CHAPTER 2

Two-Wave Mixing in Optically Active Photorefractive Sillenite Crystals

2.1 INTRODUCTION

The photorefractive effect [1-3] is a phenomenon in which the local index of refraction is changed by the spatial variation of the light intensity. The photorefractive effect [4-6] has since been observed in many electro-optic crystals [7], including BSO, BGO, BTO, LiNbO$_3$, BaTiO$_3$, SBN, GaAs, InP etc. Also in last few years the studies are going on for organic photorefractive materials such as polymers, dye-doped polymers and photosensitive glasses. Photorefractive materials are playing increasingly important roles in various applications of optical processing [8]. Recently photorefractive materials are also finding application in some of the cardinal areas like information security [9].

One of the most interesting classes of photorefractive media is that of sillenite crystals [10-11], which includes Bismuth Silicon Oxide (BSO), Bismuth Germanium Oxide (BGO), Bismuth Titanium Oxide (BTO). These crystals exhibit high photorefractive sensitivity for volume holographic grating formation [12], fast response, long storage times under dark conditions, and essentially unlimited recyclability. Such crystals are potentially useful for dynamic real-time [13-14] and time average interferometry, nonlinear optical signal processing, phase-corrected image propagation through aberrating media, optical interconnections, spatial optical switching, self-pumped laser resonators, and signal amplification.
Light diffraction from volume holograms in sillenite crystals exhibits pronounced polarization properties [15-16], including the effect of natural optical activity. Optical activity leads to rotation of the polarization planes of the interacting waves.

Two-wave mixing [17 - 20] is basically the exchange of energy between two incident beams. It is known that the natural optical activity of sillenites affects the photorefractive two-wave mixing (TWM) in these crystals strongly. This property has a potential application in spatial light modulators (SLM). A hologram has the ability to recreate an optical distribution in both amplitude and phase. In a SLM [21] one can electronically control the amplitude and/or phase over an area usually defined as a pixel structure. Hence, a SLM controlling the phase of the light could be used as an arbitrary refractive component. For each electronic update on the SLM, it could act as a completely different optical component. An SLM capable of both amplitude and phase modulation could therefore act as a dynamic hologram and thereby a 3D display. A. Marrakchi [22] have demonstrated spatial light modulation using two-beam coupling in BSO.

In this chapter polarization properties of sillenite crystals (cubic crystals of 23 symmetry) has been studied for TWM. In the case of TWM in PR crystals, energy exchange between the interacting beams depends on the factors such as thickness, absorption, electro-optic coefficient, and optical activity of the crystal, and the mutual coherence and polarization of the input beams. For some crystals like LiNbO$_3$, BaTiO$_3$ and SBN, optical activity can be neglected. However, for some highly optically active crystals like BSO, BGO and BTO, one must consider the optical activity [23] of the medium.

We have analyzed the TWM [24] in an absorbing PR medium considering the optical activity. We consider the case where the energy is transferred from the reference to
the signal beam in the diffusion regime (no external field applied). We have studied the effects of optical activity, thickness and the coupling constants on the different components of the two beams along with their polarizations. The effect of input polarization angle of pump beam is also analyzed. This analysis is valid for near collinear (very small angle between the pump and signal) interacting beams in an absorbing crystal of 23 symmetry.

2.2 THEORY

We consider two waves $A$ and $B$ incident on a Photorefractive crystal, which create a phase grating inside the medium. In this model, each beam consists of two orthogonally polarized components, designated as ‘s’ and ‘p’ components.

2.2.1 Formulation of coupled wave equations

If $A$, the reference (pump) beam and $B$, the signal beam are the two input beams (Fig. 2.1) then the electric field of the two beams can be represented as [25],

$$E_1 = (sA_s + p_1A_p) \exp(-i k_1 \cdot r) \tag{2.1}$$

and

$$E_2 = (sB_s + p_2B_p) \exp(-i k_2 \cdot r) \tag{2.2}$$

Then the total field inside the crystal is given as

$$E = E_1 + E_2 \tag{2.3}$$

Where $k_1$ & $k_2$ are the propagation vectors of the two input beams, and $s$ is the unit vector perpendicular to the plane of incidence, $p_1$ and $p_2$ are the unit vectors parallel
to the plane of incidence and perpendicular to the beam wave vector. The interference pattern [26] inside the medium can be written as

\[ I = |E_1 + E_2|^2 = A_s^* A_s + A_p^* A_p + B_s^* B_s + B_p^* B_p + \left[ (A_s B_s^* + A_p B_p^*) \exp(iK \cdot r) + c.c. \right] \]

(2.4)

Where \( K = k_2 - k_1 \) is the grating vector of the grating formed by the two input beams and \( c.c. \) represents the complex conjugate.

Fig. 2.1 Two-wave mixing in transmission geometry.
The wave equation [25] that governs the light propagation in a medium is

\[ \nabla \times \nabla \times \mathbf{E} = -\mu_0 \frac{\delta^2}{\delta t^2} \mathbf{D} \]  

(2.5)

Where , \( \mathbf{D} \) is the displacement vector expressed as

\[ \mathbf{D} = \mathbf{s} \mathbf{D}_S + \mathbf{p} \mathbf{D}_P \]  

(2.6)

For a crystal of 23-symmetry having the orientation \( \{ \mathbf{K} || <001> \} \), the effect of optical activity is to introduce off-diagonal elements into the susceptibility tensor, so that the relationship between the transverse polarization and field [27] becomes

\[ \begin{vmatrix} \mathbf{D}_S \\ \mathbf{D}_P \end{vmatrix} = \begin{vmatrix} \varepsilon & -i\beta \\ i\beta & \varepsilon_0 \end{vmatrix} \times \begin{vmatrix} \mathbf{E}_S \\ \mathbf{E}_P \end{vmatrix} \]  

(2.7)

where \( \varepsilon = \varepsilon_0 + \Delta \varepsilon \)

\( \varepsilon_0 \) is the free space dielectric constant and \( \beta = 2a/k \) is directly proportional to the optical rotatory power \( 'a' \) (deg/mm). \( \Delta \varepsilon \) is the perturbation in the dielectric constant [14] due to the PR effect.

\[ \Delta \varepsilon = -\varepsilon_0 \varepsilon \lambda [(A_S B_p^* + A_p B_s^*) \cos \theta) \cdot \exp(i\mathbf{K} \cdot \mathbf{r} + i\phi_g) + c.c.] / I_0 \]  

(2.8)

Where \( \varepsilon_1 \) is a 3 X 3 tensor, \( \phi_g \) is the spatial phase shift between the index grating and the intensity pattern, \( \theta \) is the angle between the beams inside the crystal and \( I_0 \) is the total intensity given by

\[ I_0 = A_S^* A_S + A_P^* A_p + B_S^* B_S + B_P^* B_P \]  

(2.9)

Substituting equation (2.6) in (2.5), the displacement vector \( \mathbf{D} \) can be written as

\[ \mathbf{D} = \mathbf{s}(\varepsilon \mathbf{E}_S - i\beta \mathbf{E}_P) + \mathbf{p}(i\beta \mathbf{E}_S + \varepsilon_0 \mathbf{E}_P) \]  

(2.10)

From equation (2.1) and (2.2) we can write

\[ \mathbf{E}_S = A_S \exp(-i\mathbf{k}_1 \cdot \mathbf{r}) + B_S \exp(-i\mathbf{k}_2 \cdot \mathbf{r}) \]  

(2.11)
\[ E_p = A_p \exp(-i\mathbf{k}_1 \cdot \mathbf{r}) + B_p \exp(-i\mathbf{k}_2 \cdot \mathbf{r}) \]  \hspace{1cm} (2.12)

### 2.2.2 Coupled wave equation

Substituting all above relations in Maxwell’s equation and using slowly varying approximation, \( \left| \frac{d^2 A}{dz^2} \right| \ll \left| \frac{dA}{dz} \right| \) where \( A \) is the amplitude of the respective beam such as \( A_S, A_P, B_S, B_P \), the following set of coupled wave equations are obtained:

\[
\frac{dA_S}{dz} = ie^{i\phi} GB_S \left( A_S^* B_S + A_P^* B_P \cos \theta \right) \bigg/ I_0 - aA_P \frac{1}{2} \alpha A_S \tag{2.13}
\]

\[
\frac{dA_P}{dz} = aA_S - \frac{1}{2} \alpha A_P \tag{2.14}
\]

\[
\frac{dB_S}{dz} = ie^{-i\phi} GA_S \left( A_S^* B_S + A_P^* B_P \cos \theta \right) \bigg/ I_0 - aB_P \frac{1}{2} \alpha B_S \tag{2.15}
\]

\[
\frac{dB_P}{dz} = aB_S - \frac{1}{2} \alpha B_P \tag{2.16}
\]

For simplicity, the phase shift \( \phi_g = \pi/2 \) (diffusion regime) is considered and the angle between the two beams inside the medium considered to be very small so that \( \theta \approx 0 \) and \( \cos \theta \approx 1 \).

So, now we have the coupled wave equations as

\[
\frac{dA_S}{dz} = GB_S \left( A_S^* B_S + A_P^* B_P \right) \bigg/ I_0 - aA_P \frac{1}{2} \alpha A_S \tag{2.17}
\]

\[
\frac{dA_P}{dz} = aA_S - \frac{1}{2} \alpha A_P \tag{2.18}
\]

\[
\frac{dB_S}{dz} = GA_S \left( A_S^* B_S + A_P^* B_P \right) \bigg/ I_0 - aB_P \frac{1}{2} \alpha B_S \tag{2.19}
\]
\[
\frac{dB_p}{dz} = aB_S - \frac{1}{2} \alpha B_p
\]  

(2.20)

Where \( G \) is the coupling coefficient given by

\[
G = \omega^2 \mu_0 \varepsilon_0 \varepsilon_1 / 2k
\]  

(2.21)

Where \( k \) is the propagation constant and \( \alpha \) is the absorption constant of the medium.

2.3 NUMERICAL METHOD USED

In the case of transmission geometry, the input values of \( A_S, A_P, B_S, B_P \) are known at \( z = 0 \). The coupled equations above (equations (2.17) – (2.20)) are solved by fourth order Runge-Kutta method.

The formulas used for solving coupled wave differential order [28] are given below.

For a simple pair of coupled wave equations given as:

\[
\frac{dx}{dt} = f_1(t, x, y)
\]  

(2.22)

\[
\frac{dy}{dt} = f_2(t, x, y)
\]  

(2.23)

The next approximation to the initial values of \( x \) and \( y \) is given as:

\[
r_1 = f_1(t_j, x_j, y_j)
\]  

(2.24)

\[
r_2 = f_1(t_j + \frac{h}{2}, x_j + \frac{h}{2} r_1, y_j + \frac{h}{2} s_1)
\]  

(2.25)

\[
r_3 = f_1(t_j + \frac{h}{2}, x_j + \frac{h}{2} r_2, y_j + \frac{h}{2} s_2)
\]  

(2.26)

\[
r_4 = f_1(t_j + h, x_j + hr_3, y_j + hs_3)
\]  

(2.27)
\[ x_{j+1} = x_j + \frac{h}{6}(r_1 + 2r_2 + 2r_3 + r_4) \]  

(2.28)

\[ s_1 = f_2(t_j, x_j, y_j) \]  

(2.29)

\[ s_2 = f_2(t_j + \frac{h}{2}, x_j + \frac{h}{2}r_1, y_j + \frac{h}{2}s_1) \]  

(2.30)

\[ s_3 = f_2(t_j + \frac{h}{2}, x_j + \frac{h}{2}r_2, y_j + \frac{h}{2}s_2) \]  

(2.31)

\[ s_4 = f_2(t_j + h, x_j + hr_3, y_j + hs_3) \]  

(2.32)

\[ y_{j+1} = y_j + \frac{h}{6}(s_1 + 2s_2 + 2s_3 + s_4) \]  

(2.33)

\[ t_{j+1} = t_j + h \]  

(2.34)

where \( h \) is the step size.

### 2.4 RESULTS and DISCUSSION

Results of the numerical calculations are presented in graphical form in Figs. 2.2(a) – 2.2(c) and 2.3(a) – 2.3(c).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>BTO</th>
<th>BSO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha ) (absorption coefficient) (cm(^{-1}))</td>
<td>1.0</td>
<td>0.65</td>
</tr>
<tr>
<td>( \alpha ) (optical rotatory power) (deg/cm)</td>
<td>100</td>
<td>386</td>
</tr>
</tbody>
</table>

Table 2.1 Absorption coefficient and optical rotatory power of BSO and BTO.

The variations in the intensities of the various components of the two output beams has been analyzed against various factors like depth (thickness) of the crystal (\( d \))
along z direction, coupling coefficient \((G)\), optical rotatory power \((a)\), absorption coefficient \((\alpha)\), and the input polarization angle of pump beam \(A\) (in degrees). The analysis is being done for two different values of optical activity of BSO and BTO crystals.

Parameters for BTO and BSO taken for calculations are tabulated in Table 2.1 [12]. All parameters considered at a wavelength \(\lambda=532\) nm.

2.4.1 Results for varying thickness of the crystal

In Figs. 2.2 (a) – 2.2 (c), output wave intensities are plotted for increasing depth of the crystal. Different cases have been considered, in the absence of optical activity and for strong optical activity of BSO, BTO crystals. In all the cases transfer of energy takes place from beam \(A\) to beam \(B\).

When there is no optical activity present coupling only takes place between \(s\) components of beam \(A\) and \(B\) (Fig. 2.2(a)). In case of Fig. 2.2(b) and Fig. 2.2(c), the effective coupling varies periodically along the depth of the crystal. With the presence of optical activity magnitudes of \(s\) and \(p\) components strongly depends on the optical activity of the medium.

Comparing the results for Fig. 2.2(b) and Fig. 2.2(c), it has been observed that with increasing optical activity the number of periodic oscillations increases. Hence for maximum efficiency one must take a crystal with suitable thickness.
Fig. 2.2 (a) Intensities of the four components of the two input beams plotted against z, depth in the medium for $G = 10.0 \text{ cm}^{-1}$, $d = 1 \text{ cm}$, $IA_s(0) = 1.0$, $IB_s(0) = 0.1$, $IA_p(0) = 0.0$, $IB_p(0) = 0.0$, $\alpha = 1.0 \text{ cm}^{-1}$, $a = 0 \text{ deg/cm}$. 
Fig. 2.2 (b) Intensities of the four components of the two input beams plotted against
z, depth in the medium for $G = 10.0$ cm$^{-1}$, $d = 1$cm, $I_{A_s}(0) = 1.0$, $I_{B_s}(0) = 0.1$, $I_{A_p}(0) = 0.0$, $I_{B_p}(0) = 0.0$, $\alpha = 1.0$ cm$^{-1}$, $a = 100$ deg/cm (BTO crystal)
Fig. 2.2 (c) Intensities of the four components of the two input beams plotted against $z$, depth in the medium for $G = 10.0 \text{ cm}^{-1}$, $d = 1 \text{ cm}$, $I_{A_S}(0) = 1.0$, $I_{B_S}(0) = 0.1$, $I_{A_P}(0) = 0.0$, $I_{B_P}(0) = 0.0$, $\alpha = 0.65 \text{ cm}^{-1}$, $a = 386 \text{ deg/cm}$ (BSO crystal).
2.4.2 Results for varying input polarization angle

The output beam intensity has been plotted in Figs. 2.3 (a) – 2.3 (c) with the angle of polarization of pump beam \( A \) for optical rotatory power \( a = 0 \) deg/cm (Fig. 2.3 (a)), 100 deg/cm (BTO) (Fig. 2.3 (b)) and 386 deg/cm (BSO) (Fig. 2.3 (c)).

Fig. 2.3 (a) Intensities of the four components of the output beams plotted against input polarization angle of \( A \) for \( d = 1.0 \) cm, \( G = 10.0 \) cm\(^{-1}\), \( I_{B_s}(0) = 0.1 \), \( I_{B_p}(0) = 0.0 \), \( I_A(0) = 1.0 \), \( \alpha = 1.0 \) cm\(^{-1}\), \( a = 0 \) deg/cm.
Fig. 2.3 (b) Intensities of the four components of the output beams plotted against input polarization angle of A for $d = 1.0$ cm, $G = 10.0$ cm$^{-1}$, $IB_s(0) = 0.1$, $IB_p(0) = 0.0$, $IA(0) = 1.0$, $\alpha = 1.0$ cm$^{-1}$, $a = 100$ deg/cm.
Fig. 2.3 (c) Intensities of the four components of the output beams plotted against input polarization angle of A for $d = 1.0$ cm, $G = 10.0$ cm$^{-1}$, $IB_3(0) = 0.1$, $IB_\rho(0) = 0.0$, $IA(0) = 1.0$, $\alpha = 0.65$ cm$^{-1}$, $a = 386$ deg/cm.
All the graphs are plotted for crystal of thickness 1 cm. Input polarization angle is measured with respect to the direction of the \( s \) vector. It has been observed from the graphs, that in optically active crystals by varying input polarization of pump beam we can modify behaviour of signal beam. Without optical activity one cannot control the variation of coupling among different components. One can utilize this modification in various signal processing applications.

By analysing the graphs shown in fig. 2.3(b) and 2.3(c), it can be seen that by a complete rotation of 180 degrees we are getting different components of the signal beam \( B \). This particular feature can be utilized in selecting different component of a beam according to the thickness and optical activity of the crystal.

### 2.5 CONCLUSIONS

It has been shown that by varying the polarization angle of pump beam we are able to control the polarization of the signal beam. This feature is observed in 23 symmetry crystals having a strong effect of optical activity. This feature can be used for various applications related to optical information processing, optical logic gates, spatial light modulators (which is capable of both amplitude and phase modulation therefore acting as a dynamic hologram), etc.

The polarization of the output beam components is also dependent on the thickness of the crystal. The thickness of the crystal should be optimized for the desired output components.
REFERENCES


