CHAPTER 2
MODELS FOR 1/f NOISE AND THEIR EXPERIMENTAL BASIS

Many models have been proposed to explain the 1/f noise phenomenon, but each one has its own merits and setbacks. There is no common agreement among scientific community regarding "whether all observed 1/f noise mechanisms have certain common basis OR whether there exists a common mathematical basis that explains all 1/f noise phenomena." There is a strong feeling amongst the various research groups engaged in noise research that there may exist more than one (OR more) mechanism contributing to observed 1/f noise. This conclusion requires different theories to explain all the experimental facts pertaining to 1/f noise.

2.1. HOOGO'S HYPOTHESIS

In 1969, Hooge[1] established a phenomenological relationship for resistance fluctuations

\[ \frac{S_i}{I^2} = \frac{S_R}{R^2} = \frac{\alpha}{N_{tot}} f \]  

(2.1)

where \( S_i \) is the power spectral density of current fluctuations, \( S_R \) is the power spectral density of the resistance fluctuations both being frequency dependent function, \( I \) and \( R \) are the mean current and mean resistance respectively, \( N_{tot} \) is the total number of charge carriers in the specimen, \( f \) is the frequency and \( \alpha \) is a constant that is only a slowly varying function of the temperature \( T \) called the Hooge's constant and has a value of about \( 2 \times 10^{-3} \). Therefore according to Hooge, 1/f noise stems from resistance fluctuations in a manner shown by Eq.(2.1). The original claim of Hooge that Eq.(2.1) applies to all homogeneous materials received setbacks from the studies of Dutta et.al.[2] and Eberhard and Horn [3]. Later on, Hooge and Vandamme[4] invoked a correction to Eq.(2.1). An excellent study of 1/f noise in electronic devices in terms of the Hooge's parameter was done by Van der Ziel [5]. Van der Ziel's experimental results have been compared with Handel's predictions for [6,7,8] which have been proved Handel's predictions to be correct. The Detailed treatment by Hooge [9] in a simplified form is presented in Chapter 3. However, it is equally important to briefly discuss the findings of others (sec 2.2 to sec 2.4). The objections toward the following approaches are presented in detail [9]. The salient presentation of Hooge [9] provides the necessary details.
Fig 2.1 - Noise spectrum derived from diffusion equation in sample of dimensions $l_1 \times l_2 \times l_3$; and three regions are clearly demarked.

Fig 2.2 - Hybrid model of Voss Clarke representing the three noise regions; the region with constant behaviour below $f_1$, $1/f$ noise region between $f_1$ and $f_2$, and the high frequency region above $f_2$ in which the slope is higher than
2.2. APPROACH OF VOSS AND CLAKE [10, 11]

The thermal fluctuation model was developed by Voss and Clarke who suggested that resistance fluctuations are caused by equilibrium thermal fluctuations. According to the statistical mechanics of the equilibrium state for a canonical ensemble, the average of the energy fluctuations $E$ of a system in contact with a heat bath is given by

$$\langle \delta E \rangle^2 = kT^2/C_v$$  \hspace{1cm} (2.2)

where $k$ is the Boltzmann constant, $T$ is the temperature and $C_v$ is the heat capacity of the system. Since energy fluctuations are connected with temperature fluctuations through $\delta E = C_v \delta T$, we obtain

$$\langle \delta T \rangle^2 = kT^2/C_v$$  \hspace{1cm} (2.3)

Here it should be emphasised that there is no such thing as a 'temperature fluctuation' in a canonical ensemble since the temperature is defined by the thermal reservoir and is therefore fixed. The model of Voss and Clarke is a model for spontaneous, equilibrium enthalpy fluctuations. Treating the temperature of the sample as a fluctuating quantity is an intuitive concept. We choose to retain the concept of Voss and Clarke.

Temperature fluctuations lead to resistance fluctuations through $dR/dT$ which is called the temperature coefficient of resistance of the sample. The mean square resistance fluctuations are given by

$$\langle \delta R \rangle^2 = (dR/dT)^2 \langle \delta T \rangle^2$$  \hspace{1cm} (2.4)

Defining $\beta = R^{-1} (dR/dT)$

$$\langle \delta R \rangle^2 = \beta^2 R^2 kT^2/C_v$$  \hspace{1cm} (2.5)

Thus for the voltage fluctuations

$$\langle \delta V \rangle^2 = V^2 \delta \beta^2 kT^2/C_v$$  \hspace{1cm} (2.6)

Energy being a conserved quantity, the frequency spectrum of the above model can be given by the standard diffusion equation approach. Voss and Clake have used the Langevin diffusion equation

$$\delta T/\delta t = D T + C^{-1} F$$  \hspace{1cm} (2.7)

Where $D$ is the thermal diffusivity, $C$ is the specific heat and $F$ is an uncorrelated random driving term.
On the basis of the above model, Voss and Clarke have constructed a frequency spectrum as follows. For a sample with dimensions $I_1 \times I_2 \times I_3$, four frequency regions have been identified whose spectral shapes are $f_0$, $\ln(1/f)$, $f^{-1/2}$ and $f^{-3/2}$ respectively. This is shown in Fig. (2.1). Therefore the temperature fluctuation spectra in these regions are:

- For $\omega \gg \omega_1$ : $S_R$ or $S_G$ is proportional to $\omega^{-3/2}$
- For $\omega_2 \gg \omega >> \omega_1$ : $S_R$ or $S_G$ is proportional to $\omega^{-1/2}$
- For $\omega_2 >> \omega >> \omega_2$ : $S_R$ or $S_G$ is proportional to $\omega^{-1}$ (constant - $\ln \omega$)
- For $\omega_1 >> \omega$ : $S_R$ or $S_G$ is a constant

As seen, there is no explicit $1/f$ region. Therefore Voss and Clarke constructed a hybrid model spectrum which has a $1/f$ behaviour between the frequencies $\omega_1$ and $\omega_2$. Therefore as seen in Fig. (2.2),

- For $\omega < \omega_1$ : $S_R$ or $S_G$ = constant
- For $\omega_1 < \omega < \omega_2$ : $S_R$ or $S_G$ = $f^{-1}$
- For $\omega > \omega_2$ : $S_R$ or $S_G$ = $f^{-3/2}$

In the $1/f$ region $S$ is a function of $\omega$, retaining equilibrium normalization,

$$\int S d\omega = kT^2 e^{-1}.$$

The resistance fluctuation spectrum $S_R = S_R^1$ was obtained as

$$S_R = R^2 k(\beta T)^2 / \left( C_r [3 + 2 \ln(I_2/I_1) f] \right)$$

Despite the initial success of the model, mainly in the case of $1/f$ noise in metal films in a restricted frequency range, later investigations have led to the conclusion that in general, thermal diffusion is not responsible for the observed spectra. [12,13,14]. This factor is clearly explained in Chapter 3 following Hooge.

2.3. Mc Whorter's Surface Noise Model:

Mc Whorter's surface model [15,16] is based on the idea that the charge carriers in the material interact with traps distributed in the surface oxide layer of the interface by tunneling. The occupancy of the traps therefore fluctuates. This modulates the surface potential and the carrier density in the region close to the surface. Noise is produced when a current passes through such a region in the semiconducting material. Homogeneous concentration of traps in the oxide layer of the semiconductor is presumed (The trapping probability falls exponentially from interface. This
dependence leads to a 1/f noise spectrum. Therefore Mc Whorter's noise model believes in the surface origin of 1/f noise and fluctuations of the number of free charge carriers. This model has been fully accepted for the interpretation of 1/f noise in MOSTs or other devices where the charge transport in regions near the surface predominates. In junction diodes and transistors, Mc Whorter's model is only applicable if the current is controlled by a surface potential.

This model was used in theoretical calculations by Christensson et al. [17] and Berz [18]. The physical mechanism applicable was the quantum tunneling of carriers from the Si/SiO₂ interface to traps located in the oxide. Fu and Sah [19] suggested that direct tunneling of free carriers either from the conduction band or the valance band into the oxide traps is unlikely. Instead they proposed a two step process. The free carriers first communicate with the fast surface states at the interface and then into the oxide traps elasticity resulting low frequency. The studies of Brophy and Rostoker [20] and Kleinepenning [21] favoured the number fluctuation model of Mc Whorter. Discontinuous metal films show extremely high 1/f noise due to tunneling. The 1/f noise of discontinuous metal films is the best representative of the Mc Whorter noise model.

2.4. MOBILITY FLUCTUATION NOISE MODEL:

This theory is an alternative to the number fluctuation theory. The assumption of mobility fluctuation as the origin of 1/f noise was introduced by Hooge [22] in 1972. He suggested that the mobility of a free charge carrier fluctuates as

\[ \frac{S_n}{\mu^2} = \frac{\alpha}{f} \]  

where \( S_n \) is the mobility.

In a review article D.A. Bell [23] argues in favour of mobility fluctuations as follows. Traps cause variation in the effective number of charge carriers. But, from the concept of charge neutrality of the semiconductor, the carriers still exist when trapped and so the number does not vary. On comparing this aspect with the macroscopic definition of mobility (defined in terms of averaged velocity), the immobility of charge carriers by trapping varies the mobility. This shift emphasises fluctuations in mobility rather than the surface effect. The experimental studies of Kleinepenning [21] on 1/f noise in thermal voltage, Voss [24] and Kleinepenning [25] on 1/f noise in Hall voltage, Hooge and Vandamme [26] on 1/f noise in conductance of heavily doped semicon-
ductors, Hooge and Gaal [27] on 1/f noise in electrolytes have all indicated that 1/f noise is a bulk phenomenon and is due to fluctuations in the mobility of free charge carriers.

2.5. OTHER PROMINENT MODELS:

Time and again many models for the explanation of 1/f noise have been proposed. Some of these are based on physical principles like thermodynamic equilibrium [28], movement of charge particles or their interaction with potential barriers [29]. Others make use of pure mathematical and theoretical aspects [30]. Some authors have considered 1/f noise as a random sequence pulses [31-34] while some others have proposed a 1/f noise theory based on the concept of a random-service queue [35-36] and yet a few others have treated 1/f noise as a non-stationary stochastic process [37-39].

CHAPTER 2 - REFERENCES

34 D.A. Bell, Electrical Noise (Van nastrand, London 1960).