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THEORETICAL ISSUES ON FOREIGN CAPITAL INFLOW AND SKILLED-UNSKILLED WAGE GAP IN THE PRESENCE OF INFORMAL SECTOR*

3.1 INTRODUCTION

The increase in skilled-unskilled wage gap in several countries in the last two decades has stimulated the search for explanations for the phenomenon in the economic literature. Various economists have provided different explanations for the increasing wage inequality in several countries. Some economists* believe that with the new wave of technological innovations and change in the structure of labour demand in favour of skilled labour are the main causes behind the growth in wage inequality in the last two decades. Some other causes which have been proposed to explain the increasing inequality are changes in industrial structure, decline of minimum wages (Fortin and Lemieux (1997)) etc. On the other hand, empirical evidences suggest that there exists a strong correlation between trade liberalization and skilled and unskilled wage gap of an economy. A large number of empirical studies have been carried out on this topic, but we find most of the works are confined to the developed countries. Though this issue is particularly important in the context of liberalization of developing countries, only a few studies are conducted for these economies. For example, Robbins (1994a, 1994b, 1995a, 1995b, 1996a, 1996b) and Wood (1997) have conducted some studies for the East Asian and Latin American developing countries on the wage-gap issue. The results of these studies show that liberalization has reduced the wage gap in East Asia but has widened the same in Latin America. Empirical evidences of Republic of Korea, Singapore and Taiwan show that the wage gap of these countries have narrowed down as there was an increase in demand for unskilled as compared to skilled workers. Another possible reason for the narrowing of wage gap in these economies is the post basic education. Following


* See the works of Freeman (1995), Gottschalk and Smeeding (1997).
export oriented industrialization in these nations the expansion of higher education compressed wage differentials. However, in the Latin American countries like Argentina, Chile, Colombia, Mexico, Costa Rica trade liberalization has widened the skilled-unskilled wage gap.

In recent years some attempts have been made to theoretically justify these empirical facts. Feenestra and Hanson (1995) have developed a simple model to show that increase in foreign investment in Mexico has widened the wage gap between the skilled and unskilled workers. Trade liberalization in Mexico causes the price of skill-intensive goods to rise relative to those of non-skill-intensive goods. The price changes reduce the demand for labor in non-skill-intensive industries and increase the demand for labor in skill-intensive industries. The resulting shift in employment toward skill-intensive industries contributes to an increase in the relative demand for skilled workers, which causes their wages to increase relative to those of unskilled workers. Similar results have been derived by Marjit (1998, 1999), Marjit, Broll and Sengupta (2000) and Acharyya and Marjit (2000), Marjit and Acharyya (2003) etc. The authors have argued that for a developing economy like India it is not proper to apply the conventional HOS model. Rather one should consider a hybrid of specific factor model and mobile factor HOS model.

In this chapter we consider the impact of a relatively open trade regime on the skilled-unskilled wage gap in the developing economy. Our purpose here is to focus on some of the structural features of a typical developing economy, to incorporate them in the model and then to look for the consequences of liberalization. One of the most widely observed characteristics of the labour markets in developing countries is its formidable reservoir of unskilled labour employed in the informal segment of the economy. This has been pointed out in numerous papers. But in most of the papers the informal sector has been

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18 It was also pointed out by Wood (1997).
19 In fact, straight jacket application of Stolper-Samuelson Theorem which is inherent in HOS structure gives us results that are contradictory to the empirical facts. Acharyya and Marjit (2000) and Marjit and Acharyya (2003) have argued that in order to capture the fact that India is an exporter of both skilled-intensive product like software and unskilled-intensive product like agricultural good, one should consider a structure which is a hybrid of specific-factor model and HOS type mobile factor model.
considered in a Harris-Todaro (HT) framework (1970), which has been criticized in recent years by a number of authors\(^\text{20}\). The workers migrate from the rural area to the urban area, with an expectation of higher income. But, when they fail to get a job in the formal sector of the economy, they do not return back to the rural sector, instead they search to find a job in the informal sector, though the wage income of informal sector is much less than the rural sector of the economy. To overcome such a drawback, a neo-classical full employment model has been introduced, which has been considered in the present chapter.

In the present chapter we consider a multisectoral neo-classical full employment model where the labour force of the economy is divided into two major categories — skilled and unskilled. Skilled labour is used to produce an exportable and another importable product. The agricultural product producing rural sector and the final good producing informal sector use unskilled labour. The products of both these two sectors are exported. The sector that uses skilled labour and produces an exportable product also uses sector-specific foreign capital as an input. On the other hand, (domestic) formal capital is perfectly mobile between the sector producing importable product and the sector producing agricultural product whereas informal capital is specific to the informal sector of the economy.

After specifying the basic model we consider an alternative version of the model. In the alternative version of the model we assume that the informal sector produces an intermediate product, instead of a final good, on a subcontracting basis\(^\text{21}\) for the sector that uses skilled labour and produces an exportable product. For the sake of simplicity we assume that the informal sector, instead of sector specific capital, uses the same capital as that used by the agricultural sector and the sector producing importable product.

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\(\text{20}\) See the works of Acharyya and Marjit (2000), Chaudhuri and Mukhopadhyay (2002a, b) etc.

\(\text{21}\) Empirical evidences suggest that the informal sector units mostly produce intermediate goods for the formal manufacturing sector. See for example Joshi and Joshi (1976), Bose (1978), Papola (1981), Romatet (1983) and Gupta (2002).
Our purpose here is to examine the impact of liberalization, in the form of an increase in foreign capital inflow, on the skilled-unskilled wage gap and on the level of welfare of the economy in the presence of an informal sector.

The interesting results of this chapter can be summarized in the following manner: In the basic version of our model we find that investment liberalization, in the form of increased foreign capital inflow, widens the skilled-unskilled wage gap of the economy. However, increase in foreign capital inflow increases the level of welfare of the economy. Thus we can say that the Brecher-Alejandro proposition \(^{22}(1977)\) is not valid, as there is a change in the level of welfare even in the absence of tariff. In the alternative version of our model we also find the same result under certain reasonable conditions. In other words, we can conclude that investment liberalization always widens the skilled-unskilled wage gap of a developing economy and increases welfare, irrespective of the fact that the informal sector of the economy produces a final traded good or an intermediate good on a subcontracting basis.

The chapter is organized as follows: The basic model is described in Section 3.2. Section 3.3 deals with some comparative static results related to the impact of foreign capital inflow on the skilled-unskilled wage gap and on the level of welfare of the economy. Section 3.4 describes the alternative version of the model. Section 3.5 of the chapter deals with the comparative static effects related to the alternative version of the model. Finally, the concluding remarks are made in Section 3.6.

\(^{22}\) See the work of Brecher and Alejandro (1977). The Brecher-Alejandro (1977) proposition states that if the import competing sector is capital-intensive and is protected by a tariff, then an inflow of foreign capital in the presence of full repatriation of foreign capital income reduces welfare. However, in the absence of tariff such an inflow of foreign capital leads to no change in welfare. The Brecher-Alejandro (1977) proposition has been examined by various authors in different directions. See the works of Chandra and Khan (1993), Gupta (1994, 1997b) etc.
3.2 THE BASIC MODEL

3.2.1 ASSUMPTIONS OF THE BASIC MODEL

The model considered here is a four-sector neo-classical full employment model, with a final good producing informal sector. We consider a small open economy with three sectors: the formal manufacturing sector using skilled labour, the agricultural (rural) unskilled sector and the exportable final good producing unskilled informal sector. The manufacturing sector using skilled labour is again subdivided into two sectors, one producing exportable product and another producing importable product. The products of one of the manufacturing skilled sector and the agricultural sector are also exported. All the sectors use labour and capital as inputs to produce their products. Skilled labour is perfectly mobile between the two manufacturing sectors whereas unskilled labour is perfectly mobile between the other two sectors, the agricultural and the informal sector. There exists full employment of both types of labour force. The manufacturing sector producing exportable uses sector-specific foreign capital, which is exogenously given. Similarly informal capital is specific for the informal segment of the economy. However, (domestic) formal capital is perfectly mobile between the other two sectors, the manufacturing sector producing importable and the agricultural sector producing exportable product. The small open economy assumption implies that the economy is a price taker in the international market. Thus international prices are given exogenously in our model. Finally, we have the usual assumptions of a neo-classical general equilibrium model like variable coefficient technology, constant returns to scale production function.

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23 In the literature there are many theoretical papers which formalize informal sector as an internationally traded final good producing sector. See the works of Grinols (1991), Chandra and Khan (1993) and Gupta (1997b).

24 The assumption that both the products of one of the skilled manufacturing formal sectors as well as the product of the agricultural sector are exported is quite realistic in the context of a developing economy like India, which is an exporter of both skilled-intensive product like software and unskilled-intensive product like agricultural good.

25 We assume that the rate of return on foreign capital, which the foreign capitalists receive by investing in small open economy, cannot be less than the given world rate of return on foreign capital. However, we also assume that there is control on the entry of foreign capital in the economy. The government of the economy directly regulates it, so that its stock is given exogenously. This is the experience of many developing countries in the context of liberalization. Marjit (1994) has cited the example of Asian tigers in this context. See Gupta and Gupta (1997) for details.
along with diminishing marginal productivity to the variable factors for each sector and competitive market conditions. The product of the agricultural sector is considered in this chapter as the numeraire and the world price of the agricultural product is fixed at unity.

3.2.2 NOTATIONS USED IN THE BASIC MODEL

Let $x_1$ and $x_2$ stand respectively for the manufacturing sectors using skilled labour producing exportable product and importable product. Let $y$ and $z$ stand respectively for the agricultural unskilled sector (producing agricultural product) and the unskilled informal sector (producing final traded good). Other notations of the model can be stated as follows for $j = x_1, x_2, y$ and $z$.

- $P_j$ - world price for the product of $j^{th}$ sector, $j = x_1, x_2, y$ and $z$.
- $a_{ij}$ - quantity of the $i^{th}$ input required for the production of one unit of output of the $j^{th}$ sector; $i = S, K, L, K_F, K_1$ and $j = x_1, x_2, y$ and $z$.
- $\theta_{ij}$ - share of the $i^{th}$ input in the product of the $j^{th}$ sector; $i = S, K, L, K_F, K_1$ and $j = x_1, x_2, y$ and $z$ (actually it is the value of $i^{th}$ input in the value of the $j^{th}$ product)
- $\lambda_{ij}$ - share of $i^{th}$ input used in the sector $j$ out of total endowment of that particular input; $i = S, K, L, K_F, K_1$ and $j = x_1, x_2, y$ and $z$.
- $S$ - stock of skilled labour
- $L$ - stock of unskilled labour
- $K$ - stock of (domestic) formal capital
- $K_F$ - stock of foreign capital
- $K_1$ - stock of (domestic) informal capital.
- $W_S$ - wage rate of skilled labour
- $W$ - wage rate of unskilled labour
- $r$ - rate of return on (domestic) formal capital
- $r_F$ - rate of return on foreign capital
- $r_1$ - rate of return on (domestic) informal capital
- $X_1$ - level of output of sector $x_1$
- $Y$ - level of output of sector $y$
3.2.3 THE EQUATIONAL STRUCTURE OF THE BASIC MODEL

The equational structure of the basic model is stated as follows.

The competitive equilibrium conditions are given by the following four equations:

\[ P_{x1} = a_{sx1}W_s + a_{Krx1}r_F \]  \hspace{0.5cm} (3.1)
\[ P_{x2} = a_{sx2}W_s + a_{Kx2}r \]  \hspace{0.5cm} (3.2)
\[ 1 = a_{Ly}W + a_{Ky}r \]  \hspace{0.5cm} (3.3)
\[ P_z = a_{lZ}W + a_{Kz}r_1 \]  \hspace{0.5cm} (3.4)

The product of the agricultural sector is considered as the numeraire and its price has been set equal to unity.

Factor market equilibrium conditions are given by the following equations:

\[ a_{sx1}X_1 + a_{sx2}X_2 = S \]  \hspace{0.5cm} (3.5)
\[ a_{Krx1}X_1 = K_F \]  \hspace{0.5cm} (3.6)
\[ a_{Kx2}X_2 + a_{Ky}Y = K \]  \hspace{0.5cm} (3.7)
\[ a_{lY}Y + a_{lZ}Z = L \]  \hspace{0.5cm} (3.8)
\[ a_{Kz}Z = K_1 \]  \hspace{0.5cm} (3.9)

Equations (3.5) to (3.9) imply that there exists full employment in the factor markets.

In this model we have nine equations with nine endogenous variables- \( W_s, W, r_F, r, r_1, X_1, X_2, Y \), and \( Z \). Thus the system is determinable. It is to be noted that due to the assumption of variable coefficient technology the input-output coefficients for any
particular sector are functions of factor prices (of the factors used by the particular sector)\textsuperscript{26}.

The working of the model can be explained in the following manner. From equation (3.1) we find that, given \( P_{x1} \), \( r_F \) is a decreasing function of \( W_s \). Again, from equation (3.2) we find that, given \( P_{x2} \), \( W_s \) is a decreasing function of \( r \). Thus it can be concluded that both \( r_F \) and \( W_s \) are functions of \( r \). The decomposability property is not applicable here\textsuperscript{27}.

i.e. \( r_F = r_F(r) \) and \( W_s = W_s(r) \) where \( (\partial r_F / \partial r) > 0 \) and \( (\partial W_s / \partial r) < 0 \)

Using equation (3.6), equation (3.5) can be written as

\[
(a_{x1} / a_{Kx1}) K_F + a_{sx2} X_2 = S
\]

\[
\Rightarrow \Psi(r) K_F + a_{sx2} X_2 = S \tag{3.5.1}
\]

where \( \Psi = (a_{sx1} / a_{Kx1}) \). As \( r_F = r_F(r) \) and \( W_s = W_s(r) \) we can write

\[
a_{sx1} = a_{sx1} (W_s / r_F) = a_{sx1} (r) \quad \text{and} \quad a_{Kx1} = a_{Kx1} (W_s / r_F) = a_{Kx1} (r)
\]

Since \( a'_{sx1} > 0 \) and \( a'_{Kx1} < 0 \)

Therefore \( \Psi = \Psi(r) \) where \( \Psi'' > 0 \).

The combination of \( r \) and \( X_2 \) which maintains equilibrium in the market for skilled labour is given by the SS locus, in figure 3.1. The slope of this locus, as obtained from equation (3.5.1) is given by

\[
(dr/dX_2) \bigg|_{SS} = -a_{sx2} / (K_F \Psi'' + X_2 a_{sx2}')
\]

Since the input-output ratio is positive and the function \( \Psi \) along with the input-output ratio \( a_{sx2} \) varies directly with \( r \), therefore the curve SS is negatively sloped. This is shown in figure 3.1.

Given the price of the rural sector as unity and also given \( P_{sz} \), we can write from equations (3.3) and (3.4) \( W = W(r) \) and \( r_1 = r_1 (W(r)) \) respectively. Thus, \( r_1 = r_1 (W(r)) = r_1 (r) \). Here,

\textsuperscript{26} Thus, for example \( a_{ly} = a_{ly} (W/r) \). It is to be noted that \( a_{ly} \) can be interpreted as demand for labour to produce one unit of the product of sector 'y'. As (input) demand functions are homogeneous of degree zero we can write \( a_{ly} = a_{ly} (W/r) \).

\textsuperscript{27} Decomposability property implies that the factor prices are determined independently of capital stocks and labour endowments. See Corden and Findlay (1975) in this context.

\textsuperscript{28} One can consider the unit demand function as \( a_{sx2} = a_{sx2} (W_s(r)/r) \). Now, as \( r \) decreases, \( W_s \) increases, therefore \( (W_s/r) \) increases due to which the producers will substitute labour by capital, which in turn will increase the demand for capital and decrease the demand for labour. Thus, \( a_{sx2} > 0 \).
(\partial W/\partial r)<0 and (\partial r_1/\partial r)>0. Substituting equation (3.9) in equation (3.8), Y can be derived\textsuperscript{29} as a decreasing function of r. Thus equation (3.7) can be rewritten as

\begin{equation}
Y = a_{Kx2}(r)X_2 + a_{Ky}(r)Y(r) = K
\end{equation}

The locus of r and X_2, which maintains the equilibrium of the domestic capital market, is given by the KK curve in figure 3.1. The slope of this curve as obtained from equation (3.7.1) is given by

\begin{equation}
\frac{dr}{dX_2} \bigg|_{KK} = -\frac{a_{Kx2}}{X_2 a_{Kx2} + Ya_{Ky} + a_{Ky}Y'}
\end{equation}

Since the input–output ratios are positive and the functions a_{Kx2}, a_{Ky} and Y vary inversely with r, the curve KK is positively sloped. This is shown in figure 3.1.

The intersection of SS and KK locus in figure 3.1 gives us the equilibrium values of r and X_2. In other words, r and X_2 can be determined from equations (3.5.1) and (3.7.1), when solved simultaneously. Once r is known, all other factor prices and the input-output coefficients are known. It is to be noted that W_s, r_f, W and r_1 can be determined with the help of equations (3.1), (3.2), (3.3) and (3.4), once the value of r is determined. Again when X_2 is determined, one can easily determine X_1 from equation (3.5) and Y from equation (3.7). When Y is known, Z can be determined easily from equation (3.8). This completes the working of our model.

\textsuperscript{29} From equation (3.9) we get \( Z = K_1/a_{Klz} \), substituting the value of Z in equation (3.8) we get

\begin{align*}
& a_{ly}(W(r)/r)Y + a_{lz}(W(r)/r_1 (W(r))) \{K_l/a_{Klz} (W(r)/r)\} = L \\
\Rightarrow & a_{ly}(r)Y + \{a_{lz}(r)/a_{Klz}(r)\} K_l = L
\end{align*}

Since L and K_l are labour endowment and informal capital stock, they are given exogenously. Therefore Y is a function of r only i.e. \( Y = f(r) \)

Again \( a'_{ly}, a'_{lz}>0 \) and \( a'_{Klz}<0 \), which implies \( \delta (a_{lz}(r)/a_{Klz}(r))/\delta r > 0 \), therefore \( (\delta Y / \delta r < 0) \)
Figure 3.1
Before going to comparative statics, it is important to mention that our measure of welfare in this small open economy is the NI measured at world prices\textsuperscript{30}, \( \Omega \), which is expressed as follows

\[
\Omega = WL + W_S + rK + r_1K_1
\]  

(3.10)

Here we have assumed that foreign capital income is fully repatriated. In equation (3.10), \( WL \) is the total wage income of the unskilled labourers, who are employed in the rural sector and the informal sector. \( W_S \) is the total wage income of the skilled labourers employed in the manufacturing sectors. \( rK \) and \( r_1K_1 \) are the rental income from the domestic and informal capital respectively.

### 3.3 THE COMPARATIVE STATIC EFFECTS

In this section of the chapter, our main focus is to analyze the impact of liberalization on the skilled- unskilled wage gap of the economy. It is captured through an increase in the stock of foreign capital inflow into the economy. With larger inflow of foreign capital into the economy, the level of output of the sector using it increases. For given \( r \) and hence for given input-output ratios \( a_{Kx1}, a_{Sx1} \) and \( a_{Sx2} \), an increase in \( K_F \) implies an increase in the level of output of sector \( X_1 \). As the endowment of skilled labour force, \( S \), is given it implies a contraction of \( X_2 \). In other words, from equation (3.5.1) we find that for given \( r \), an increase in \( K_F \) implies a reduction in \( X_2 \) i.e. a leftward shift of \( SS\) locus. However there will be no effect on the \( KK\) curve. Hence a new equilibrium has been obtained, where both \( r \) and \( X_2 \) fall. As a result of decrease in \( r \), both \( W \) and \( W_S \) rise (as \( P_{x2} \) and \( P_f \) are given). Hence from competitive equilibrium conditions (3.1) and (3.4) we find that both \( r_F \) and \( r_1 \) fall and in order to determine the impact of liberalization on the skilled-unskilled wage gap of the economy, we have to examine the movement of \( (W_S/W) \) with a rise in \( K_F \) i.e. \( (\dot{W}_S - \dot{W})/\dot{K}_F \)

\textsuperscript{30} Here NI can be considered as a proxy for social welfare. See for example, Chaudhuri (2001a, 2001b, 2003), Chaudhuri and Mukhopadhyay (2002a, 2002b), Gupta (1994, 1997) etc. It has been explained in details in chapter 2, section 2.3 of the study.
Taking total differentiation of equations (3.2) and (3.3) and using envelope condition\textsuperscript{31} then dividing equations (3.2) and (3.3) by $P_{x_2}$ and $P_y$ respectively we get

$$
\hat{P}_{x_2} = \theta_{sx_2} \hat{W}_s + \theta_{Kx_2} \hat{r}
$$

(3.2.1)

$$
\hat{P}_y = \theta_{ly} \hat{W} + \theta_{Ky} \hat{r}
$$

(3.3.1)

where $\theta_{ij}$ implies the share of $i^{th}$ input in the product of $j^{th}$ sector or $\theta_{ij}$ implies the value of $i^{th}$ input in the value of $j^{th}$ product (for example $\theta_{sx_2} = (W_s S_{x_2} / P_{x_2} X_2)$. We denote $\hat{k} = dk/k$ for $k = P_{x_2}, P_y, r$ etc.

Putting $\hat{P}_{x_2} = \hat{P}_y = 0$ (as $P_{x_2}$ and $P_y$ are given due to the small open economy assumption), the equations can be further transformed as (for detailed derivations see appendix 3.1)

$$
\hat{W}_s = -(\theta_{Kx_2} / \theta_{sx_2}) \hat{r}
$$

(3.2.2)

$$
\hat{W} = -(\theta_{Ky} / \theta_{ly}) \hat{r}
$$

(3.3.2)

Subtracting equation (3.3.2) from equation (3.2.2) we get

\textsuperscript{31} To explain the envelope condition for illustrative purpose let us first consider the production function for sector 'x'. In sector 'x', output depends on skilled labour and capital i.e. $X_2 = f(S_{x_2}, K_{x_2})$. Using the assumption of CRS we can write $\lambda X_2 = f(\lambda S_{x_2}, \lambda K_{x_2})$. Without loss of generality we consider $\lambda = (1 / X_2)$ and get $1 = f(a_{sx_2}, a_{Kx_2})$. We refer to the corresponding isoquant as the unit isoquant.

The slope of the unit isoquant is $-(da_{Kx_2} / da_{sx_2})$ and when we minimize unit cost subject to unit isoquant the cost-minimizing condition is $-(da_{Kx_2} / da_{sx_2}) = (W_s / r)$. From a different angle we can interpret that producers in each industry choose techniques of production so as to minimize unit costs. This leads to the condition that the distributive-share weighted average of changes in input-output coefficients along the unit isoquant in each industry must vanish near the cost-minimizing point. This states that an iso-cost line is tangent to the unit isoquant. In mathematical terms we can analyze it in terms of sector 'x'.

$$
dP_{x_2} = a_{sx_2} dW_s + a_{Kx_2} dr + W_s da_{sx_2} + r da_{Kx_2}.
$$

Now, cost minimizing point for given unit isoquant implies $-(da_{Kx_2} / da_{sx_2}) = (W_s / r)$ or $W_s da_{sx_2} + r da_{Kx_2} = 0$. Thus, $W_s da_{sx_2} + r da_{Kx_2} = 0$ implies the cost minimizing condition. Again, $W_s da_{sx_2} + r da_{Kx_2} = 0 \Rightarrow \theta_{sx_2} \hat{a}_{sx_2} + \theta_{Kx_2} \hat{a}_{Kx_2} = 0$. It implies that the distributive-share weighted average of changes in input-output coefficients along the unit isoquant for each industry must vanish near the cost-minimizing point. This is the envelope condition. Using the envelope condition we can write for sector 'x', $dP_{x_2} = a_{sx_2} dW_s + a_{Kx_2} dr$ or $\hat{P}_{x_2} = \theta_{sx_2} \hat{W}_s + \theta_{Kx_2} \hat{r}$.

Similarly, for sector 'y' we can write $\hat{P}_y = \theta_{ly} \hat{W} + \theta_{Ky} \hat{r}$. The envelope conditions are widely used in general equilibrium trade models. For details see Caves, Frankel and Jones (1990), pg 732-738.
\[(W_s - W) = \hat{\theta}[({\theta_y / \theta}_{I_y}) - ({\theta_xz / \theta}_{sxz})]\]

\[\Rightarrow (W_s - W) / K_F = (\hat{\theta} / \hat{K}_F) [(\theta_{Ky / \theta}_{I_y}) - (\theta_{Kxz / \theta}_{sxz})]\]

In an economy where agricultural sector is backward and manufacturing sector is highly capital intensive we would expect the value of \((\theta_{Ky / \theta}_{I_y})\) to be very low and the value of \((\theta_{Kxz / \theta}_{sxz})\) to be very high. In such an economy we would expect

\[(\theta_{Ky / \theta}_{I_y}) < (\theta_{Kxz / \theta}_{sxz})\]

\[\Rightarrow (\hat{\theta} / \hat{K}_F) (0 ; (W_s - W) / \hat{K}_F) > 0.\]

Thus it can be concluded that, under some reasonable conditions, increase in foreign capital inflow raises the skilled-unskilled wage gap of the economy.

We now consider the impact of foreign capital inflow on the NI of the economy when the foreign capital income is fully repatriated\(^{32}\). It is given by

\[\Omega = WL + W_S S + r K + r_1 K_I\]

(3.10)

Differentiating equation (3.10) with respect to \(K_F\) we get

\[(d\Omega / dK_F) = L(dW / dK_F) + (dW_S / dK_F) + K (dr / dK_F) + K_I (dr_1 / dK_F)\]

We can rewrite it as

\[(d\Omega / dK_F) = L(dW / dr_1)(dr_1 / dK_F) + S (dW_S / dr) (dr / dK_F) + K (dr_1 / dK_F) + K_I (dr / dK_F)\]

Using the Shephard-Samuelson relations\(^{33}\) we find from competitive equilibrium conditions

\[(dW / dr_1) = - (a_{Ki2 / a_{L2}}) \text{ and } (dW_S / dr) = - (a_{Kx2 / a_{sx2}})\]

Therefore

\[(d\Omega / dK_F) = L (dr_1 / dK_F) [K_I / L] - (a_{Ki2 / a_{L2}}) + S (dr / dK_F) [K / S] - (a_{Kx2 / a_{sx2}})\]

\(^{32}\) In case of a small open economy, NI or factor income is considered as a measure of welfare. See footnote 30. See also the works of Gupta and Gupta (1997), Gupta (1997b) etc.

\(^{33}\) Actually these are same as that of envelope conditions. See Dixit and Norman (1980).
As \( \frac{d\rho}{dK_F} < 0 \) and \( \frac{dr}{dK_F} < 0 \)

Under sufficient conditions\(^{34} \) \((K / S) < (a_{K_{z2}} / a_{S_{z2}})\) and\(^{35} \) \((K_1 / L) < (a_{K_{z2}} / a_{L_{z2}})\)
we find that \( \frac{d\Omega}{dK_F} > 0 \)

Hence we find that under the given conditions, an increase in foreign capital inflow increases the level of NI (welfare) of the economy.

We summarize our results in the form of following proposition.

**Proposition 3.1:** Trade liberalization in the form of an increase in the foreign capital inflow in an economy raises the skilled- unskilled wage gap of the economy. Such an inflow also increases the level of NI (welfare) in the presence of informal sector and full repatriation of foreign capital income under some reasonable conditions.

Finally, we are interested to examine the impact of foreign capital inflow on the output of the informal sector. We have shown that as a result of an increase in \( K_F \), both \( r \) and \( r_F \) falls. Hence from equation (3.9) it implies that \( a_{K_{z2}} \) increases. Given the stock of informal capital, \( K_1 \), we can thus conclude that the output of the informal sector falls.

We can thus write the following proposition

**Proposition 3.2:** In an economy with skilled – unskilled division of workforce an increase in foreign capital inflow reduces the output of the informal sector when it produces internationally traded final good.

\(^{34} \) This assumption is feasible in the context of a developing economy, like India, where we generally expect \( a_{K_{z2}} \) to be quite high and methods of production are highly capital intensive. It is to be noted that \( K \) is shared between sectors ‘x’ and ‘y’. So it is reasonable to assume that unit requirement of capital per unit of skilled labour is higher than the average capital-skilled labour endowment of the economy.

\(^{35} \) As \( (a_{K_{z2}} / a_{L_{z2}}) = (a_{K_{z2}} Z / a_{L_{z2}} Z) = (K_1 / a_{L_{z2}} Z) \) and as \( L > a_{L_{z2}} Z \), we can write \( (K_1 / L) < (a_{K_{z2}} / a_{L_{z2}}) \)
3.4. **THE ALTERNATIVE VERSION OF THE MODEL**

3.4.1 **ASSUMPTIONS AND THE EQUATIONAL STRUCTURE OF THE ALTERNATIVE VERSION OF THE MODEL**

Informal sectors of a developing economy mostly produce non-traded intermediary products for the formal sectors of the economy, instead of final traded goods. Empirical evidences also support this fact.³⁶ In order to capture this characteristic of developing countries, we have modified the basic version of our model and tried to examine the impact of trade liberalization on the skilled-unskilled wage gap of the economy, within this framework. In this version of our model one can assume that the product produced by the informal segment of the economy is used both by sectors ‘x₁’ and ‘x₂’, as an intermediate product. But for the sake of simplicity, we assume that the product of sector ‘z’ is used as an intermediate input by sector ‘x₁’ to produce its product. Thus, sector ‘x₁’ uses skilled labour, foreign capital and intermediate product, ‘z’, to produce its product. It implies that the competitive equilibrium condition of sector ‘x₁’ is modified, but the competitive equilibrium conditions of the other two sectors, ‘x₂’ and ‘y’ remains the same (as given by equations (3.2), and (3.3) respectively). The modified competitive equilibrium of sector ‘x₁’ in the extended version of the model is expressed as

\[ P_{x_1} = a_{x_1} W_S + a_{Kx_1} r_F + a_{Zx_1} P_z \]  

(3.1.1)

To simplify matters, in the present model we assume that the informal sector uses the same domestic capital that is used by the formal part of the economy, instead of sector specific capital. We refer to it as domestic capital. Equation (3.4) of the basic model is thus transformed as

\[ P_z = a_{l_z} W + a_{Kz} r \]  

(3.4.1)

³⁶ In India, for example, many of the large industries like leather bag and shoe manufacturing industries, garments industries etc. use intermediate inputs, which are produced by the informal sectors of the economy. For example, in Kolkata the informal segment of the economy carries out leather-tanning process for the shoe and bag manufacturing industries. Similarly, for the garment industry the dyeing and stitching of garments are done by the informal sector of the economy on a subcontracting basis.
Equation (3.7) of the basic model is thus modified in the extended version of the model, as domestic capital is perfectly mobile among sector ‘y’, ‘x2’ and ‘z’ instead of ‘y’ and ‘x2’.

\[ a_{Ky}Y + a_{Kz}Z + a_{Kx2}X_2 = K \]  
(3.7.2)

Since the product Z is a non-traded intermediate product (used on a subcontracting basis), its price, \( P_z \), is endogenously determined and its demand-supply equation is given as

\[ a_{z1}X_1 = Z \]  
(3.9.1)

Again for the sake of simplicity it is assumed that the unit requirement of informal sector product by the formal sector, ‘x1’, is fixed i.e. the input-output coefficient, \( a_{z1} \) is fixed.\(^{37}\)

Thus in the extended version of the model, we have nine equations, (3.1.1), (3.2), (3.3), (3.4.1), (3.5), (3.6), (3.7.2), (3.8) and (3.9.1), with nine endogenous variables \( r, r_F, W_S, W, P_z, X_1, Y, Z \) and \( X_2 \) which implies the system is determinable. It is to be noted that in this version of the model we cannot determine the factor prices separately from the competitive equilibrium conditions. In other words, this version of the model implies an indecomposable structure. We need to solve for the above mentioned nine endogenous variables from the above mentioned nine equations.

\(^{37}\) It implies that fixed amount of the product of the sector ‘z’ is needed as an intermediate product by the sector ‘x1’ to produce one unit of its product. It rules out the possibility of substitution between the non-traded intermediary and other factors of production in sector ‘x1’. It is a reasonable assumption from the point of view of the nature of subcontracting between the formal and informal firms, as experienced in India. In industries like shoe making and garments, large formal sector firms shift their production to small informal sector firms under the system of sub contracting. So the production is done in the informal sector firms while the formal sector firms do packaging and marketing. One pair of shoes produced in the informal sector does not change in quantity when it is marketed by the formal sector as a final commodity. Thus there remains a fixed proportion between the use of intermediary and the quantity of final commodity produced and marketed by the formal sector.
3.5 THE COMPARATIVE STATIC EFFECTS OF THE ALTERNATIVE VERSION OF THE MODEL

In context of this model, we want to examine the impact of trade liberalization i.e. larger foreign capital inflow into the economy on the level of skilled-unskilled wage gap. In other words, we want to examine the movement of \( (W_S / W) \) as a result of increase in \( K_F \), i.e. \( (\hat{W}_S - \hat{W}) / \hat{K}_F \).

Differentiating equations (3.3) and (3.4.1) and then using envelope conditions and hat mathematics, we get (for detailed derivations see appendix 3.2, subsection 3.2.1)

\[
\hat{W} = -(\theta_{K_y} / \theta_{Lu}) \hat{P}_z
\]

\[
\hat{F} = (\theta_{Ly} / \theta_{Lu}) \hat{P}_z
\]

where \( |\theta| = \{(\theta_{K_x} / \theta_{Lu}) - (\theta_{K_y} / \theta_{Ly})\} \)

In a developing economy like India, the agricultural sector has already experienced mechanization in the form of Green Revolution whereas the informal sector is mainly a labour absorbing sector. Thus it can be assumed that \( (\theta_{K_x} / \theta_{Lu}) < (\theta_{K_y} / \theta_{Ly}) \). It implies that the capital-labour ratio of sector ‘z’ is less than the capital-labour ratio of sector ‘y’ both in physical and value terms.

Therefore \( |\theta| < 0 \).

Similarly the values of \( \hat{W}_S \) and \( \hat{F} \) are obtained from equations (3.2) and (3.1.1) respectively, after differentiating and using envelope condition and hat mathematics, as follows

\[
\hat{W}_S = -\{(\theta_{Kx2} \theta_{Ly}) / (\theta_{Sx2})\} \hat{P}_z
\]

\[
\hat{F}_S = -\{(\theta_{Sx1} \theta_{Sx2} \theta_{Kx2} \theta_{Ly}) / (\theta_{Sx2} \theta_{Kx2})\} \hat{P}_z
\]

This can be viewed in the Punjab area of India, where the agricultural sector has already experienced mechanization in the form of Green Revolution.
We have already expressed the change in factor prices in terms of the change in the price of the product of non-traded informal sector. We now want to express the gap between change in the levels of wage rates (both skilled and unskilled) and the rates of return on capital (domestic and foreign) in terms of the change in price of the product of the non-traded informal sector. The purpose is that once we can predict the movement of the price of the product of the non-traded informal sector as a result of foreign capital inflow it will help us to predict the movement of factor prices along with the movements in the gaps between change in wage rates and rates of return on capital.

\[(\hat{W}_s - \hat{r}) = -((\theta_{Kx2}\theta_{Ly})/(\theta_{Sx2}) - (\theta_{Ly})/(\theta_{Sx2}))\hat{P}_z\]

\[= (-\hat{P}_z \theta_{Ly})/(\theta_{Sx2})
= B\hat{P}_z\]

where \(B = ((-\theta_{Ly})/(\theta_{Sx2})\}

and

\[(\hat{W}_s - \hat{r}_f) = -((\theta_{Kx2}\theta_{Ly})/(\theta_{Sx2}))\hat{P}_z + ((\theta_{Z1}\theta_{Sx2}|\theta| - \theta_{Sx1}\theta_{Kx2}\theta_{Ly})/(\theta_{Sx2}\theta_{KFx1}))\hat{P}_z\]

\[= A\hat{P}_z\]

where

\[A = -((\theta_{Kx2}\theta_{Ly}\theta_{KFx1} - \theta_{Z1}\theta_{Sx2}|\theta| + \theta_{Sx1}\theta_{Kx2}\theta_{Ly})/(\theta_{Sx2}\theta_{KFx1}))\]

and

\[(\hat{W} - \hat{r}) = -\hat{P}_z (\theta_{Kx} + \theta_{Ly})\}/\theta|

\[= -\hat{P}_z /|\theta|\]

\(A > 0, B > 0\) as \(|\theta| < 0\)

After total differentiation of equation (3.8) we get

\[\lambda_{Ly}\hat{Y} + \lambda_{lz}\hat{Z} = -(\lambda_{ly}\hat{\lambda}_{Ly} + \lambda_{lz}\hat{\lambda}_{lz})\]  \hspace{1cm} (3.8.1)

where \(\lambda_{ij}\) implies the share of \(i\)th input used in sector \(j\) out of total endowment of that particular input.
Assuming that the production function of sector y is Cobb-Douglas\textsuperscript{39} production function, we can write the elasticity of substitution as
\[ \sigma_y = \frac{\hat{a}_{Ky} - \hat{a}_{Ly}}{(\hat{W} - \hat{r})} = 1 \]

Applying envelope condition to equation (3.3), (detailed derivations are available in appendix 3.2, subsection 3.2.2), we get
\[ \hat{a}_{Ky} = -\left(\frac{\theta_{Ky}}{\theta_{Ky}}\right)\hat{a}_{Ly} \]

Putting the value of \( \hat{a}_{Ky} \) in the equation showing the expression for \( \sigma_y \), we get
\[ (\hat{W} - \hat{r})\theta_{Ky} = -\hat{a}_{Ly} \]

Similarly, the value of \( \hat{a}_{Lz} \) can be obtained from equation (3.4.1) as
\[ -\hat{a}_{Lz} = \theta_{Kz}(\hat{W} - \hat{r}) \]

Substituting the values of \( \hat{a}_{Ky} \) and \( \hat{a}_{Lz} \) in the equation (3.8.1), the equation can be further transformed as
\[ \lambda_{ly}\dot{Y} + \lambda_{lz}\dot{Z} = (\lambda_{ly}\theta_{Ky} + \lambda_{lz}\theta_{Kz})(\hat{W} - \hat{r}) \] (3.8.2)

Substituting the value of \( (\hat{W} - \hat{r}) \) in equation (3.8.2), the equation can be finally transformed as (see appendix 3.2, subsection 3.2.2 for detailed derivations)
\[ \lambda_{ly}\dot{Y} + \lambda_{lz}\dot{Z} = -(\lambda_{ly}\theta_{Ky} + \lambda_{lz}\theta_{Kz})\dot{P}_z/|\theta| \] (3.8.3)

Similarly by differentiating equation (3.5) and by incorporating the value \( \hat{a}_{S_{x1}} \) and \( \hat{a}_{S_{x2}} \) (as obtained from equations (3.1.1) and (3.2)) (see appendix 3.2, subsection 3.2.3, for detailed derivations), we get

\textsuperscript{39} This is just a simplifying assumption. See Chaudhuri and Gupta (2004). Elasticity of substitution of the production function is given as
\[ \sigma_y = \frac{\hat{a}_{Ky} - \hat{a}_{Ly}}{(\hat{W} - \hat{r})} = 1 \] when the production function is Cobb–Douglas.

Even if we do not consider a Cobb-Douglas production function we get the same results that we have derived in this chapter. The assumption of Cobb-Douglas production function makes our algebra much simpler.
\[ \lambda_{\text{Sr1}} \hat{X}_1 + \lambda_{\text{Sr2}} \hat{X}_2 = \lambda_{\text{Sr1}} \theta_{K_{F1}} (W_S - \hat{r}_F) + \lambda_{\text{Sr2}} \theta_{K_{K2}} (W_S - \hat{r}) \quad (3.5.2) \]

Similarly by putting the values of \((\hat{W}_S - \hat{r}_F)\) and \((\hat{W}_S - \hat{r})\) in equation (3.5.2) we get

\[ \begin{align*}
\lambda_{\text{Sr1}} \hat{X}_1 + \lambda_{\text{Sr2}} \hat{X}_2 &= (\lambda_{\text{Sr1}} \theta_{K_{F1}} A + \lambda_{\text{Sr2}} \theta_{K_{K2}} B) \hat{P}_z \\
&= C \hat{P}_z 
\end{align*} \quad (3.5.3) \]

where \( C = (\lambda_{\text{Sr1}} \theta_{K_{F1}} A + \lambda_{\text{Sr2}} \theta_{K_{K2}} B) \)

\( C > 0 \) as \( A > 0 \) and \( B > 0 \).

Using equation (3.6) we can solve for \( \hat{X}_1 \) and then by substituting the value of \( \hat{X}_1 \) in equation (3.5.3), the above equation can be further transformed and from that we can express \( \hat{X}_2 \) in terms of \( \hat{P}_z \) (see appendix 3.2, subsection 3.2.4 for details). The expression for \( \hat{X}_2 \) is as follows

\[ \hat{X}_2 = \left( C \hat{P}_z - \lambda_{\text{Sr1}} (K_F - \theta_{\text{Sr1}} A \hat{P}_z) \right) / \lambda_{\text{Sr2}} \]

Again by differentiating equation (3.7.2) it can be written as (detailed derivations are shown in appendix 3.2, subsection 3.2.5)

\[ \lambda_{K_y} \hat{Y} + \lambda_{K_z} \hat{Z} + \lambda_{K_{K2}} \hat{X}_2 = (\hat{P}_z / \theta) (\lambda_{K_y} \theta_{L_y} + \lambda_{K_z} \theta_{L_z}) - \lambda_{K_{K2}} \theta_{\text{Sr2}} B \hat{P}_z \quad (3.7.3) \]

Pre multiplying equation (3.8.3) by \( \lambda_{K_y} \) and equation (3.7.3) by \( \lambda_{L_z} \) and then subtracting equation (3.7.3) from (3.8.3) after some mathematical manipulation and by incorporating the value of \( \hat{X}_2 \) (see appendix 3.2, subsection 3.2.6)

we get

\[ \hat{Z} | \lambda = \alpha \hat{P}_z - ((\lambda_{\text{Sr1}} \lambda_{K_{K2}} \lambda_{L_y} \hat{K}_F) / \lambda_{\text{Sr2}}) \quad (3.12) \]

where

\[ \alpha = [D / \theta] + E + ((\lambda_{K_{K2}} \lambda_{L_y} C) / (\lambda_{\text{Sr2}})) + ((\lambda_{\text{Sr1}} \lambda_{K_{K2}} \lambda_{L_y} \theta_{\text{Sr1}} A) / (\lambda_{\text{Sr2}})) \]

\[ D = -(\lambda_{L_y} \lambda_{K_y} \theta_{K_y} + \lambda_{K_z} \lambda_{K_y} \theta_{K_z} + \lambda_{K_z} \lambda_{l_y} \theta_{L_z} + \lambda_{K_{K2}} \lambda_{L_y} \theta_{L_y}) \]

\[ E = (\lambda_{K_{K2}} \lambda_{L_y} B) \]

\[ | \lambda = (\lambda_{K_y} \lambda_{L_z} - \lambda_{K_{K2}} \lambda_{L_y}) \]
As \((\lambda_K / \lambda) > (\lambda_K / \lambda)\) i.e. sector ‘y’ is more capital intensive than sector ‘z’.

Therefore \(|\lambda| > 0\)

On the basis of the assumption that the agricultural sector of the economy is much more capital intensive than the informal sector of the economy, it can also be concluded that \(|\theta| < 0\) and \(A > 0\). Since \(|\theta| < 0\) and all \(\theta_{ij}\)s and \(\lambda_{ij}\)s are positive

Therefore \(B > 0\), \(C > 0\), \(E > 0\) and \(D < 0\).

Since \(a_{xz1}\) is fixed, it implies \(\hat{X}_1 = \hat{Z} \). Thus, equation (3.6) implies \(\hat{Z} = (\hat{K}_F - \hat{a}_{KF1})\). It can be shown (see appendix 3.2, subsection 3.2.4) that \(\hat{a}_{KF1}\) can be expressed as \(\theta_{st1}A\hat{P}_z\)

Therefore by incorporating the value of \(\hat{Z} = (\hat{K}_F - \theta_{st1}A\hat{P}_z)\) in equation (3.12) we get (see subsection 3.2.7 of appendix 3.2 for detailed derivations)

\[
\begin{align*}
\hat{P}_z / \hat{K}_F &= \{[|\lambda| + (\lambda_{st1}\lambda_{xz2}\lambda_{st2})/(\lambda_{st2})]/\{[|\theta|\alpha + |\lambda|\theta_{st1}A]\} \\
(\hat{P}_z / \hat{K}_F) &= 0 \\
\end{align*}
\]

Thus, as a result of foreign capital inflow price of the product of informal non-traded intermediate good sector rises.

Putting the value of \(\hat{P}_z\) in the expression (as obtained from equation (3.3)) we get (one can see appendix 3.2, subsection 3.2.8 for the detailed derivations)

\[
\begin{align*}
\hat{W}/\hat{K}_F &= -[\theta_{st2}\{[|\lambda| + (\lambda_{st1}\lambda_{xz2}\lambda_{st2})/(\lambda_{st2})]/\{[|\theta|\alpha + |\lambda|\theta_{st1}A]\} \\
\end{align*}
\]

Similarly, by incorporating the value of \(\hat{P}_z\) in the expression (as obtained from equation (3.2)) we get

\[
\begin{align*}
\hat{W}_x / \hat{K}_F &= -[(\theta_{xz2}\theta_{st2})/\{[|\theta|\alpha + |\lambda|\theta_{st1}A]\}] \\
\end{align*}
\]

Subtracting \((\hat{W}/\hat{K}_F)\) from \((\hat{W}_x / \hat{K}_F)\) we get

\[
\begin{align*}
= G\{(\theta_{st2}\theta_{xz2} - \theta_{xz2}\theta_{st2})/\{[|\theta|\alpha + |\lambda|\theta_{st1}A]\} \\
\end{align*}
\]
where \( G = \left[ \left\{ \lambda \right\} + (\lambda_{Kx2} \lambda_{Ly})/(\lambda_{Ly2}) \right\} / \{ \alpha + [\lambda_{Sx1} A] \} \right) \\

Since \(|\lambda| > 0, \alpha > 0, A > 0, |\theta| < 0 \)
Therefore \( G > 0 \)
Thus
\[
(\hat{W}_s - \hat{W})/\hat{K}_F \geq 0
\]
if \((\theta_{Kx2}/\theta_{Sx2}) > (\theta_{Ky}/\theta_{Ly})\)

In this model, it has been assumed that the sector 'x2' uses skilled labour to produce its product whereas the sector 'y', which produces agricultural product, absorbs the unskilled labour to produce its product\(^{40}\). The assumption \((\theta_{Kx2}/\theta_{Sx2}) > (\theta_{Ky}/\theta_{Ly})\) implies that for sector 'x2' capital per unit of skilled labour is higher than capital per unit of unskilled labour for sector 'y'. Generally the sector that is dependent on skilled labour invests more on capital than the sector that is dependent on unskilled labour. In other words, capital used per unit of skilled labour is generally higher than the capital used per unit of unskilled labour. The reason is that the skilled workers are more trained and also more familiar, compared to that of the unskilled workers, to work with modern machineries and equipments. So the sector that uses skilled labour finds it profitable to invest more on capital equipments per unit of skilled labour than the sector that uses unskilled labour. This is true in both physical and value terms. Under this assumption it can be concluded that the skilled-unskilled wage gap of the economy rises with an increase in the foreign capital inflow.

We now consider the impact of foreign capital inflow on NI (welfare) when the foreign capital income is fully repatriated. It is given by
\[
\Omega = WL + rK + SW_s \\
(3.13)
\]
Differentiating equation (3.13) with respect to \(K_F\) we get
\[
(d \Omega / dK_F) = L (dW/dK_F) + K (dr/dK_F) + S (dW_s/dK_F)
\]
\(^{40}\) It is to be noted that here the capital-intensities of the two sectors cannot be directly compared as the two sectors use different types of labour as inputs. However, one can infer about the values of the capital-intensities on the basis of empirical facts.
\[ \frac{d \Omega}{d K_F} = (dr/dK_F) [K + L \frac{dW}{dr} + S \frac{dW_s}{dr}] \]

Since we already know from competitive conditions (using Shephard-Samuelson relations) that

\[ (dW/dr) = - (a_{K_y}/a_{L_y}) \] and \[ (dW_s/dr) = - (a_{Kx2}/a_{Sx2}) \]

therefore

\[ \frac{d \Omega}{d K_F} = (dr/dK_F) [K - L \frac{(a_{K_y}/a_{L_y})}{K_y/L_y} - S \frac{(a_{Kx2}/a_{Sx2})}{Kx2/Sx2}] \]

\[ = (dr/dK_F) [K - \{(K_y/L_y) L^+ (Kx2/Sx2) S\}] \]

\[ = (dr/dK_F) [K - \{(K_y/K)/(L_y/L)\} - K \{(Kx2/K)/(Sx2/S)\}] \]

\[ = (dr/dK_F) K \{1 - \{\lambda_{K_y}/\lambda_{L_y} + \lambda_{Kx2}/\lambda_{Sx2}\}\} \]

\[ = (dr/dK_F) K \{(\lambda_{L_y}(\lambda_{Sx2} - \lambda_{Kx2}) - \lambda_{K_y} \lambda_{Sx2})/(\lambda_{L_y} \lambda_{Sx2})\} \]

Thus \( \frac{d \Omega}{d K_F} > 0 \) if \( \lambda_{Sx2} < \lambda_{Kx2} \), as \( (dr/dK_F) < 0 \).

As it is empirically quite realistic to assume that for the manufacturing sector, which produces importable, the share of domestic capital in total endowment of (domestic) capital is higher than the share of skilled labour in total skilled labour endowment.

We summarize the results in the form of following proposition

**Proposition 3.3:** Trade liberalization in the form of foreign capital inflow in an economy widens the skilled–unskilled wage gap of the economy, in the presence of a non-traded informal sector that produces an intermediate product on a subcontracting basis, if the following assumptions are satisfied (i) the rural sector of the economy is much more capital intensive than that of informal sector and (ii) sector 'x' uses more capital per unit of skilled labour as compared to the capital used by sector 'y' per unit of unskilled labour. The foreign capital inflow with full repatriation of foreign capital income also raises the level of welfare of the economy in the presence of informal sector if \( \lambda_{Kx2} > \lambda_{Sx2} \).
3.6 CONCLUDING REMARKS

This chapter has analyzed the impact of liberalization on the skilled-unskilled wage gap as well as on the level of welfare of the economy, in the presence of an informal sector. It has been considered both as final goods producing and as intermediate goods producing sector.

In the first part of the model we have considered that the informal sector produces a final good so that its price is internationally determined. While in the second part of the chapter the informal sector produces an intermediate product for the formal sector, \( x_1 \) so that its price is endogenously determined. In either case we observe that liberalization widens the skilled-unskilled wage gap of the economy and raises welfare under reasonable conditions.

Some authors have analyzed the problem of impact of foreign capital inflow on skilled-unskilled wage gap. The importance of the present exercise is that we have introduced the role of the informal sector to analyze the problem. For example Ghosh and Gupta (2001) have considered a HT framework and analyzed the same problem in the absence of informal sector and obtained opposite results. However for a developing economy one cannot ignore the role of informal sector. Once it is introduced one can obtain interesting results as well as empirically valid results for less developed countries. Our model can be considered as a generalization of Acharyya and Marjit's (2000) model. Here we have considered the issue of foreign capital inflow and also both final goods producing and intermediate goods producing informal sectors. Acharyya and Marjit (2000) have considered only intermediate goods producing informal sector. Moreover, in our study we have considered a four-sector division of the economy to capture more clearly the features of a developing economy.

Our study would have been more interesting if both final goods producing and intermediate goods producing sectors would have been considered simultaneously to analyze the problem. Apart from this one can take into account of the issue of skill
formation of the unskilled labour force in the presence of informal sector. We want to take all these issues in our future research agenda.
APPENDICES

APPENDIX 3.1

Derivation of the sign of \((\dot{W}_S - \dot{W})/\dot{K}_F) > 0\)

Totally differentiating equation (3.5.1) and (3.7.1) and rearranging in matrix form we get

\[
\begin{pmatrix}
\psi' K_F + a'_{Sx2} X_2 \\
X_2 a'_{Kx2} + a'_{Ky} Y + a_{Ky} Y'
\end{pmatrix}
\begin{pmatrix}
\frac{dr}{dK_F} \\
\frac{dX_2}{dK_F}
\end{pmatrix}
= \begin{pmatrix}
-\psi dK_F \\
0
\end{pmatrix}
\]

The value of determinant matrix is given by

\[
\Delta = \psi' K_F a_{Kx2} + a'_{Sx2} X_2 a_{Kx2} - a'_{Kx2} X_2 a_{Sx2} - a'_{Ky} Y a_{Sx2} - a_{Ky} Y' a_{Sx2}
\]

Since all the input-output ratios are positive and \(\psi' > 0, a'_{Sx2} > 0, a'_{Kx2} < 0, a'_{Ky} < 0\) and \(Y' < 0\).

Therefore

\(\Delta > 0\).

Now using Cramer’s rule we have

\[
\frac{dr}{dK_F} = - \frac{\{a_{Kx2}\}}{\Delta}
\]

or \(\frac{dr}{dK_F} < 0\) i.e. \((\dot{r} / \dot{K}_F) < 0\)

Similarly

\[
\frac{dX_2}{dK_F} = \frac{\{a'_{Kx2} X_2 + a'_{Ky} Y + a_{Ky} Y'\}}{\Delta}
\]

or \(\frac{dX_2}{dK_F} < 0\)

In order to examine the impact of liberalization on the skilled-unskilled wage gap of the economy \((W_S / W)\), we have to compare the change in \(W_S\) and \(W\) due to increase in \(K_F\).

i.e. \(\log W_S - \log W = \log y\)
Totally differentiating equation (3.2) and (3.3), using the envelope conditions and then dividing the equations by $P_{x2}$ and $P_y$ respectively we get

$$(dP_{x2} / P_{x2}) = (a_{sx2} Ws / P_{x2}) (dWs / Ws) + (a_{Kx2} r / P_{x2}) (dr / r)$$

$$(dP_y / P_y) = (a_{ly} W / P_y) (dW / W) + (a_{Ky} r / P_y) (dr / r)$$

It is to be noted that in the above derivations, using the envelope conditions we get

$$Ws da_{sx2} + r da_{Kx2} = 0$$

and

$$W da_{ly} + r da_{Ky} = 0$$

Again, since $P_{x2}$ and $P_y$ are exogenously given

$$(dP_{x2} / P_{x2}) = (dP_y / P_y) = 0$$

Therefore the above equations can be transformed as

$$0 = \theta_{sx2} \hat{W}_s + \theta_{Kx2} \hat{r}$$

$$0 = \theta_{ly} \hat{W} + \theta_{Ky} \hat{r}$$

where

$$\hat{W}_s = (dW_s / W_s), \hat{W} = (dW / W)$$

and $\hat{r} = (dr / r)$

Thus we get

$$\hat{W}_s = -(\theta_{Kx2} / \theta_{sx2}) \hat{r}$$

and

$$\hat{W} = -(\theta_{Ky} / \theta_{ly}) \hat{r}$$

These expressions are same as obtained in the equations (3.2.2) and (3.3.2) of the present chapter.

Subtracting $\hat{W}$ from $\hat{W}_s$, we get

$$(\hat{W}_s - \hat{W}) = \hat{r} [(\theta_{Ky} / \theta_{ly}) - (\theta_{Kx2} / \theta_{sx2})]$$

$$(\hat{W}_s - \hat{W}) / \hat{K}_F = (\hat{r} / \hat{K}_F) [(\theta_{Ky} / \theta_{ly}) - (\theta_{Kx2} / \theta_{sx2})]$$

From the sufficient condition

$$(\theta_{Ky} / \theta_{ly}) < (\theta_{Kx2} / \theta_{sx2})$$

and

$$\left( \hat{r} / \hat{K}_F \right) < 0$$

(already proved)
we get 

\((\hat{W}_s - \hat{W}) / \hat{K}_F \rangle 0\)

APPENDIX 3.2

3.2.1 Derivation of the expressions for \(\hat{W}, \hat{r}, \hat{r}_F, (\hat{W}_s - \hat{r}_F), (\hat{W}_s - \hat{r})\) and \((\hat{W} - \hat{r})\)

Differentiating equation (3.3) and equation (3.4.1), using the envelope conditions and also using hat mathematics we can rewrite the equations as

\[
0 = \theta_{ly} \hat{W} + \theta_{Kz} \hat{r} \\
\hat{P}_z = \theta_{lz} \hat{W} + \theta_{Kz} \hat{r} \\
\text{(Since } P_y \text{ is exogenously given, therefore } P_y = 0)\]

By Cramer’s rule we get

\[
\hat{W} = -\left(\frac{\theta_{Kz}}{\theta_{ly}} \hat{P}_z\right) \\
\text{and} \\
\hat{r} = \left(\frac{\theta_{lz}}{\theta_{ly}} \hat{P}_z\right)
\]

where

\[
|\theta| = \left(\theta_{ly} \theta_{Kz} - \theta_{Kz} \theta_{lz}\right) \\
= \left(\theta_{lz} / \theta_{ly}\right) - \left(\theta_{Kz} / \theta_{ly}\right)
\]

Since the rural sector is more capital intensive than the informal sector therefore \((\theta_{Kz} / \theta_{lz}) < \left(\theta_{Kz} / \theta_{ly}\right)\)

i.e. \(|\theta| < 0\)

Similarly differentiating equation (3.2), using the envelope condition and also using hat mathematics we get

\[
\theta_{zs2} \hat{W}_s + \theta_{Kz2} \hat{r} = 0
\]
Putting the value of \( \dot{r} \) (as derived above) we get
\[
\dot{W}_s = -\{(\theta_{x21}\theta_{x2y})/(\theta|\theta_{x2})\} \dot{P}_z
\]

Similarly from equation (3.1.1) and by using the value of \( \dot{W}_s \) (as obtained above) we get
\[
\dot{r}_F = -\{(\theta_{x21}\theta_{x2y})/(\theta_{x2})\} \dot{P}_z
\]
By using the value of \( \dot{W}_s, \dot{r}_F \) and \( \dot{r} \) we can calculate
\[
(\dot{W}_s - \dot{r}_F) = -\{(\theta_{x21}\theta_{x2y} - \theta_{x21}\theta_{x21}\theta_{x2y})/(\theta_{x2})\} \dot{P}_z + ((\theta_{x21}\theta_{x2y} - \theta_{x21}\theta_{x21}\theta_{x2y})/(\theta_{x2})\} \dot{P}_z
\]

\[
= -\{(\theta_{x21}\theta_{x2y} - \theta_{x21}\theta_{x21}\theta_{x2y}) + \theta_{x21}\theta_{x21}\theta_{x2y})/(\theta_{x2})\} \dot{P}_z
\]

\[
= A \dot{P}_z
\]
where
\[
A = -\{(\theta_{x21}\theta_{x2y} - \theta_{x21}\theta_{x21}\theta_{x2y}) + \theta_{x21}\theta_{x21}\theta_{x2y})/(\theta_{x2})\}
\]

therefore \( A > 0 \) as \( |\theta| < 0 \)

and
\[
(\dot{W}_s - \dot{r}_F) = -\{(\theta_{x21}\theta_{x2y})/(\theta_{x2})\} \dot{P}_z - \{(\theta_{x21}\theta_{x2y})/(\theta_{x2})\} \dot{P}_z
\]

\[
= (\dot{P}_z \theta_{x2y})/(\theta_{x2}) = B \dot{P}_z
\]

where
\[
B = -(\theta_{x2y})/(\theta_{x2}) \text{ as } |\theta| < 0
\]

therefore \( B > 0 \)

and
\[
(\dot{W} - \dot{r}) = -\{(\theta_{x2y} + \theta_{x2y})/(\theta_{x2})\} \dot{P}_z = -\dot{P}_z |\theta|
\]

3.2.2 Derivation of the expression numbered (3.8.3) in the present chapter

Differentiating equation (3.8) we get
\[
\lambda_2 \dot{\hat{y}} + \lambda_2 \dot{\hat{z}} = -(\lambda_2 \dot{\hat{a}}_{ly} + \lambda_2 \dot{\hat{a}}_{l2}) \tag{3.8.1}
\]
where \( \lambda_{ij} \) represent the share of \( i^{th} \) input used in sector \( j \) out of total endowment of that particular input. We know that the elasticity of substitution of the sector ‘y’ is given by
\[ \sigma_y = (\hat{a}_{Ky} - \hat{a}_{Ly})(\hat{W} - \hat{r}) \]

or \[ \sigma_y(\hat{W} - \hat{r}) = (\hat{a}_{Ky} - \hat{a}_{Ly}) \]

Applying envelope theorem, equation (3.3) can be rewritten as

\[ Wda_Ly + rda_{Ky} = 0 \]

or \[ \theta_{Ly} \hat{a}_{Ly} + \theta_{Ky} \hat{a}_{Ky} = 0 \]

Therefore \[ \hat{a}_{Ky} = -\frac{\theta_{Ly}}{\theta_{Ky}} \hat{a}_{Ly} \]

Thus \[ \hat{a}_{Ky} - \hat{a}_{Ly} = -\left( \frac{\theta_{Ly}}{\theta_{Ky}} \right) \hat{a}_{Ly} - \hat{a}_{Ly} \]

\[ = -\hat{a}_{Ly} \left( \frac{\theta_{Ly} + \theta_{Ky}}{\theta_{Ky}} \right) \]

\[ = -\left( \frac{\theta_{Ly}}{\theta_{Ky}} \right) \]

Replacing \( -\left( \frac{\theta_{Ly}}{\theta_{Ky}} \right) \) in place of \( (\hat{a}_{Ky} - \hat{a}_{Ly}) \) in the expression of elasticity of substitution of sector 'y', we get

\[ \sigma_y(\hat{W} - \hat{r}) = \frac{\theta_{Ly}}{\theta_{Ky}} \]

therefore \[ \sigma_y(\hat{W} - \hat{r}) \theta_{Ky} = -\hat{a}_{Ly} \]

Since the production function is Cobb-Douglas production we have \( \sigma_y = 1 \)

Thus,

\[ (\hat{W} - \hat{r}) \theta_{Ky} = -\hat{a}_{Ly} \]

Similarly with the help of equation (3.4.1) we get the value of \( \hat{a}_{Lz} \)

\[ -\hat{a}_{Lz} = \theta_{Kz} (\hat{W} - \hat{r}) \]

Substituting the values of \( \hat{a}_{Ly} \) and \( \hat{a}_{Lz} \) in equation (3.8.1) we get

\[ \lambda_{Ly} \hat{Y} + \lambda_{Lz} \hat{Z} = \left( \lambda_{Ly} \theta_{Ky} + \lambda_{Lz} \theta_{Kz} \right) (\hat{W} - \hat{r}) \]  \hspace{1cm} (3.8.2)

Putting the value of \( (\hat{W} - \hat{r}) \) (as derived earlier) we get
\[ \lambda_{t_y} \dot{y} + \lambda_{t_z} \dot{z} = -\{ (\lambda_{l_y} \theta_{k_y} + \lambda_{l_z} \theta_{k_z}) \hat{P}_z \} / |\theta| \]  

(3.8.3)

3.2.3. Derivation of the expression numbered (3.5.3)

Similarly by differentiating equation (3.5) and by incorporating the values of \( \hat{a}_{sr1} \) and

\( \hat{a}_{sr2} \) (as obtained from equation (3.1.1) and (3.2)) we get

\[ \lambda_{sr1} \dot{x}_1 + \lambda_{sr2} \dot{x}_2 = \lambda_{sr1} \theta_{k_{r1}} (\hat{W}_s - \hat{r}_f) + \theta_{k_{r2}} \lambda_{sr2} (\hat{W}_s - \hat{r}_f) \]  

(3.5.2)

Putting the values of \( (\hat{W}_s - \hat{r}_f) \) and \( \hat{W}_s - \hat{r}_f \) in equation (3.5.2) we get

\[ \lambda_{sr1} \dot{x}_1 + \lambda_{sr2} \dot{x}_2 = (\lambda_{sr1} \theta_{k_{r1}} A + \theta_{k_{r2}} \lambda_{sr2} B) \hat{P}_z \]

\[ = CP_z \]  

(3.5.3)

where \( C = (\lambda_{sr1} \theta_{k_{r1}} A + \theta_{k_{r2}} \lambda_{sr2} B), A = - \{(\theta_{k_{r2}} \theta_{l_y} \theta_{k_{r1}} - \theta_{z_{r1}} \theta_{z_{r2}} |\theta| + \theta_{sr1} \theta_{k_{r2}} \theta_{l_y}) / (\theta_{sr2} |\theta| \theta_{k_{r1}}) \} \)

and \( B = \{- (\theta_{l_y}) / (\theta_{sr2} |\theta|) \} \)

since \( A > 0 \) and \( B > 0 \) (as \( |\theta| < 0 \))

therefore \( C > 0 \)

3.2.4. Derivation of the value of \( \dot{x}_2 \).

From equation (3.6) we get

\[ \hat{a}_{k_{r1}} + \dot{x}_1 = \dot{K}_F \]

Therefore

\[ \dot{x}_1 = \dot{K}_F - \hat{a}_{k_{r1}} \]

Thus by replacing the value of \( \dot{x}_1 \) in equation (3.5.3) we get
\[ \lambda_{s_{x_1}}(\hat{K}_F - \hat{a}_{KFx_1}) + \lambda_{s_{x_2}} \hat{X}_2 = C \hat{P}_z \]

Since \( \hat{a}_{KFx_1} = \theta_{s_{x_1}}(\hat{W}_S - \hat{r}_F) \) using the definition of (partial) elasticity of substitution between \( K_F \) and \( S \) for sector 'x_1' and assuming the value of the elasticity of substitution to be one we get from appendix 3.2, subsection (3.2.1) that \( (\hat{W}_S - \hat{r}_F) = A \hat{P}_z \). Thus, \( \hat{a}_{KFx_1} = \theta_{s_{x_1}} A \hat{P}_z \). So we can rewrite the expression \( \lambda_{s_{x_1}}(\hat{K}_F - \hat{a}_{KFx_1}) + \lambda_{s_{x_2}} \hat{X}_2 = C \hat{P}_z \) as follows

\[ \lambda_{s_{x_1}}(\hat{K}_F - \theta_{s_{x_1}} A \hat{P}_z) + \lambda_{s_{x_2}} \hat{X}_2 = C \hat{P}_z \]

\[ \hat{X}_2 = \{C \hat{P}_z - \lambda_{s_{x_1}}(\hat{K}_F - \theta_{s_{x_1}} A \hat{P}_z)\} / \lambda_{s_{x_2}} \]

3.2.5. Derivation of the expression numbered (3.7.3) in the present chapter

Differentiating equation (3.7.2) we get

B = \lambda_{K_y} \hat{Y} + \lambda_{K_z} \hat{Z} + \lambda_{Kx_2} \hat{X}_2 = -\lambda_{K_y} \dot{a}_{K_y} - \lambda_{K_z} \dot{a}_{K_z} - \lambda_{Kx_2} \dot{a}_{Kx_2}

By putting the values of \( \dot{a}_{K_y} \), \( \dot{a}_{K_z} \) and \( \dot{a}_{Kx_2} \) as obtained from equation (3.3), (3.4.1) and (3.2) in the above equation, we get

\[ \lambda_{K_y} \hat{Y} + \lambda_{K_z} \hat{Z} + \lambda_{Kx_2} \hat{X}_2 = -((\hat{W} - \hat{r})(\lambda_{K_y} \theta_{l_y} + \lambda_{K_z} \theta_{l_z}) + \lambda_{Kx_2} \theta_{s_{x_2}} (\hat{W}_S - \hat{r})) \]

Incorporating the values of \( (\hat{W} - \hat{r}) \) and \( (\hat{W}_S - \hat{r}) \) (see appendix 3.2, subsection 3.2.1) in the above equation, we get

\[ \lambda_{K_y} \hat{Y} + \lambda_{K_z} \hat{Z} + \lambda_{Kx_2} \hat{X}_2 = \{\hat{P}_z (\lambda_{K_y} \theta_{l_y} + \lambda_{K_z} \theta_{l_z})\} / \theta - \lambda_{Kx_2} \theta_{s_{x_2}} B \hat{P}_z \]

(3.7.3)

3.2.6. Derivation of the expression numbered (3.12) in the present chapter

Multiplying equation (3.8.3) by \( \lambda_{K_y} \) we get

\[ \lambda_{K_y} \lambda_{l_y} \hat{Y} + \lambda_{K_y} \lambda_{l_z} \hat{Z} = -((\lambda_{K_y} \lambda_{l_y} \theta_{K_y} + \lambda_{K_y} \theta_{K_z} \lambda_{l_z}) \hat{P}_z) / \theta \]

(3.8.4)
Multiplying equation (3.7.3) by \( \lambda_{ly} \) we get

\[
\lambda_{ky} \lambda_{ly} \dot{y} + \lambda_{kz} \lambda_{ly} \dot{z} + \lambda_{kx2} \lambda_{ly} \dot{x}_2 = \{ \dot{P}_z (\lambda_{ky} \lambda_{ly} \theta_{ly} + \lambda_{ly} \lambda_{kz} \theta_{lz}) \} / \theta - \lambda_{ly} \lambda_{kx2} \theta_{sz2} B \dot{P}_z (3.7.4)
\]

Subtracting equation (3.7.4) from equation (3.8.4) we get

\[
\lambda_{ky} \lambda_{ly} \dot{y} + \lambda_{ky} \lambda_{lz} \dot{z} - \lambda_{ly} \lambda_{ky} \dot{y} - \lambda_{kz} \lambda_{ly} \dot{z} - \lambda_{kx2} \lambda_{ly} \dot{x}_2
\]

\[
= (\dot{P}_z / \theta) [-\lambda_{ky} \lambda_{ly} \theta_{ky} - \lambda_{ky} \lambda_{lz} \theta_{lz} - \lambda_{ly} \lambda_{ky} \theta_{ly} - \lambda_{ly} \lambda_{kz} \theta_{lz}] + \lambda_{ly} \lambda_{kx2} \theta_{sx2} B \dot{P}_z
\]

or

\[
\dot{z} | A - \lambda_{ly} \lambda_{kx2} \dot{x}_2 = D(\dot{P}_z / \theta) + E \dot{P}_z
\]

(3.11)

where

\[
| \lambda | = [\lambda_{ky} \lambda_{lz} - \lambda_{ly} \lambda_{kz}]
\]

\[
D = [-\lambda_{ky} \lambda_{ly} \theta_{ky} - \lambda_{ky} \lambda_{lz} \theta_{lz} - \lambda_{ly} \lambda_{ky} \theta_{ly} - \lambda_{ly} \lambda_{kz} \theta_{lz}]
\]

\[
E = \lambda_{ly} \lambda_{kx2} \theta_{sx2} B
\]

Since \( B > 0, (\lambda_{ky} / \lambda_{lz}) > (\lambda_{kz} / \lambda_{lz}) \), therefore \( |\lambda| > 0, D < 0 \) and \( E > 0 \).

Putting the value of \( \dot{x}_2 \) in equation (3.11) we get

\[
\dot{z} | A = D(\dot{P}_z / \theta) + E \dot{P}_z + \{ (\lambda_{ly} \lambda_{kx2} \mathcal{C}_2 \dot{P}_z) / \lambda_{sx2} \} - \{ \lambda_{sx1} \lambda_{kx2} \lambda_{ly} \theta_{sx1} A \} / \lambda_{sx2} \} K_F + \{ (\lambda_{sx1} \lambda_{kx2} \lambda_{ly} \theta_{sx1} A) / \lambda_{sx2} \} \dot{K}_F
\]

\[
= \dot{P}_z [D / \theta] + E + \{ (\lambda_{ly} \lambda_{kx2} \mathcal{C}) / (\lambda_{sx2}) \} + \{ \lambda_{sx1} \lambda_{kx2} \lambda_{ly} \theta_{sx1} A / (\lambda_{sx2}) \} \dot{K}_F
\]

\[
\dot{z} | A = a \dot{P}_z - \{ \lambda_{sx1} \lambda_{kx2} \lambda_{ly} / \lambda_{sx2} \} \dot{K}_F
\]

(3.12)

where \( a = [D / \theta] + E + \{ (\lambda_{ly} \lambda_{kx2} \mathcal{C}) / (\lambda_{sx2}) \} + \{ \lambda_{sx1} \lambda_{kx2} \lambda_{ly} \theta_{sx1} A / (\lambda_{sx2}) \} \)

Since \( D < 0, E > 0, C > 0, A > 0 \) and \( |\theta| < 0 \), therefore \( a > 0 \)

3.2.7. Derivation of the sign of the expression \( (\dot{P}_z / \dot{K}_F) \)

Since \( \dot{a}_{z1} \) is fixed, therefore \( \dot{a}_{z1} = 0 \)
Thus $Z = \hat{X}$ (from equation (3.9.1))

By replacing the value $Z = (\hat{K}_F - \theta_{Sx1} A \hat{P}_z)$ in equation (3.12) we get

$$\hat{K}_F |\lambda| - |\lambda| \theta_{Sx1} A \hat{P}_z = \alpha \hat{P}_z - \{(\lambda_{Sx1} \lambda_{Kx2} \lambda_{iy}) / \lambda_{Sx2}\} \hat{K}_F$$

Therefore

$$\frac{\hat{P}_z}{\hat{K}_F} = \frac{[|\lambda| + (\lambda_{Sx1} \lambda_{Kx2} \lambda_{iy}) / (\lambda_{Sx2}) / \{\alpha + |\lambda| \theta_{Sx1} A\}]}{\hat{K}_F}$$

Since $|\lambda| > 0$, $\alpha > 0$ and $\Lambda > 0$

Therefore $(\frac{\hat{P}_z}{\hat{K}_F}) > 0$

### 3.2.8. Derivation of the sign of the expression

$(\hat{W}_s - \hat{W}) / \hat{K}_F$

Now we consider the value of $(\hat{W} / \hat{K}_F)$

Putting the value of $\hat{P}_z$ in the expression $\hat{W} = -(\theta_{Ky} / |\theta|) \hat{P}_z$ (as obtained from equation (3.3), we get

$$\hat{W} = -\{(\theta_{Ky} / |\theta|) [|\lambda| + (\lambda_{Sx1} \lambda_{Kx2} \lambda_{iy}) / (\lambda_{Sx2}) / \{\alpha + |\lambda| \theta_{Sx1} A\}] \hat{K}_F$$

Therefore $(\hat{W} / \hat{K}_F) = -\{(\theta_{Ky} / |\theta|) [|\lambda| + (\lambda_{Sx1} \lambda_{Kx2} \lambda_{iy}) / (\lambda_{Sx2}) / \{\alpha + |\lambda| \theta_{Sx1} A\}]$)

Similarly we get

$(\hat{W}_s / \hat{K}_F) = -\{(\theta_{Kx2} \theta_{iy}) / |\theta| \theta_{Sx2}) [\{|\lambda| + (\lambda_{Sx1} \lambda_{Kx2} \lambda_{iy}) / (\lambda_{Sx2}) / \{\alpha + |\lambda| \theta_{Sx1} A\}]$)

Subtracting $(\hat{W} / \hat{K}_F)$ from $(\hat{W}_s / \hat{K}_F)$, we get

$$(\hat{W}_s - \hat{W}) / \hat{K}_F = (\hat{W}_s / \hat{K}_F) - (\hat{W} / \hat{K}_F)$$

$$= [\{\lambda| + (\lambda_{Sx1} \lambda_{Kx2} \lambda_{iy}) / (\lambda_{Sx2}) / \{\alpha + |\lambda| \theta_{Sx1} A\}]$$

$[-\{(\theta_{Kx2} \theta_{iy}) / |\theta| \theta_{Sx2}) + \theta_{Ky} / |\theta|}]$

$$= G[\{\theta_{Sx2} \theta_{Kx} - \theta_{Kx2} \theta_{iy}\} / |\theta| \theta_{Sx2})]$$
where $G = \frac{\|\lambda\| + (\lambda_{S1} \lambda_{K2} \lambda_{L2})/(\lambda_{S2})}{(\alpha + |\lambda|\theta_{S1} A)}$

since $|\lambda| > 0, \alpha > 0, A > 0$

therefore $G > 0$ and $|\theta| < 0$

In this economy it has already been assumed that

$$(\theta_{K2} / \theta_{S2}) > (\theta_{K2} / \theta_{L2})$$

Therefore $$(\hat{W}_s - \hat{W}) / \hat{K}_r > 0$$