CHAPTER I

INTRODUCTION

In the fascinating modern computerized world where we face a lot of complicated problems, there is no definite or exact solution as there are hundreds and thousands of variables which will go to decide the exact solution to the problem. But an approximate solution can always be attempted using the ‘priors’ in such a way that these solutions can be of some practical use to the decision maker. Human beings have to make decisions at each and every stage of life. For instance, the expected length of human life may be required for a hospital to give an advertisement in a local newspaper to build up its image. We all know that the health and human life depends upon several variables, some of which are indicated here:

(i) Food intake
(ii) Physical exercise
(iii) Type of occupation
(iv) Sleeping hours
(v) Number of children
(vi) Parents age/grand parents age and
(vii) Walking speed etc.,

The above variables are only indicative and not exhaustive and therefore capturing all the relevant and related variables will be nearly impossible and to get the exact solution regarding the exact life of a particular human being, one may take several years to collect and study these variables and still one may not get to reach a satisfactory solution and this is where ‘priors’ come handy to give better solutions to these problems. In real life, one need not waste one’s time in collecting all the details of a ‘human’ to tell the expected length of life but it is sufficient if the prior information about the earlier humans who lived in similar/identical conditions will
give the approximate solution to this ‘expected age’ estimation problem. Thus, we can
gain a lot by properly choosing the priors, their types and their behavioral parameters
and adopting to the given situation or problem, so that some approximate solution
becomes available to the problem, as problems are normally complicated. Some such
complicated problems are:

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<tr>
<th>Problem Area</th>
<th>Prior information that can be chosen to solve it</th>
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| Oil discovery      | 1. Previous wells drilled in that area.  
                      2. The structural pattern of the sub-surface.  
                      3. The depth at which oil was discovered earlier or in the recent past. |
| Rocket launching   | 1. Earlier experience of the scientists with in the country.  
                      2. Experiences of other countries in similar launches.  
                      3. Type of scientists deployed for the project etc.,                              |
| Fish population    | 1. Already established fish breeding rate in the world.  
                      2. Fish availability in similar sea-depths in other places.  
                      3. Ratio between availability and catch ability.                                  |

We can see from the above examples that the given problems can be solved
with an acceptable approximation using earlier/prior information related to
similar/identical cases. Thus, if prior information is properly selected and applied, we
can get solutions to each and every real life problem though it is approximate and not
exact but the cost-effectiveness is very high in these approximate solutions as getting
the exact solution will not only take several hours days, months and years but also a
lot of expenses will be incurred in the collection of fresh and complete data/details
where as in the case of prior information, the cost and time involved will be
abysmally low rather it will be negligible.
Thus, all approximate solutions needed for real life problems can be obtained through Bayesian methodology which is based upon prior information and such prior information can be transformed into distributions and treated as prior distributions and then one can use the Bayesian logic to find posterior solutions. Let us now see the literature on the type of prior distributions and their behavior in a detailed manner in chapter II which will now form the basis of the approximate posterior solutions as explained here.

Modern computer world and real life problems; solutions to real life problems and statistical methods; statistical methods and statistical inference; statistical inference and Bayesian inference; Bayesian inference and prior information are all inseparable in today’s problem solving context. Therefore prior information and the modern computerized world are inseparable. Now the problem of obtaining a reliable solution to real life problems lies in the selection of the right kind of prior information and thereby the prior distribution. Prior information is one that is available before sampling which is obtained through expert opinion, advice, intuitive decision, hunch, prediction etc., Bayesian-priors (or) statistical priors view of the subjectivist is that the prior represents the “belief” of the statistician or “expert knowledge” pertaining to the measurements which reduces the ambiguity in problem-solving and thus setting the stage for a heated debate between “pure Bayesian” and “pure frequentists” concerning the philosophical merits of either school of thought within statistics. The subjectivists argue that the prior information emanates from one’s personal beliefs. Kass and Wasserman (1996) have mentioned that “subjectivism has become the dominant philosophical foundation for Bayesian inference. Most Bayesian analyses are performed with the so-called “Non-informative” priors that is, priors constructed by some formal rule”. “The prior distribution is a key part of Bayesian Inference (Bayesian methods and modeling) and represents the information about an uncertain parameter \( \theta \). When that is combined with the probability distribution of a new data, it yields the posterior distribution which in turn is used for future inferences and decisions involving \( \theta \)” this was reported by Gelman (2002).
Thus, prior information (or) prior probability distribution can be described as given below

(i) Prior distribution is one that would express one’s uncertainty about the population parameter ‘p’ before the data are taken into account.

(ii) Prior information of an uncertainty or uncertain quantity p, where p is the proportion of voters who will vote for the politician named G.V. RAJ in an election.

(iii) Prior is an attribute to uncertainty rather than randomness to the uncertain quantity.

(iv) Any useful information available for decision before sampling.

(v) Results of non Bayesian observations.

(vi) Prior is often the purely subjective assessment of an experienced expert. Some will choose a conjugate when they can make computation of posterior easier.

(vii) Assessment of the most likelihood of parameters.

(viii) It is common to assume a uniform distribution over the appropriate range of values for the prior (parameters) distribution.

(ix) Prior parameter values are chosen to reflect the existing belief of information.

(x) Prior information may be the information converted from past data relating to the uncertainty of a parameter.

In general, prior distributions can be classified on the basis of its nature, Structure and the characteristics for example, Informative, Non-informative, Uninformative, Weakly informative and Strongly informative and also Proper and Improper priors.

First and foremost, Laplace (1812) described the Non-informative prior of the parameter of normal sequence like measure of prior ignorance about a parameter and was reported by Jeffreys (1961). A Non-informative prior is defined as the square root of the information measure/matrix by Jeffreys (1961). An uninformative prior expresses vague or general information about a variable as described by Jaynes.
The uninformative prior is called not a very informative prior and some authorities prefer the term “objective prior”. An informative prior expresses specific, definite information about a variable. Regarding proper prior and improper priors, Berger (1985) described that if the priors follow the properties of probability functions, then they are called proper priors otherwise improper priors.

Venkatesan and Nathiya (2008) made a survey and reported about a total of 30 and more priors like Informative, Non-informative, Proper, Improper, Probability density functions based, Uniform, Conjugate, Hierarchical, Dominated likelihood, Robust, Flat-tailed, Laplace, Jeffreys, Reference, Matching, MaxEnt, Weakly informative, Block, Folded non central-t, Half-t, Half-Cauchy, Inverted-Gamma etc, in their study. In addition to this plenty of priors (a total of more than 100 priors) reported by several authors in the recent past. Some of them, other than the above mentioned ones are remarkably mentioned as Subbotin prior described by Subbotin (1923), reported in Einmahl, Kumar and Magnus (2011), Matching priors by Welch and Peers (1963), Indifference prior by Movik and Hay (1965) which reported in Kass and Wasserman (1996), Restricted class of priors by Berger (1985), Parametric Approximation prior by Robert (1994), Partial sample information as priors by Moreno, Giron and Martinez (1998), Default priors by Ghosh (1999), Compatible priors by David and Laxritzen (2001), Ordered Group reference priors by Roverato and Consonni (2001), Expected posterior prior (EPP) by Parez and Berger (2002), Democratic prior by Wright (2011), Confronting prior by Lopes and Tobias (2011) etc. These priors have been studied and reported in detail and are elaborated in Chapter II under the section: “Review of priors”.

Selection of priors and selection and specification of values for the parameters of the prior distribution is a key problem and it should be given more attention in the research field of Bayesian methodology and as such due importance has been given by many authors and good amount of work has been done in this area and some of them are remarkably mentioned in the following sections.
The choice of priors or selection of priors is the major research work under which many Bayesians have studied this problem. In this context, selection of priors using Bayesian Testing of hypothesis procedure is normally applied and utilizing the Bayes Factor techniques, solutions are obtained for the problems and this has been done by many authors using their own criterion and some of these are notably mentioned as Kass and Wasserman (1996), Lieshout (1997), Moreno, Giron and Martinez (1998), Chick (1999), David and Laxritzen (2001), Roverato and Consonni (2001), Snoussi and Djafari (2002), etc., and it was reviewed and reported in detail which are elaborated in Chapter II under the section: “Selection of priors”.

Liang, Paulo, Glyde and Berger (2008) studied the problem of model selection using mixtures of g priors as an alternative to default g priors that resolve many of the problems with the original formulation while maintaining the computational tractability that has made the g prior. Wright (2011) studied the problem of evaluating real time Vector Autoregressive Process (VAR) forecasts with an Informative democratic prior through Bayesian Methodology in which the author assigned an independent prior for the Autoregressive vector parameter A and precision matrix $\Sigma$. Einmahl, Kumar and Magnus (2011) studied the problem on the choice of prior in Bayesian model averaging in which attempts were made by combining the parameter estimation and model uncertainty in one coherent framework. In this work they introduced five different types of priors and selection of priors by studying the performance of the priors through growth empirics real life data based on measures like MSE, RMSE and SD by comparison.

Li Ma (2012) studied the problem of a new model space prior for Bayesian variable selection in linear regression. This prior is designed based on a recursive constructive procedure that randomly generates models by including variables in a stage wise fashion and illustrated that how the prior can be specified to take into account model space redundancy arising from strong correlation among
predictors. Bayesian Model Averaging (BMA) under the Forward Stagewise (FS) prior can also be conveniently carried out using a sequence of recursive computation. Carbonetto and Stephens (2012) studied the problem of the Bayesian approach to variable selection in regression and retain useful features of Bayesian variable selection at a reduced cost. Using simulations designed to mimic genetic association studies, this simple variational approximation yields posterior inferences in some settings that closely match exact values. To validate the variational approximation, two simulation studies are carried out such as the first one is an idealized simulation in which all variables are independent and the second one is a more realistic case study in which many variables are strongly correlated.

Venkatesan and Nathiya (2008) studied the problem of finding posterior solution to the parameter of normal sequence through Bayesian logic using proper priors such as Double exponential distribution for the location parameter and Half-normal prior for the scale parameter. Venkatesan and Nathiya (2010) introduced the Double Exponential Taylor Series Approximation prior for the location parameter of normal sequence and obtained the posterior solution. Related work has also been done by Venkatesan and Nathiya (2011). All these works done above are filled with complications as it is nearly impossible to obtain analytical derivations of Bayes estimates and even numerical integral solutions were not possible. To overcome these difficulties, Venkatesan and Nathiya (2011a, 2011b, and 2011c) have developed a new methodology for numerical integrations and introduced a new technique for analytical integration to find the posterior solutions and Bayes estimates for the parameter of normal sequence and studied the problem through Bayesian Methodology.

In many real life situations which imitates the problem of target tracking of sparsely distributed event with abrupt random fluctuations, the usual type of conjugate priors normal-Inverted Gamma for the normal parameters will not suitably describe the situation and it needs a heavy-tailed distribution as the prior for the parameter(s)
like Double Exponential and Half-Normal distributions can suitably describe the location and scale parameters respectively. Bayesian estimates are obtained through Bayesian Methodology. Further to represent some realistic situations, a mixture of distributions may be adopted as priors to describe the situation suitably. The authors carried out the Bayesian analysis of the normal sequence with various combinations of priors consisting of normal, Double Exponential, Gamma, Inverted Gamma, Extended Inverted Gamma and Bimixture of these with suitable range of conceptual combinations.
Organization of the Research Report

In this Research, introduction and importance of prior distributions or informations and its pros and cons are discussed.

A Review of the literature has been done and selection or choice of prior distribution and specification of prior parameters are elaborated in Chapter II.

In Chapter III, novel types of prior distributions have been assumed for suitably describing the complex situation. Based on this, Bayes estimates of the parameters of normal sequence have been studied and reported.

The mixture of distributions as mixture-priors have been introduced and the posterior solutions of the parameters of normal sequence have been studied and reported in Chapter IV.

Bayesian computations of algorithmic procedure for finding numerical integration and solution to the parameters with infinite range has been derived and reported in Chapter V.

Numerical illustration for the newly developed methodology has been carried out through simulation study and results are reported in detail in Chapter VI.

Summary and Conclusion of the report are described in Chapter VII.