4.1 Introduction

In 1988, Yoshizawa & others introduced a new class of iron based alloys, named nanocrystalline, which exhibit superior soft magnetic behavior [66]. The properties were a unique combination of the low losses, high permeability and near zero magnetostriction. Compared with all previously-known soft magnetic materials, nanocrystalline type materials have higher products of relative permeability and saturation flux density, as shown in Fig. 4.1. The higher the value of the product, smaller is the size and lighter is the weight of magnetic components. Certainly good high frequency behavior, low losses and the good thermal stability are also important factors to affect the component density.

"The fact that an extremely fine grained structure leads to good magnetic properties actually came as a surprise since for conventional crystalline magnetic materials the coercivity increases with decreasing grain size, as illustrated in Fig. 4.2. Yet excellent soft magnetic properties are re-established when the grain size is below about 20 nm." Somehow, the nanocrystalline materials are fitting the gap between the amorphous and crystalline materials [67].

Fig. 4.1 Typical initial permeability and saturation flux density for soft magnetic material
Typical and so-far optimal nanocrystalline material composition is $\text{Fe-Si-B-Nb-Cu}$, which is also adopted by two major commercial nanocrystalline materials, Hitachi Finemet® $\text{Fe}_{73.5}\text{Si}_{13.5}\text{B}_{9}\text{Nb}_{3}\text{Cu}_1$ and Vaccumschmelze Vitroperm® $\text{Fe}_{73.5}\text{Si}_{13.5}\text{B}_{9}\text{Nb}_{3}\text{Cu}_1$ [82]. Nanocrystalline materials are prepared based on amorphous precursors and the nanocrystalline state is achieved by annealing at a temperature typically between 500 and 600 °C; which leads to primary crystallization. The resulting microstructure is characterized by randomly oriented, ultra fine grains of $\text{Fe-Si}$ with a typical grain size 10-15 nm embedded in a residual amorphous matrix which occupies about 20-30% of the volume and separates the crystallites at a distance of about 1-2 nm. These features are the basis for the excellent soft magnetic properties indicated by high values of initial permeability of about 10 and correspondingly low coercivity of less than 1A/m [67].

4.2.1. Loss Performance

As one of the most important indicators to evaluate a soft magnetic material, core loss density can be obtained through the measured B/H loops. Since core loss is the function of frequency and flux density, the B/H loops of the material is measured with exciting frequency changing from 50 Hz to 100 kHz and flux levels up to...
saturation $B_s$, as shown in Fig. 4.3. It is as expected that loss will increase as frequency increases, because of the effect of the eddy current inside the core.

The enclosed area by the B/H loop represents the energy during each cycle trapped in the unit volume of the magnetic core. This energy is finally dissipated as heat loss. Therefore, the loss density of the magnetic material can be calculated as,

\[
P_{cv} = f \cdot \int H \cdot dB = f \cdot \sum_0^N \{H(i) \cdot \left[ B(i) - B(i-1) \right] \} \tag{4.1}
\]

The calculated core loss density $P_{cv}$, in mW/cm$^3$, of the Finemet material is plotted against flux density and frequency in Fig. 4.4.

Another way to interpret the loss characteristic of the magnetic material is the complex permeability, $\mu = \mu' - j\mu''$ with an imaginary part representing loss and a real part for inductance. The complex permeability is very explicit for inductor and choke designs. To obtain the complex permeability (also termed as impedance
permeability), the above measured current and voltage waveforms of the core can be processed as:

\[ L = \frac{V_{\text{max}}}{\omega I_{\text{max}}} = \mu \mu_0 \frac{n_i^2 A_i}{l_e} \]  

(4.2.4)

\[ \Rightarrow \mu = \frac{V_{\text{max}} l_e}{\mu_0 \omega I_{\text{max}} n_i^2 A_i} \]  

(4.2.5)

\[ \mu^- = \mu \sin(\phi) \quad \mu^+ = \mu \cos(\phi) \]  

(4.2.6)

\( \phi \) is the angle of the voltage leading the current waveform. Basically, each set of complex permeability values are function of both frequency and exciting flux level. Loss and inductance are both affected by measurement of flux level, so it is important to plot the complex permeability frequency dependence at the same flux level. In the measurement it is important to keep the measuring flux level low, so distortions of the current and voltage waveforms do not affect the calculations. The calculated complex permeability of the material is plotted in Fig. 4.5.
4.2.2. B/H Curve

The most important characteristics of soft magnetic materials are saturation induction $B_s$, relative permeability $\mu_r$, coercivity $H_c$, and core loss density $P_c$, all of which can be visualized in the B/H curve of the material. To characterize the nanocrystalline material, the test circuitry shown in Fig. 4.6 was setup [36].

Two close coupled windings are wound onto a toroidal core, which is made of the nanocrystalline material, Finemet®, to be characterized. A sinusoidal signal is applied to the primary winding through a wideband power amplifier and induced voltage on the secondary winding is fed into an integrator, the flux density inside the material can be obtained. Practically, the flux compensation has to be considered, since the initial value of captured voltage could be any point between plus and minus maximum [37]. There is a constant offset error at the output of the integrator due to
the initial value of the voltage waveform $\sin(\omega t + \phi)$, i.e. taking $\phi = 0$ as the starting point. The output of the integrator will have a DC offset as $\frac{V}{\omega}$.

$$B = \frac{\int v(t) \cdot dt}{n_s A_c}$$

(4.2.7)

Exciting current is picked up using a sensing resistor and magnetic field intensity is calculated by

$$H = \frac{n_s \cdot i}{I_c}$$

(4.2.8)

Where $A_c$ notes a cross-section area of the core and is the magnetic length. The measured B/H loop curve under 60 Hz and 25 °C of Finemet® is shown in Fig.4.7. With exciting level increases, the core is finally driven into saturation, so a series of minor and major B/H loops are recorded. Due to the output current limit of the amplifier and the size of the core used in the test, the maximum magnetic field density obtained is around 5 A/m. Therefore, the real saturation flux density cannot be read directly, but it is clear that the material can hardly be used beyond 1 Tesla for transformer or inductor applications.

The interpretation of the measured B/H curves gives a deeper understanding of the magnetization process of the nanocrystalline material. The black dotted curve (normal magnetization curve) connecting all tips of B/H loops clearly shows three magnetization scopes: initial magnetization, which happens below 0.1 Tesla, then irreversible magnetization with a much higher incremental permeability and finally rotation magnetization before saturation. The corresponding incremental permeability can be obtained based on the normal magnetization curve and is plotted against the magnetic field strength, as shown in Fig. 4.8. The following results were derived from the experiment conducted on the above B/H curve measurement setup.
4.2.3. Temperature Dependence Performance

To characterize the temperature dependency of the nanocrystalline material. The core loss density as the function of temperature is shown in Fig. 4.9. At different flux levels, the core loss has a different temperature dependency.
4.2.4. Cut Core Issues

Similar to amorphous metals, the nanocrystalline material is in the shape of a thin tape, which is the result of the rapid solidification process. After annealing, the tape is very brittle and hard to handle, so there are several ways to assemble the thin tape into different shapes of cores to meet different application requirements. For common-mode chokes and small power transformers those do not require air gaps, cased toroidal cores would be the most convenient solution. Metal or nylon cases, enclosing a roll of nanocrystalline material tape, actually can act as winding bobbins. In this manner, the magnetic performance of the nanocrystalline material can be improved for higher values of flux density. However, the toroidal core cannot include air gap and is hard to be applied to high power transformers [68].

Air gaps are required to store energy for inductors and transformers. Besides, a tiny air gap will improve the operating ruggedness of the magnetic component. First, gapping the core would prevent saturations. A higher DC current value can be sustained by the transformer. Secondly, the gapping effect would reduce the sensitivity of permeability to the temperature variation. However, a gap would reduce the magnetizing inductance of the transformer, which means more loss to the circuit and cause more winding losses due to fringing field around the gap. Therefore, small air gap has been introduced into the magnetic path formed by the core, which can be realized by using cut core structures [38].
Nanocrystalline materials are inherently brittle and hard to cut, so the preparation of cut cores needs special care. Usually, nanocrystalline tapes are wound into the expected shape and the core is impregnated by an insulation resin. Stress is induced during the casting process, so the molded core needs to be annealed to release the stress. Up to this stage, the core can be machined and cut into halves and a special treatment is required to improve the smoothness of the cutting surface and to remove the short circuit between tape layers. As can be seen, the magnetic properties of the material would be changed due to these processes. The characteristics of cut cores were investigated to provide insight into the transformer design based on cut cores.

To study the effect of air gaps, C-cores made of the same Finemet nanocrystalline material were prepared and put into the test circuit shown in Fig. 4.6. For the C-core used for this demonstration, the minimum achievable air gap length is $L_g = 2360 \text{ pm}$, which is mainly determined by the quality of the cutting and grinding processes during the C-core preparation [appendix-III].

The relationship between air gap length and effective permeability can be derived as

$$NI = H_{core} \cdot l_e + H_{gap} \cdot l_g = \frac{B}{\mu_0} \left( \frac{l_e}{\mu_e} + l_g \right) = \frac{B \cdot l_e}{\mu_0 \mu_e}$$

$$l_g = l_e \left( \frac{1}{\mu_e} + \frac{1}{\mu_0} \right)$$

When the gap length is varied from 18 \text{ pm} to 160 \text{ pm}, core loss densities are measured for different frequency, as shown in Fig. 4.10. For the gap length range analyzed, the increase of core loss density is not proportional and can be omitted.
4.3. Winding Loss Calculation

The transformer windings are formed by conductors in different shapes of plates, round wires, strand wires and Litz wires. The resistance of the winding at a low frequency can be easily calculated as follow, with \( N \) for turn’s number, \( \rho \) for conductor resistivity at certain temperature, \( MLT \) for mean length per turn and \( A_w \) for conductor cross-section area.

\[
R_{dc} = \frac{N\rho MLT}{A_w} \tag{4.9}
\]
The winding loss is equal to the product of DC resistance and the current square value. However, for high frequency power electronics converter systems, skin and proximity effects cannot be omitted. Both of them change the current distribution inside the conductor, so the resistance is increased when compared with the DC value. Therefore, the challenge of calculating winding loss really means how to count the eddy current effect in the conductor.

4.4. Litz Wire Optimal Design

The AC-to-DC resistance ratio is the function of strand numbers and strand diameters for same winding layers and certain frequency. For a selected core window area and fixed fill factor, the larger diameter means fewer strands needed. There should be an optimal selection of the Litz wire strand number and diameter. Sullivan [39] has addressed this issue, but his eddy current effect consideration for the Litz wire is in prime stage. The first relationship is that the total copper area is fixed, if the core window has been selected and the fill factor is kept constant. So,

\[ \text{Const} = N_0 \cdot d_{\text{strand}}^2 \]  

(4.10)

So the DC resistance is fixed for the case discussed here. Therefore, the optimal \( y_{op} \) is given by

\[ y_{op} = 4 \cdot \sqrt{ \frac{\pi^2 \cdot N_0 \cdot \eta}{4} \left( \frac{3}{1 + \frac{\pi^2 \cdot N_0 \cdot \eta}{16 m^2 - 1 + \frac{24}{\pi^2}}} \right)} \]  

(4.11)

\[ y_{op} = \frac{d_{\text{strand}}}{\delta} = d_{\text{strand}} \cdot \sqrt{ \frac{\pi \cdot f \cdot \mu_r \cdot \mu_0}{\rho} } \]  

(4.12)

Since the above equations are non-linear, a program has been developed to optimize the Litz wire parameters [Appendix-II].

4.5 Step-Up Transformer Design

Primary and secondary windings are wound on two different legs of the core, thereby simplifying enormous insulation problems. Dielectric strength required between primary and core is 1.3 kV while between secondary and core is 10 kV. Coil formers for both windings are custom made out of 1.5-mm thick acrylic sheet and are covered with one layer of Kapton or Mylar tape before winding. The primary winding includes also an electrostatic shield (which is grounded) to reduce capacitive coupling.
between primary and secondary and to avoid high-voltage to reach the bridge switch circuitry in case of a dielectric breakdown at the secondary. The transformer requirements are listed in Table

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>Primary RMS voltage</td>
<td>560 V</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Secondary RMS voltage</td>
<td>5 kV</td>
</tr>
<tr>
<td>$I_1$</td>
<td>Primary RMS current</td>
<td>3.7 A</td>
</tr>
<tr>
<td>$I_2$</td>
<td>Secondary RMS current</td>
<td>0.8 A</td>
</tr>
<tr>
<td>$f$</td>
<td>Switching frequency</td>
<td>50 kHz</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Primary leakage inductance</td>
<td>70 uH</td>
</tr>
</tbody>
</table>

A safe value $B_{\text{max}}$ of 400 mT is selected for the maximum flux density. To avoid core saturation, $B_{\text{max}}$ must not be exceeded. The minimum number of primary turns is, therefore, restricted to satisfy this requirement. The correct calculation of the core flux density requires the application of the Faraday’s law:

$$ dB = \frac{d\phi}{A_c} = \frac{10^4}{A_c N} \int e \cdot dt $$  \hspace{1cm} (4.13) 

*Where*

$dB$ = change in flux density [T]

$d\phi$ = change in flux [maxwell]

$A_c$ = core magnetic cross section [cm$^2$]

$N$ = number of turns

$e$ = voltage applied [V]

The value of the above integral is positive for the half-cycle where the voltage is positive and negative for the other half-cycle of voltage. When the positive half-cycle begins, $B$ is at its maximum negative value therefore its maximum value is only one half of its total change.

$$ B_{\text{max}} = \frac{dB}{2} = \frac{10^4}{2 A_c N} \int e \cdot dt $$  \hspace{1cm} (4.14)
\[ B_{\text{max}} = \text{MAX} \left[ \frac{10^4}{2A_N} \int_0^t e \cdot dt \right] = \frac{10^4}{2A_N} \text{MAX} \int_0^t e \cdot dt \] \hspace{1cm} (4.15)

The largest integral value can be found near the charging end. It is

\[ \text{MAX} \int_0^t e \cdot dt = 3.16 \cdot 10^{-3} \text{ V.s} \] \hspace{1cm} (4.16)

Practical tests indicate that 15 turns for the primary winding allows achieving exactly the required inductance

\[ N_1 = 15 \]

So \( N_2 = N_1 \frac{5000}{560} = 135 \text{ Turns} \)

### 4.6.1 Winding Capacitance Calculation

Winding capacitances, also generally defined as stray capacitances, of a transformer arise from the distributed electrical coupling between any two conductors in or around the transformer. Winding capacitances are all related to transformer windings but not only between winding conductors. The capacitance occurs between the windings and between winding and magnetic cores, in transformer. They are heavily geometry-dependent and distributed in nature and to most power converter applications, lumped models of winding capacitance are sufficient [69].

It has become widely aware that winding capacitances in high-frequency transformers have significant effects on the performance of the components as well as the entire power electronic systems. The winding capacitance results in current spikes and slow rise times which would cause more stress and loss on semiconductor switches. Furthermore, it is responsible for the propagation of conducted EMI noises in converter systems. Resonant topologies can utilize the winding capacitance which becomes a part of the resonant capacitance required for the operation. Therefore a winding capacitance calculation is an important aspect of the transformer design.

Substantial attention has been drawn to the modeling of winding capacitance of transformers. The lumped winding capacitance of transformers has been found. The main approaches can be categorized into three groups: 1) experimental methods - the transformer is treated and measured as a single or two port network and the lumped capacitances would be calculated according to the measured impedance
together with inductances. 2) theoretical calculation based on the field analysis - sophisticated field distribution can be obtained on the detailed transformer winding geometries and electrostatic energy in the structure would be integrated to have the lumped terminal capacitance [40],[41],[42]. 3) analytical expressions - simplified electrostatic field analysis makes the closed-form solution to the equivalent lumped capacitance possible, for regular winding structures [43],[44]. The fundamental equation of all energy base approaches is as:

$$W = \frac{1}{2} \iint \varepsilon_r \cdot \varepsilon_0 \cdot E^2 \cdot dy = \frac{1}{2} C \cdot V^2$$  \hspace{1cm} (4.17)

To integrate the winding capacitance calculation into the transformer design procedure, the preferred method is analytical expressions. 2-D or 3-D field analysis approaches could give accurate results but definitely require a great amount of information about the geometry and boundary conditions. It is not possible to include this into design iterations due to its extensive computation resource requirements. The simplified energy-base approach could be the solution if the electric field distributions are regular in the transformer, which requires that distances between the windings should be much smaller than the height of windings.

4.6.2. Simplified Energy Base Calculation Method

Since voltages induced in all the turns of a given winding layer are identical, the potential varies linearly from one end of the layer to the other, as illustrated in Fig. 4.11. To make the illustration simple, three independent voltages are defined between the four terminals. Therefore, the 1-D electrical field distribution can be found between the two layers as:

$$E_r(y) = \frac{1}{d} \left( \frac{V_2 \cdot y}{h} + \frac{V_3 \cdot y}{h} \right)$$ \hspace{1cm} (4.18)

Now, the energy stored between two layers, which have a unit length, can be expressed as where the dielectric material in between has permittivity of $\varepsilon_r$:

$$W = \frac{1}{2} \int \varepsilon_r \cdot \varepsilon_0 \cdot E_r^2 \cdot dy = \frac{\varepsilon_r \cdot \varepsilon_0}{2} \int_0^h \left( E_r(y) \right)^2 \cdot dy$$ \hspace{1cm} (4.19)
After putting (4.19) into (4.20), the relationship between the energy and the terminal voltage is:

\[ W = \frac{C_0}{2} \left( \frac{V_1^2}{3} + \frac{V_2^2}{3} + V_3^2 - \frac{2V_1 \cdot V_2}{3} + V_3 \cdot V_2 - V_3 \cdot V_1 \right) \]  

(4.20)

Where \( C_0 = \frac{\varepsilon_r \cdot \varepsilon_0 \cdot h}{d} \) which can be looked as the structure capacitance formed by the two conductor layers.

This structure can be applied to the particular transformer windings to calculate capacitance. First, determine the connection between the two layers and correspondingly correlate terminal voltages with the particular values. The corresponding capacitance can be obtained by the relationship in (4.17). Two of the most popular winding structures are shown in Fig. 4.12. For the wave-type winding, the terminal voltage is correlated as:

\[ V_1 = -V_2 = \frac{1}{n} \cdot V_{\text{winding}}, V_3 = 0 \]  

(4.21)

Similarly, the leap-type winding has the relationship like:

\[ V_1 = V_2 = \frac{1}{n} \cdot V_{\text{winding}}, V_3 = -\frac{1}{n} \cdot V_{\text{winding}} \]  

(4.22)
The above discussion is valid for the situation of parallel flat winding layers. If the winding is cylindrical or if the curvedness of the winding can not be omitted, the equation (4.21) still holds true except the structure capacitance is expressed as:

\[
C_0 = \frac{2\pi \cdot \varepsilon \cdot \varepsilon_0 \cdot h}{\ln\left(\frac{r_2}{r_1}\right)}
\]  

(4.23)

**4.6.3. Transformer Winding Capacitance Calculation**

With the energy based method discussed above, calculate the winding capacitance of a magnetic core transformer is calculated. For a transformer with secondary floating, three terminal voltages can be defined as shown in Fig. 4.13 (a). As in equation (4.25), the total electrostatic energy stored in the transformer could be expressed in six terms which correspond to all capacitance between any two terminals.

\[
W = \frac{1}{2} \left( C_{11} \cdot V_1^2 + C_{22} V_2^2 + C_{33} V_3^2 + 2C_{12} \cdot V_1 \cdot V_2 + 2C_{13} \cdot V_1 \cdot V_3 + 2C_{23} \cdot V_2 \cdot V_3 \right)
\]  

(4.25)

However, these three voltages are not independent and the voltage between primary and secondary windings, as the function of primary and secondary terminal voltages \( V_1 \) and \( V_2 \), are to be determined.

\[
V_3 = \frac{C_{13} \cdot V_1 + C_{23} \cdot V_2}{C_{33}}
\]  

(4.26)
So, the high-frequency equivalent circuit of the transformer can be drawn as in Fig. 4.13 (b). If winding resistances are omitted for the analysis, the energy stored in transformer is expressed as:

\[
W = \frac{1}{2} \left( C_1 V_1^2 + C_2 \left( \frac{V_1}{n} \right)^2 + C_3 \left( V_1 - \frac{V_2}{n} \right)^2 \right) \tag{4.27}
\]

By comparing equation (4.27) and (4.25) with (4.26), the lumped equivalent capacitances as:

\[
C_1 = C_{11} - \frac{C_{13}^2}{C_{33}} \tag{4.28}
\]

\[
C_2 = n^2 \left( C_{22} - \frac{C_{33}^2}{C_{33}} \right) \tag{4.29}
\]

\[
C_2 = -n^2 \left( C_{12} - \frac{C_{23} \cdot C_{13}}{C_{33}} \right) \tag{4.30}
\]

![Transformer terminal voltages (a) high-frequency equivalent circuit (b)](image-url)
Actually, the equivalent lumped capacitor has been further reduced into one, by expressing secondary voltage with respect to primary one or vice versa. Although it is true, from an energy storage point of view, that the distributed characteristic of the winding capacitance makes the equivalent circuit capable of representing the transformer behavior to a high frequency range. Another point is that the transformer turns ratio in fig. 4.13 (b) is the real ratio between the primary and secondary turn numbers, but it is not equal to the voltage ratio. Especially for resonant converter applications, leakage inductances cause a voltage drop which cannot be omitted.

The magnetic core would have certain voltage potential, if the material is conductive, like most ferro-magnetic materials. Usually the core will be grounded just like the primary winding.

If other winding connection configurations are adopted, such as common ground for primary and secondary windings, then to find out the value of inductance by correlating equation (4.27) with (4.25).

4.7 Leakage Inductance Calculation

Leakage inductance is one of the important design parameters of a high-frequency transformer. Leakage inductance plays a very important role in pulse-width-modulated (PWM) converters, limiting the upper frequency of operation. In switched-mode converters, the leakage inductance causes undesirable voltage spikes that can damage circuit components [70]. For the PWM full bridge converter shown in Fig. 4.14, the effect on the switch voltage stress is demonstrated. Here it clearly shows that the voltage ringing is more severe as the leakage inductance of the transformer increases. The overshoot voltage could destroy the semiconductor devices and the extra losses due to the ringing would reduce the system efficiency. Therefore, the leakage inductance needs to be controlled in the transformer design.
Since it is difficult to have small leakage values, especially for high power applications, soft switching and resonant operation schemes have been proposed to absorb the inevitable leakage inductance [45],[46]. In these cases, the leakage inductor participates in the circuit operation through resonating with certain capacitance along or together with extra inductors.

From the discussions above, it has been concluded that accurate calculation of leakage inductance is critical to high power density converters. The leakage inductance is actually a lumped equivalent representation of the energy stored in the leakage field, which consists of the portion of the magnetic flux not linking with both primary and secondary windings. Therefore, the calculation of the leakage inductance can be performed as:

$$W = \frac{1}{2} L_{\mu} I^2 = \frac{1}{2} \mu_0 \int H^2 \cdot dV$$

(4.31)
4.8.1. Leakage Inductance Calculation Method Survey

a) Simplified 1-D calculation

The well known way to calculate leakage inductance assumes that the current distribution in the winding is linear and the core permeability is infinite [71]. The magnetic field in the winding space is assumed to be parallel to the leg of the transformer core. This field distribution, as a one-dimensional problem, is as shown in Fig. 4.15. The 1-D leakage filed distribution. \( H_z(r) \) is shown in the cross-section of the pot-core transformer. The leakage inductance of unit winding length can be obtained by putting \( H_z(r) \) into equation (4.31).

\[
\frac{L_{zk}}{l_w} = \frac{\mu_0 N^2 b_w}{3} \tag{4.32}
\]

Where, \( b_w \) is the core window width and \( l_w \) is mean length of the winding. This equation has been used for other types of cores and winding structures with the validation of the assumptions. Even for the transformer with perfect fit structure, this method is only accurate for low frequencies since it omits high-frequency eddy current effects inside the transformer winding conductors.

b) Numerical methods

The numerical method is powerful to solve complex structure and asymmetrical problems. Significant research has been done to analyze and model eddy current effects in transformers. Skutt [47] applied 3-D Finite Element Analysis method to a high-power (500 W) planar winding transformer. The impact that the secondary winding terminations have on the leakage characteristics of the device is examined. It is shown that the effective inductance, the transformer presents to the circuit, can be several times the inductance calculated or measured based on an ideal short-circuited winding. This is a perfect example where 1-D simplification cannot be hold due to the structure. In this case, the sophisticated FEA calculation is performed initially to identify the discrepancy between ideal calculation and the real situation. However, the FEA calculation needs expensive computation resources and the most important issue is that it is hard to integrate into a design procedure.
Instead of working in frequency domain, the method by Lopera [48] is constructed in time domain. 1-D distributions of magnetic and electric fields are assumed and from Maxwell’s equations an equivalent electric circuit is easily obtained. This equivalent circuit can be included in analog simulators. The leakage inductance calculated by this method represents eddy current effects, but it is hard to apply the method to Litz wire windings, because of its inherent 1-D plate structure basis.

To capture all the irregularities of the leakage field distribution due to edge effects and asymmetries, FEA is probably the only appropriate way to calculate leakage inductance, but the lengthy computation makes it unsuitable to be integrated into the design procedure. Another drawback of the numerical method is that the Litz wire, especially the one with many strands, is hard to depict and calculate. The radial and azimuthally transposition of strands in the Litz wire actually requires 3-D solvers. It is impractical to calculate detailed field distribution of each tiny strand and to sum them up in order to obtain the macro winding inductance. The FEA method is really insufficient because of the number of strands and the small diameter of each strand.
The high frequency requires that each strand be very thin, while the high power means that more strands are needed; thus these requirements make the FEA method impractical.

c) Close-form methods considering eddy current effects

The closed-form expression of leakage inductances is preferable for power electronics designers. Dowell [49] did pioneering work on including high-frequency eddy current effects into 1-D impedance solutions and Venkatraman [50] expanded the method to non-sinusoidal waveforms. This particular approach, being one dimensional in rectangular coordinates, is in principal applicable to foil conductors having a magnetic field parallel to the conductor surface and is therefore subject to certain restrictions.

Magnetizing current of transformers cannot be included, as it results in a magnetic field component that is not parallel to the foil conductor surface. Even when considering transformers with negligible magnetizing current, it should be realized that, strictly speaking, the analysis is valid for infinitely long solenoid windings. When the windings fill the window length completely or if the distance between primary and secondary is small, the resultant field approaches that of infinite solenoid windings.

Error is introduced by replacing round conductors with square-shaped conductors of an equal cross-sectional area. At DC, the resistances are equal, although at high frequencies the square representation of round conductors becomes inaccurate.

Another shortcoming is that the method cannot be applied to stranded or Litz wire windings, since it requires the series connection of conductors carrying same current. The parallel conductor in stranded wires would not have the same current flowing, due to the eddy current effects.

Hurley [51] thoroughly derived leakage inductance calculation for toroidal ferrite core transformers. The method is generic for windings with or without magnetic core, but it is too complicated to be applied to Litz wire windings. Goldberg [52] studied planar pot core transformers and more general structures (Pot or EE core with cylindrical windings) were considered by Niemela [53].
For high-frequency high-power transformers, Litz wires are used to reduce the winding loss. Ideally, there should be only a skin effect and no proximity effects due to the adoption of the Litz wire, so the current distribution and correspondingly the leakage field distribution should be easier to derive. However, the Litz wire is still not immune to the proximity effect because of its local field created by neighboring strands within one bundle. No method has been developed to calculate the leakage inductance for Litz wire winding transformers. Although Cheng [54] considered the stranded wire and applied the Dowell method to obtain the frequency-dependent leakage inductance, there is no clear analysis of the Litz wire mechanism and the limitations of the method.

4.8.2. Proposed Leakage Inductance Calculation Method

A closed-form method is proposed here to overcome the problem. To calculate unit length leakage inductance of the high-power high-frequency transformers, first the leakage inductance is divided into two parts - one part is frequency-independent components and the other is frequency-dependent components. The first part is mainly due to the stored energy in the interlayer and inter-strand gaps. The latter part of the leakage inductance is due to the flux crossing the winding conductors. The electric potential is produced according to the Faraday's Law and correspondingly eddy current generated in the conductor, which would change the filed distribution inside the conductor space. The computation of the frequency-independent component is simple. The frequency-dependent component is the theme of the work as it affects the performance of a transformer significantly and has been discussed in detail.

a) 1-D structure eddy current effects

The typical transformer structure is revised here, as in Fig. 4.16. The coordinate system has been used for all discussions in this chapter. Several assumptions have been made the analysis explicit but without losing much generalness and accuracy.

The first assumption is all wires in the same layer could be treated as a plate with the same area and width (in y direction), but an equivalent height (in z direction). The second assumption is that permeability of the core is infinite, so that the magnetizing current is neglected. This will ensure the leakage field in the winding would be parallel to the limbs of transformer core. The third assumption is that the
current distribution is unchanged for each whole turn, which means the edge effect of the hang-out portion of the winding is omitted. The current and field distribution for the winding portion within the core window is calculated and the result is multiplied by the mean turn length to get the overall inductance of the winding. All of these assumptions will bring in errors to some extent, but the analysis is greatly simplified. Actually, for high-power transformer applications, these assumptions are acceptable and realistic.

With the above mentioned assumptions made, the eddy current effect of the transformer winding can be analyzed. Both skin and proximity effects change the MMF field distribution.

If the transformer shown in Fig. 4.16 is cut along the x-y plane, the winding conductors can be considered as a semi-infinite current-carrying plate of thickness 2b in y direction and width h in z direction, as illustrated in Fig. 4.17. Here, considering the skin effect only, a unit current I flow through the plate in x direction. The magnetic
field in $z$ direction has been induced. The current density and magnetic field
distribution along $y$ direction inside the plate as, $H_z(y)$ and $J_z$ is found out [55].

If the linear current distribution in the conductor plate is assumed, the ideal
current density can be obtained as

$$J_{x,av} = \frac{l}{4b \cdot h}$$  \hspace{1cm} (4.33)

Now the Maxwell’s equations are applied to the problem shown above to
obtain the skin effect [72]. For conductor with conductivity $\sigma$, the sinusoidal current
with frequency $\omega$ and no displacement current considered, the famous elliptic partial
differential equations are;

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu_0 \cdot \mu \frac{\partial H}{\partial t} = -j \cdot \mu_0 \cdot \mu_r \cdot w \cdot H$$  \hspace{1cm} (4.34)

$$\nabla \times H = J + \epsilon_0 \frac{\partial E}{\partial t}$$  \hspace{1cm} (4.35)

$$J = \sigma \cdot E$$  \hspace{1cm} (4.36)

$$\nabla \times (\nabla \times E) = -j \cdot \sigma \cdot \nabla \cdot \mu_0 \cdot \mu_r \cdot w \cdot H$$  \hspace{1cm} (4.37)

$$\nabla \cdot H = 0$$  \hspace{1cm} (4.38)
\[ \nabla^2 H = \nabla \cdot H - \nabla \times (\nabla \times H) = j \cdot \sigma \cdot \mu_0 \cdot \mu_r \cdot w \cdot H \]  
(4.39)

In the Cartesian coordinates, the diffusion equation in (4.39) is in the form of

\[ \nabla^2 H = \frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = j \cdot \sigma \cdot \mu_0 \cdot \mu_r \cdot w \cdot H \]  
(4.40)

According to the 1-D assumptions and the boundary conditions,

\[ H_{sz}(b) = -H_{sz}(-b) = \frac{I}{2h} \]  
the magnetic field can be solved as (4.41), which gives the description of the magnetic field distribution along the y-axis:

\[ H_{sz}(y) = \frac{I}{2h} \frac{\text{Sinh}(\alpha y)}{\text{Sinh}(\alpha b)} \]  
(4.41)

\[ \alpha^2 = j \cdot \sigma \cdot \mu_0 \cdot \mu_r \cdot w \cdot H \quad \text{and} \quad \alpha = \frac{1+j}{\delta}, \]  
in which skin depth is expressed as

\[ \delta = \sqrt{\frac{2}{\sigma \cdot \mu_0 \cdot \mu_r \cdot w}}. \]  
The current density distribution can be obtained by putting (4.41) into (4.35) and is expressed in (4.42).

\[ J_{sz}(y) = \frac{\alpha I}{2h} \frac{\text{Cosh}(\alpha y)}{\text{Sinh}(\alpha b)} \]  
(4.42)

The skin effects on current density and magnetic field distribution within the conductor plate space are shown in Fig. 4.18, for a 1 cm thick copper plate under different operating frequencies. It is clear that the current distribution is "crowding" beneath the surface, as it is called the skin effect. The higher the frequency is, the more severe the skin crowding degree is.
Similarly, the proximity effect of the transformer winding is analyzed. According to the 1-D assumption, the winding conductor is under the leakage effect.
magnetic field, which is parallel to it’s surface along the z direction, as illustrated in Fig. 4.19.

The conductor, put in a varying magnetic field, will generate voltage potential and then eddy current, which in turn creates an internal magnetic field to oppose the change of the external field. Therefore, to solve the magnetic field and current distributions inside the conductor space, Maxwell’s equations are adopted again and the same diffusion equation as (4.40) can be derived for the proximity effect. The solution to the equation would be different, due to different boundary conditions,

\[ H_x(b) = H_x(-b) = H_e, \]

The magnetic field can be solved as (4.41) which gives the description of magnetic field distribution along the y-axis

\[ H_{pr-y}(y) = H_e \cdot \frac{\cosh(\alpha y)}{\cosh(\alpha b)} \]  

(4.43)

The current density distribution can be obtained by putting (4.43) into (4.35) and is expressed in (4.44).

\[ J_{pr-y}(y) = \alpha H_e \cdot \frac{\sinh(\alpha y)}{\cosh(\alpha b)} \]  

(4.44)

The proximity effects on current density and magnetic field distribution within the conductor plate space are shown in Fig. 4.20, for a 1 cm thick copper plate under 1 Tesla external magnetic field in different operating frequencies.
Once the skin and proximity effects are known separately, those can be applied to a typical transformer winding configuration which has primary and secondary windings, as shown in Fig. 4.21(a). The normalized magnetic field distribution is plotted for different frequencies in Fig. 4.21(b). The eddy current would result in a
smaller magnetic energy stored which means smaller inductance in the conductor spaces and the enclosed area decreases greatly as the conductor thickness to skin depth ratio $\frac{b}{\delta}$ reaches 3.4. It is obvious that the simple 1-D calculation method would over predict the leakage inductance, even for the design which chooses the conductor thickness as two times that of the skin depth.

The ultimate objective is to find out the inductance, which is directly related to the total energy stored by the leakage field. Therefore, the power integral of magnetic field intensity is calculated. From the above analysis, the magnetic field distributions due to skin and proximity effects are calculated separately. The total leakage field energy can be calculated as described below. Since the integral of $Sinh \ cosh$ is zero, the cross product terms, $H_{SK} \times H_{P}$ and $H_{SK} \times H_{r}$ are cancelled for the final total energy calculated [73]. This is called the orthogonality that exists between the skin and proximity effects under the condition discussed above.
The greatness of this relationship is that it can now be considered separately and the mathematic derivation would be simpler. If the applied field and current flowing directions are not perfectly perpendicular to each other, the orthogonal relationship is not valid any more.

\[
W = \frac{1}{2} \int \mu \left( H \cdot H' \right) dV = \frac{1}{2} \int \mu \left( \left( H_s + H_p \right) \cdot \left( H_s' + H_p' \right) \right) dV \\
= \frac{1}{2} \mu \left( H_s \cdot H_s' + H_p \cdot H_p' + H_s \cdot H_p' + H_p \cdot H_s' \right) dV \\
= \frac{1}{2} \mu \left( H_s^2 + |H_p|^2 \right) dV
\]

(4.45)

Fig. 4.18 and Fig. 4.20 clearly show the eddy current effect. Thus the calculated inductance from the linear magnetic field distribution, as for low frequency cases, would be larger than the actual magnetic field distribution. This is the major motivation for considering eddy current effects into leakage calculation. The winding loss at high frequency also is determined by the leakage field.
b) Litz wire effects

Litz wires are originally developed to result in lower ac resistances than solid wires. Eddy current effects drive the current through a solid wire close to the surface of the conductor at high operating frequencies. If a multi-strand conductor is used, the overall cross-sectional area is spread among several conductors with a small diameter. For this reason, a Litz wire would result in a more uniform current distribution across the wire section. Moreover, Litz wires are assembled so that each single strand, in the longitudinal development of the wire, occupies all the positions in the wire cross section. Therefore, not only the skin effect but also the proximity effect is drastically reduced [56], [57].

However, the following limitations occur when using Litz wires: 1) the utilization of the winding space inside a bobbin width is reduced with respect to a solid wire. 2) The dc resistance of a Litz wire is larger than that of a solid wire with the same length and equivalent cross-sectional area because each strand path is longer than the average wire length.
c) Litz wire winding leakage inductance calculation

With the above derivations, now the leakage inductance of the Litz wire is calculated, which ideally should present even current distribution among each strand. However, current distribution within each strand is still affected by the local skin effect of the individual stand and the local proximity effect of all its neighboring strands.

For high-power applications, the Litz wire used usually has a huge number of strands, so the Litz wire can be approximated by a square array of strands without sacrificing accuracy [58]. Furthermore, all strands in one row are packed into an equivalent copper plate. Therefore, the analysis that resulted during the above section can be adopted here to solve the leakage inductance within Litz wire area. Each layer is combined into one solid foil as shown in Fig. 4.22.

![Fig. 4.22. Litz wire approximation](image)

Therefore, the energy stored in the $k_{th}$ layer can be derived as:

$$W = \frac{1}{2} \mu \int \left( |H_s|^2 + |H_p|^2 \right) dV$$

$$W = \frac{MLT \cdot h \cdot \delta}{2} \int \left\{ \frac{2I^2}{h^2} \cdot \frac{\sinh v - \sin v}{\cosh v - \cos v} + \frac{2(k-1)^2 I^2}{h^2} \cdot \frac{\sinh v + \sin v}{\cosh v + \cos v} \right\}$$
The leakage inductance can be calculated through summing all layers together.

\[
L_{\text{leak}} = \frac{2 \cdot MLT \cdot \delta}{m^2 h} \sum_{k=1}^{m} \mu \left\{ \frac{\sinh v - \sin v}{\cosh v - \cos v} \right\} \left\{ \frac{\sinh v + \sin v}{\cosh v + \cos v} \right\}
\]

\[(4.47)\]

4.8.3. Verifications

The comparison of leakage inductance by the proposed method, simplified method and measurement for the developed transformer using the Litz wire has been analyzed. A simplified 1D method generates only a frequency-independent result. It proves that the proposed method is more accurate and avoids the time-consuming FEA method.

Fig. 4.23  Leakage inductance by the proposed method (blue solid), the simplified method (pink solid) and measurement (black dots)

4.9 Minimum-Size Design Procedure

Fig. 4-24 illustrates the procedure for a minimum-size transformer design for the given application [Appendix-II]. Clearly the relationships between design variables, constraints and conditions must be established for a successful design. The key relationships include the loss models and parasitic models discussed earlier.
For a certain core, the core and winding loss can be expressed as the function of flux density as shown in Fig. 4.25. It is obvious that the core loss increases as the flux density increases. The opposite trend can be observed for the corresponding winding loss, since higher flux density means less turns and smaller resistance for the same core.

$$P_c = V_c \times K \times f_{eq-fixed} \times (\Delta B)^6$$  \hspace{1cm} (4.31)

$$P_w = K_r \times I_{rms}^2 \times \frac{\lambda^2 \cdot \rho \cdot (MLT)}{4A_{window} \cdot K_n \cdot A_c^2 \cdot \left( \frac{1}{\Delta B} \right)^2}$$  \hspace{1cm} (4.32)
When the dielectric loss is omitted, the total loss of the transformer can be obtained by adding the core loss and the winding loss together. Therefore, an optimal flux density can be found, for which the total loss is minimal. This means that with...
this core the minimum loss can be achieved. The core design has been designed running at the $B_{op}$ as shown in Fig. 4.26. The optimal flux density $B_{op}$ for each core can be found under the minimum loss condition, $\frac{\partial P_{core}}{\partial B_{ph}} = 0$. The most interesting aspect of this is that the optimal operating flux density does not necessarily happen at the point where core loss equals winding loss.

![Fig. 4.26 Optimal flux density for minimum total loss](image)

A set of total losses is shown in Fig. 4.26, for transformers using different FT-3M nanocrystalline. The U-core dimension can be changed continuously for a full customized design.

### 4.10. Prototyping and Testing Results

Prototype transformer has been made for the converter charger. Nanocrystalline material cut cores and Litz wire windings are used for the prototypes. The structure of the transformer is illustrated in Fig. 4.27. Two sets of identical windings are put on each leg of the C-cores, interleaving winding are used to reduce the eddy current effect. Actually, the resonant inductance is fully realized by the leakage inductance of the transformer, so the distance between primary and secondary is tuned to ensure the leakage inductance and the 10 kV insulation requirements can be satisfied.
The C-core is chosen mainly for availability and for better balanced secondary outputs, multiple secondary outputs by suitable tapping and they are required to be identical to reduce the mismatch effect on resonant operation. A small inevitable air gap is introduced, which can reduce the temperature dependence of the nanocrystalline material and improve saturation proneness.

The results obtained for developed prototype transformer are given below:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>Primary RMS voltage</td>
<td>560 V</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Secondary RMS voltage</td>
<td>5 kV</td>
</tr>
<tr>
<td>$I_1$</td>
<td>Primary RMS current</td>
<td>3.7 A</td>
</tr>
<tr>
<td>$I_2$</td>
<td>Secondary RMS current</td>
<td>0.8 A</td>
</tr>
<tr>
<td>$f$</td>
<td>Switching frequency</td>
<td>50 kHz</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Primary leakage inductance</td>
<td>70 uH</td>
</tr>
</tbody>
</table>
The output voltage after the rectification is shown in fig. 4.28 which is 5 kV. The applied voltage is 560 Volt so the results are according to the design.

![Graph showing output voltage across Transformer](image)

**Fig. 4.28 Output voltage across Transformer**

### 4.11 Conclusion

The design issues of high-density transformer for high-frequency high-power resonant converters have been studied in this work and several critical technical barriers have been identified and solved. Prototype transformer for a 5 kV charging converter has been developed successfully and tested. The proposed modeling and calculation methods have been verified.

According to characteristics discussed previously, the nanocrystalline material is superior to ferrites and amorphous materials for high-frequency high-power applications. The introduction of nanocrystalline magnetic materials with relatively low loss density, high saturation flux density and high Curie temperature has shown promise for high density magnetic design.