raised cosine window with different roll-off factors.](image)
With Nyquist windowing, the whole filter bank is less sensitive to frequency deviations, disturbances, etc. The reason for the improvement can also be explained through a decrease of the FFT-leakage. Since the leakage is responsible in several cases for OFDM signal degradation, an overall improvement in demodulation is expected.

A number of different windows (Hanning, Nyquist, Kaiser etc.) have been described in the literature. All of the windows give some reduction in the sensitivity to frequency offset. But only Nyquist windows (of which the Hanning window is one particular example) have no ICI for the case of no frequency offset.

### 3.3.1 Realization of Time Domain Windowing

In Time domain equalization technique, a window function is applied to the data in time domain, obtained after performing IFFT operation. The application of the windowing function tapers the start and ends of waveform reducing the transients and consequently the spectral spreading.

The application can be divided into two groups. In the first group, windowing is used to reduce the sensitivity to linear distortions. In the second group, windowing is used to reduce the sensitivity to frequency errors. In this thesis, the second approach is used. In this case, the window function improves the spectral efficiency and reduces the BER of the OFDM system.

According to the circular convolution property, to realize the frequency domain circular convolution process, an equivalent time-domain windowing operation can be done. For the correlative polynomial used in frequency domain scheme, the window function is proposed in this work to improve the performance of the OFDM system. For the correlative polynomial \((1-D)\) is
used, the window function proposed in this scheme is expressed as $(1 - \exp(j2\pi n/N))$.

ICI can be suppressed well by applying the proposed window function, then the remaining task that affects the BER performance would be decided by the demodulation technique. In the proposed method the decoding process does not require a prior information of the transmitted data. For convenience, the proposed window function is applied at the transmitter. The subcarrier spectrum can be observed for the various order of the windowing function.

Figure 3.5 Block diagram of Time Domain Windowing scheme equivalent to Correlative Coding scheme

Figure 3.2 shows that the proposed correlative coding scheme can be realized in the time domain by using a window function. When the time domain window function $(1 - \exp(j2\pi n/N))$ is applied, the data samples transmitted on the $k^{th}$ subcarrier can be expressed as
\[ b_k = a_k (1-\exp(j2\pi n/N)) \] (3.10)

The window function used is optimized to obtain a maximized Carrier to Interference ratio (CIR). So, the order of the window function is optimized to be 1, assuming the modulation system to be BPSK.

\[ r_k = C_k + I_k \] (3.11)

where

\[ C_k = b_k S(-\varepsilon) \] (3.12)

and

\[ I_K = \sum_{l=0, l\neq k}^{N-1} b_l S(l-k-\varepsilon) \] (3.13)

The theoretical CIR for the order 1 of the window function is given by Zhao.Y et al (1998) as

\[
\text{CIR} \frac{|S(-\varepsilon)|^2}{\sum_{l=0}^{N-1} |S(k-\varepsilon)|^2 - \sum_{l=0, l\neq k}^{N-1} \text{Re}(2S^*(k-\varepsilon)S(l-k-\varepsilon))} \] (3.14)

The correlative pairs would in average contribute to the smallest ICI only when the weighting magnitudes of the elements of each pair are equal or symmetric. This also agrees with the observation that the distributions of ICI coefficients are circularly symmetric.

When a higher order correlative polynomial is selected, better ICI suppression performance can be obtained in equivalent to that if the order of the window function is varied in the proposed window function a stronger main lobe and smaller side lobes in each subcarrier spectrum can be achieved.
3.4 SIMULATION AND RESULT ANALYSIS

Simulations are performed in MATLAB using the parameters list in Table 3.1. The BER performance under Time Domain Windowing method are studied for different frequency offset values with reference to $E_b/N_o$ values.

Table 3.1. Simulation Parameters for Time Domain Windowing method

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of carriers (N)</td>
<td>52</td>
</tr>
<tr>
<td>Modulation (M)</td>
<td>BPSK</td>
</tr>
<tr>
<td>Frequency offset ($\varepsilon$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Bits per OFDM symbol (BPS)</td>
<td>$N\times\log_2(M)$</td>
</tr>
<tr>
<td>$E_b/N_o$</td>
<td>1:5</td>
</tr>
<tr>
<td>IFFT, FFT size</td>
<td>64 - point</td>
</tr>
<tr>
<td>Total Guard Interval size</td>
<td>FFT size / 4</td>
</tr>
<tr>
<td>Channel</td>
<td>AWGN</td>
</tr>
</tbody>
</table>
The BER performance of the time domain windowing scheme and correlative coding method is shown in Figure 3.6. The BER of time domain windowing technique is less compared to correlative coding method and standard OFDM.

It is found that for other values of $\varepsilon$ also, the BER performance of time domain windowing method shows better performance than the other two schemes.

Further, simulations are carried out for the study of CIR performance of the three schemes by varying the frequency offset values.
Figure 3.7 shows the CIR versus normalized frequency offset for the correlative coding and proposed windowing scheme. From the simulated curves, it can be observed that, when the normalized frequency offset is zero the time domain windowing technique behaves same as the standard OFDM.

The effect of the time domain windowing can be explicitly seen when the frequency offset value is increased above 0.1. Thus the time domain windowing scheme outperforms the correlative coding scheme, due to its better capabilities in suppressing ICI and preventing error propagation through OFDM symbols.

Figure 3.7 CIR performance of Time domain windowing and Correlative coding scheme for $E_b/N_0 = 4.5$
3.5 CONCLUSION

The ICI suppression scheme using time domain windowing for OFDM systems, provides a better BER performance compared to the correlative coding. The windowing technique used also avoids the propagation of errors in the demodulation process. From the Figure 3.7 it is found that there is a 3dB improvement in CIR in the proposed windowing scheme compared to the existing correlative coding scheme. The CIR is reduced when the frequency offset value increases in the case of all ICI suppression schemes.