List of Figures

1.1 Basins of attraction of a simple damped pendulum along with the equilibrium points. ........................................ 15

1.2 Bifurcations from equilibrium point: (a) SN bifurcation; (b) transcritical bifurcation; (c) and (d) are pitch fork bifurcations. .... 30

1.3 Hopf bifurcation from an equilibrium point: (a) shows supercritical and (b) the subcritical cases. In (b), a stable equilibrium coexists with stable and unstable periodic orbits. ....................... 32

1.4 Neimark bifurcation or secondary Hopf bifurcation .................. 34

1.5 Period doubling bifurcations. (a) is the supercritical and (b) the subcritical form. .................................................. 35

2.1 Poincaré section of a trajectory undergoing period doubling bifurcation. On the left is the original periodic trajectory, giving the intersection point q*. On the right this is changed to two points. 38

2.2 Geometrical meaning of P (q) ........................................ 40

2.3 Schematic representation of the period doubling route, using eigen values. The different figures correspond to (a) a stable cycle, (b) an unstable cycle, (c) a nonstable cycle and (d) an unstable cycle with all λ_j's outside the unit circle. .............................. 41

2.4 Bifurcation to double period. ........................................ 42
2.5 Bifurcation diagram of the simple damped pendulum [Eqn. (2.7)] obtained numerically. Here \( \theta \) values are plotted corresponding to \( \Sigma^{\omega} \) with \( t_0 = 0 \), against \( \lambda \). Parameters are \( \omega = 1.0, q = 0.2 \). 42

2.6 Schematic representation of the quasiperiodic route. Fixed point in the Poincaré map is a limit cycle. As the control parameter is changed a second frequency appears. Quasiperiodic behaviour follows when the frequencies are incommensurate. 45

2.7 Arnold tongues: frequency locking occurs in the shaded regions. 49

2.8 The Complete Devil's Staircase. The winding number given by Eq. (2.14) plotted against \( \Omega \) for sine-circle map. 50

2.9 Intermittency shown by the logistic map. Fig. (a) is a plot of a period - 5 behaviour for \( \mu = 0.935 \); in (b), for \( \mu = 0.9342 \) intermittent behaviour is seen. 52

2.10 The orbits of the surface of section map is shown as a function of the parameter \( p \). Stable orbits are represented by solid lines and dashed curves represent unstable orbits. 54

2.11 Heteroclinic tangency crisis illustrated in (a). The homoclinic version is shown in (b). 58

2.12 Phase portraits showing (a) homoclinic orbit \( \Gamma \), which is also the separatrix and (b) heteroclinic orbits \( \Gamma^1 \) and \( \Gamma^2 \) forming separatrix, along with other phase trajectories. 63

2.13 The stable and unstable manifolds of an unperturbed system joining smoothly. 65

2.14 Appearance of Homoclinic tangle 66

2.15 Variation of \( R^0 (\omega) \) as a function of \( \omega \) for the Duffing oscillator in Eq. (2.47). 71
LIST OF FIGURES

2.16 The stability regions of Mathieu equation $\frac{d^2y}{dt^2} + (a - 2b \cos 2t) y = 0$. 77

2.17 The evolution of the principal axes of an $n$-sphere is shown in (a).

The application of the GSR is illustrated in (b). ............... 82

3.1 Schematic diagram of the driven Frode pendulum. ............. 85

3.2 Basin of attraction of the undriven Frode pendulum. Parameters are $q_1 = 0.1, q_2 = 0.7$ and $\omega = 0.2$. ...................... 88

3.3 A limit cycle of the undriven FP is shown in (a), and (b) shows a chaotic trajectory of the driven system (3.6). .................... 89

3.4 Bifurcation diagrams for the driven FP. $\theta$ is plotted as a function of $f$ in (a) and $\omega$ in (b). ......................... 90

3.5 Plot of maximal LE for the driven FP. The parameters are $q_1 = 0.1, q_2 = 0.7$ and $\omega = 0.2$. ............................. 91

3.6 Blow up of figure 3.5, for $f < 0.34$ shown. ..................... 92

3.7 A Poincare attractor for $f = 0.1$ is shown in (a); (b) is the winding number plot. Parameters are $q_1 = 0.1, q_2 = 0.7, \omega = 0.2$. 93

3.8 The period doubling cascades shown for same parameters as in previous figures. ............................................. 94

3.9 The crisis route to chaos. Figure above shows a boundary crisis while interior crisis can be seen below. Parameters are as in Fig. 3.8. 95

3.10 The Melnikov threshold $f_{th}$ plotted as a function of $\omega$ for the FP. Parameters are $q_1 = 0.3$ and $q_2 = 0.5$. ................... 97

3.11 Phase space trajectories of the FP (a) below and (b) above the Melnikov threshold in Fig. 3.10. ................................. 98

3.12 Chaotic attractor of the FP for $f=1.7$; other parameters are as in the previous figure. ........................................... 99
LIST OF FIGURES

3.13 The limit cycle of the system (3.6) obtained by numerical integration, compared to the one got by harmonic balance method. 100

4.1 Response curves of the FP: (a). The response curves for \( f = 0.08, 0.1, \ldots (0.02) \).
(b). Curves for \( f = 0.094, 0.114, 0.134 (f_*) \), 0.154. 106

4.2 The values of \( \beta_*, f_* \) and \( \omega_* \) for different \( q_2 \) values are plotted in figure. 108

4.3 The regions of stability identified for all the three cases above. Different shades represent different no. of coexisting solutions. 115

5.1 (a) The bifurcation diagram, b) the plot of maximal LE and (c) chaotic trajectories; parameters are \( q_1 = 0.3, q_2 = 0.5 \) and \( \omega = 0.7 \). 124

5.2 \( \Delta \eta \) for modulation of drive term plotted as a function of \( f \). (b) is bifurcation diagram. (c) is LE plot and (d) trajectories of modulated pendulum. 127

5.3 Modulation amplitude \( \Delta \eta \) for modulation of damping term plotted against \( f \). Parameters and numbers as in previous figure. 129

5.4 Modulation amplitude \( \Delta \eta \) for modulation of restoring term plotted against \( f \). Parameters and numbers as in previous figure. 130

5.5 \( \Delta \eta \) on adding a secondary forcing, plotted against original amplitude \( f \); details are as in figure. 5.2.a 132