Preface

Most of the systems in the universe are non-linear in nature. These can therefore be described by non-linear differential or difference equations. A few of these systems are integrable and hence can be solved analytically. However the majority of these are non-integrable, exhibiting diverse global and local complex patterns such as spatiotemporal chaos, homogeneous states, localised coherent structures, phase locking, quasiperiodicity and stochastic resonance. The chaotic phase is perhaps the most fascinating and most widely studied among these phenomena. These phenomena and the associated dynamics offer fine examples of study in various fields like physics, mathematics, engineering, chemistry, biology and economics. In the study of such problems, a mathematical model using non-linear differential equations is first constructed. If the describing differential equation is non-integrable, a change in parameters of the problem may result in a state, which is irregular, showing a sensitive dependence on initial conditions, which is the hallmark of chaos. Sensitiveness to initial conditions lead to unpredictability in the system evolution, which is not altogether random. Instead, this unpredictability is quite deterministic in that it is controlled by the describing equations. This phenomenon of deterministic chaos is one of the epoch-making discoveries of the twentieth century.

The work that forms this thesis is mainly centred on the occurrence of these complex phenomena, their stability and characterization in an analytical way.
This is a field where numerical techniques form a strong basis and most often is the only procedure of approach. An attempt is now made to stretch the very few analytic tools like harmonic balance method, stability analysis by reduction to Mathieu equation and Melnikov analysis, to a wider extent to provide useful results. The analysis is concentrated in the specific case of a non-linear mechanical system called the Froude pendulum (FP).

In the mathematical modeling of nonlinear systems, pendulum usually has the status of a paradigm, from a dynamical point of view. The different types of damping such as dry friction, coulomb friction, linear, quadratic or higher degree damping, cause diversity in the dynamical behaviour of the pendulum. This leads to variety in applications and investigations. A typical effect of the nonlinear damping is to lead to an asymptotic dynamic state called 'limit cycle'. The van der Pol oscillator, Lotka Volterra equations and the Froude pendulum are a few examples for this.

The Froude pendulum is a typical nonlinear dynamical system, showing periodic behaviour, quasiperiodicity, phase locking and chaos. Of these different phenomena, we study chaos in more detail. This is analysed with the help of phase plots (where the state vectors are plotted in two or three dimensions), phase space attractors in the Poincaré plane, bifurcation diagrams, and Lyapunov Exponents (LE), from which the dynamic state can be inferred. Phase space attractors in the Poincaré plane are employed to study the asymptotic state to which the system is eventually led. Bifurcation diagrams are drawn with
one of the parameters such as the driving frequency, drive term amplitude or either of the damping constants, as the independent variable (keeping all other parameters constant) and the angular displacement or angular velocity of oscillations as the dependent variable. These are very useful in tracking the different periodic, quasi-periodic and chaotic oscillations.

The LE technique is a widely used tool in the study of chaos. Here an initial small difference, in the state vector (n-dimensional) from the actual value obtained by numerical integration methods is allowed to evolve in time. Depending on whether the difference remains constant, dies down or grow (exponentially) we have n LE's which may be zero, negative or positive. A positive value for the maximum LE characterizes chaos, negative value indicates asymptotic periodic behaviour and zero value reveals quasiperiodic behaviour. Parameter space plot, showing periodic, quasiperiodic or chaotic regions in the space of parameters provide valuable information in the diagnosis of nonlinear dynamics of the system. Finally the Melnikov method, the only analytical tool for estimating parameter values corresponding to transition from chaos to periodicity or vice versa, is also used to study the dynamics of the pendulum.

The periodic behaviour of a nonlinear system gives way to chaotic state, through loss of stability of the steady state. The harmonic balancing method is made use of to study the changes in stability of the system. Here the nonlinear resonance curves are analysed for regions of stability. Employing Mathieu equation method, regions of stability are identified.
The loss of stability, eventually bringing in chaotic behaviour can be checked using different techniques. Feedback techniques, addition of quasiperiodic driving, modulation of different parameters of the pendulum, can all be used for suppressing chaos. In the Froude pendulum suppression of chaos is achieved by modulation of damping parameter, restoring term and drive amplitude.

To sum up the present work is an exhaustive study of the possible dynamics of the Froude pendulum system. The studies are based mainly on analytical techniques, supported further by relevant, detailed numerical works.