3.0 Introduction

It is known that in the crisp situation the absolute of a topological space can be constructed using the open ultrafilters also \([P; W]\). The set \(\tau(X)\) of all fuzzy open subsets of \(X\) forms a pseudo complemented lattice with 
\[ \mu = 1 - f-cl(\mu) = f-int(1 - \mu). \]
Therefore as in the case of topological spaces fuzzy absolutes also can be constructed using f-open ultrafilters. So in the first section of this chapter we introduce fuzzy open filters. Fuzzy open ultrafilters are defined and an equivalent formulation for such ultrafilters has been given. Using the concept of adherence of an f-open filter we define fixed f-open ultrafilters and they are used to characterize the convergence property of a filter. Here also some results in the crisp topology are not true in the fuzzy context. Counter examples are given where strict analogous results are not possible.

A space \(X\) is said to be H-closed if it is closed in every Hausdorff space containing \(X\) as a subspace. In section 3.2 we give the fuzzy analogue of this concept "Fuzzy Hausdorff closed" or (f-H closed) spaces. The characterization for f-H closed spaces given here establishes the relationship between the f-H closed spaces and their f-open filters.

* Some results of this chapter were presented in the Annual Conference of Kerala Mathematical Association at Kottayam, December 2001.
In the third section we introduce an s-continuous mapping from a topological space to a fuzzy topological space and prove that the image of an H-closed space under an s-continuous map is f-H closed. Here we have also proved that the arbitrary product $\prod_{i} f_{i}$ and the sum $\bigoplus f_{i}$ of the s-continuous maps $f_{i}$ are also s-continuous.

3.1 f-open filters:

3.1.1 Definition: Let $(X, \delta)$ be a fuzzy topological space. Then $P \subseteq \delta$ is said to be a fuzzy open filter or f-open filter if it satisfies the following conditions.

i) $\lambda \in P$ and $\mu \in \delta$ such that $\mu > \lambda$ imply $\mu \in P$

ii) $\lambda, \mu \in P$ implies $\lambda \land \mu \in P$.

iii) $0 \notin P$.

3.1.2 Note: The f-open filter basis and f-open ultrafilter can be defined as in 2.2.5.

3.1.3 Definition: Let $P$ be an f-open filter on $X$. Then a crisp open subset $Y$ of $X$ is said to be included in $P$ if and only if every f-open subset of $X$ with support $Y$ is an element of $P$.

3.1.4 Theorem: Let $F$ be an f-open filter on $(X, \delta)$. Then the following are equivalent.

i) $F$ is an f-open ultrafilter.

ii) Let $\mu \in \delta$. If $\mu \notin F$, then there is some $\gamma \in F$ such that $\mu \land \gamma = 0$.
iii) Let \( Y \) be a crisp open subset of \( X \). Then either \( Y \) or \( Y^c \) is included in \( \mathcal{F} \) where \( Y^c = X \setminus \text{cl}(Y) \)

**Proof:** Similar to that of theorem 2.2.7.

3.1.5 **Note:** Propositions 2.2.8 and 2.2.9 also hold good for \( f \)-open ultrafilters.

3.1.6 **Definitions:**

a. An \( f \)-open ultrafilter \( \mathcal{U} \) on \( X \) is said to be fixed if \( a(\mathcal{U}) = \bigwedge \{ \mu : \mu \in \mathcal{U} \} \neq 0 \) and free if \( a(\mathcal{U}) = 0 \). \( a(\mathcal{U}) \) is called the adherence of \( f \)-open ultrafilter \( \mathcal{U} \).

b. If \( x_p \) is a fuzzy point in \( (X, \delta) \) with support \( x \), then \( N(x_p) = \{ \mu \in \delta : p \leq \mu(x) \} \) is called the set of neighbourhoods of \( x_p \).

3.1.7 **Definition:** Let \( x_p \) be an \( f \)-point in \( X \). Then an \( f \)-open filter \( \mathcal{F} \) on \( X \) is said to converge to \( x_p \) if \( N(x_p) \subset \mathcal{F} \) and it is denoted as \( \mathcal{F} \rightarrow x_p \). The point \( x_p \) is said to be a cluster point of \( \mathcal{F} \) if \( \mu \land F \neq 0 \) for all \( F \in \mathcal{F} \) and \( \mu \in N(x_p) \).

3.1.8 **Lemma:**

Let \( X \) be a fuzzy topological space and \( \mathcal{U} \) be a fixed \( f \)-open ultrafilter on \( X \). Let \( x_p \) be a fuzzy singleton in \( X \). Then \( x_p \in a(\mathcal{U}) \Rightarrow N(x_p) \subset \mathcal{U} \).

**Proof:**

Suppose \( x_p \in a(\mathcal{U}) \) where \( a(\mathcal{U}) = \bigwedge \{ \mu : \mu \in \mathcal{U} \} \)

Then \( a(\mathcal{U})(x) = 1 \).

That is \( \bigwedge \{ \mu : \mu \in \mathcal{U} \}(x) = 1 \)
Therefore, $\bar{\mu}(x) = 1$ for every $\mu \in \mathcal{U}$.

Let $\delta \in N(x_p)$ and $\gamma$ be any arbitrary element of $\mathcal{U}$. Then $\delta(x) = 1$ and $\bar{\gamma}(x) = 1$.

Therefore $(\delta \land \bar{\gamma})(x) = 1$ and so $\delta \land \bar{\gamma} \neq 0$.

If $\delta \land \gamma = 0$, then $\gamma \leq \delta^c$. i.e. $\bar{\gamma} \leq \delta^c$.

$\bar{\gamma}(x) \leq \delta^c(x)$, for every $x \in X$ which is not possible.

Therefore $\delta \land \gamma \neq 0$.

i.e. $\delta \land \gamma \neq 0$ for every $\gamma \in \mathcal{U}$.

Therefore $\delta \in \mathcal{U}$ (by the theorem 3.1.4(ii))

Therefore $N(x_p) \subseteq \mathcal{U}$.

3.1.9 Remark: In the case of non crisp sets the converse of 3.1.8 need not be true as is seen from the following example.

3.1.10 Example: Let $X = \{a, b, c\}$.

Define $\mu_1, \mu_2$ and $\mu_3$ from $X$ to $[0,1]$ as

$\mu_1(a) = \frac{1}{2}$, $\mu_1(b) = \mu_1(c) = 0$

$\mu_2(b) = \frac{1}{2}$, $\mu_2(a) = \mu_2(c) = 0$

$\mu_3(c) = \frac{1}{2}$, $\mu_3(a) = \mu_3(b) = 0$

Then $\tau = \{0, 1, \mu_1, \mu_2, \mu_3, \mu_1 \lor \mu_2, \mu_1 \lor \mu_3, \mu_2 \lor \mu_3, \mu_1 \lor \mu_2 \lor \mu_3\}$ is a fuzzy topology on $X$. Then the closed sets are $\{0, 1, \mu_1', \mu_2', \mu_3', (\mu_1 \lor \mu_2)', (\mu_2 \lor \mu_3)', (\mu_1 \lor \mu_2 \lor \mu_3)'\}$,
Let $\mathcal{U} = \{1, \mu_1, \mu_1 \lor \mu_2, \mu_1 \lor \mu_3, \mu_1 \lor \mu_2 \lor \mu_3\}$. Then $\mathcal{U}$ is an f-open ultrafilter.

$$a(\mathcal{U}) = \bigwedge \{ \overline{\mu} : \mu \in \mathcal{U} \}$$

$$= \bigwedge \{1, \mu_1 \lor \mu_2 \lor \mu_3\} = \mu_1 \lor \mu_2 \lor \mu_3.$$

Now, consider the fuzzy singleton $a_1$. Then

$$N(a_1) = \{ \mu \in \tau(X) : \mu(a) = 1 \} = \{1\}$$

$$\therefore N(a_1) \subseteq \mathcal{U}.$$

But $a(\mathcal{U})(a) = (\mu_1 \lor \mu_2 \lor \mu_3)(a) = \frac{1}{2} \neq 1$

$$\therefore a_1 \notin a(\mathcal{U}).$$

### 3.1.11 Result: Let $\mathcal{U}$ be an f-open ultrafilter and $x_p \in a(\mathcal{U})$. Then $x_p$ is a cluster point of $\mathcal{U}$.

**Proof:**

If $\mathcal{U}$ is an f-open ultrafilter and $x_p \in a(\mathcal{U})$ then by lemma 3.1.8

$$N(x_p) \subseteq \mathcal{U}.$$

Therefore $\mu \land F \neq 0$ for every $\mu \in N(x_p)$ and $F \in \mathcal{U}$.

That is $x_p$ is a cluster point of $\mathcal{U}$.

### 3.1.12 Lemma: Let $C(\mathcal{U})$ denote the set of all cluster points of $\mathcal{U}$. If $\mathcal{U}$ is a fixed f-open ultrafilter then $C(\mathcal{U})$ contains exactly one point.

**Proof:** Since $\mathcal{U}$ is fixed $a(\mathcal{U}) \neq 0$ and so $C(\mathcal{U}) \neq \emptyset$. 
Let \( x_1 \) and \( y_1 \) be two distinct fuzzy singletons such that both belong to \( C(\mathcal{U}) \). Since \( X \) is \( fT_2 \), we can have \( \mu \in N(x_1) \) and \( \gamma \in N(y_1) \) such that \( \mu \land \gamma = 0 \). If \( x_1 \) and \( y_1 \) are cluster points, then \( \mu \land U \neq 0 \) and \( \gamma \land U \neq 0 \) for every \( U \in \mathcal{U} \). Therefore \( \mu \in \mathcal{U} \) and \( \gamma \in \mathcal{U} \) which is not possible.

Hence \( C(\mathcal{U}) \) contains exactly one point.

### 3.2 \( f \)-H closed spaces:

**3.2.1. Definition:** A fuzzy topological space \((X, \delta)\) is said to be closed in \((Y, \tau)\) if \( Cl_Y X = X \). (Closure is with respect to the fuzzy topology of \( Y \)).

\( X \) is said to be fuzzy Hausdorff closed or \((f\text{-}H \text{ closed})\) if \( X \) is closed in every fuzzy Hausdorff space containing \( X \) as a subspace.

**3.2.2. Theorem:** For a fuzzy topological space \( X \), the following are equivalent.

1. \( X \) is \( f\text{-}H \) closed
2. Every \( f \)-open filter on \( X \) has a non empty adherence.
3. For every \( f \)-open cover of \( X \), there is a finite subfamily whose union is dense in \( X \)

**Proof:**

i)\(\Rightarrow\) ii) Let \((X, \delta)\) be \( f\text{-}H \) closed and \( \mathcal{F} \) be an \( f \)-open filter on \( X \) such that \( a(\mathcal{F}) = 0 \) where \( a(\mathcal{F}) = \land\{ \mu : \mu \in \mathcal{F} \} \).

Let \( Y = X \cup \{ \mathcal{F} \} \).
Define $\mu$ to be f-open in $Y$ and write $\mu \in \tau$ if $\mu \land X$ is open in $X$
and $F \in \mu$ implies $\mu \land X \in F$.

Then $\mu_1$ and $\mu_2 \in \tau \Rightarrow \mu_1 \land X \in \delta$ and $\mu_2 \land X \in \delta$.

\[ \therefore (\mu_1 \land \mu_2) \land X \in \delta. \]
\[ F \in \mu_1 \land \mu_2 \Rightarrow F \in \mu_1 \text{ and } F \in \mu_2 \]
\[ \Rightarrow (\mu_1 \land X) \in F \text{ and } (\mu_2 \land X) \in F. \]
\[ \Rightarrow (\mu_1 \land \mu_2) \land X \in F. \]

\[ \therefore \mu_1 \land \mu_2 \in \tau. \]

$\mu_i \in \tau \Rightarrow \mu_i \land X \in \delta$ and $F \in \mu_i$ implies $\mu_i \land X \in F$.

Then $(\lor \mu_i) \land X = \lor (\mu_i \land X) \in \delta$ and $(\lor \mu_i \land X) \in F$.

Therefore $\lor \mu_i \in \tau$.

That is $(Y, \tau)$ is a fuzzy topological space and $X$ is not closed in $Y$.

Since $X$ is fuzzy Hausdorff, for every distinct fuzzy points in $X$ we can have disjoint f-open sets in $Y$.

Let $x_p$ be a fuzzy singleton in $X$ with support $x$. Since $a(F) = 0$, $a(F)(x) = 0$ for every $x \in X$. i.e. there exists some $\gamma \in F$ such that $\tilde{\gamma}(x) = 0$, i.e. $\gamma(x) = 0$. Then there exists an open neighbourhood $\eta$ of $x_p$ such that $\gamma \land \eta = 0$.

Therefore, $\gamma \cup \{F\}$ and $\eta$ are disjoint open neighbourhoods of $F$ and $x_p$ in $Y$.

Therefore $Y$ is Fuzzy Hausdorff. But $X$ is not closed in $Y$. Thus $X$ is not f-H closed.
Therefore, $X$ is $f$-H closed $\Rightarrow a(F) \neq 0$.

ii)$\Rightarrow$i) Suppose every $f$-open filter on $X$ has a non empty adherence. To show that $X$ is $f$-H closed.

Suppose $X$ is not $f$-H closed. i.e., there exists a fuzzy Hausdorff space $Y$ such that $X$ is a subspace and $\text{cl}_Y( X ) \neq X$. Let $x_p$ be a fuzzy singleton in $\text{cl}_Y( X ) \setminus X$ with support $x$.

Let $F = \{ \mu \cap X : \mu \in \tau, \mu(x) = 1 \}$. Then $0 \notin F$.

If $\gamma_1, \gamma_2 \in F \Rightarrow \gamma_1 = \mu_1 \cap X, \mu_1(x) = 1

\gamma_2 = \mu_2 \cap X, \mu_2(x) = 1

\gamma_1 \land \gamma_2 = (\mu_1 \land \mu_2) \land X, (\mu_1 \land \mu_2)(x) = 1$

Therefore $\gamma_1 \land \gamma_2 \in F$.

If $\delta > \gamma$ and $\gamma \in F$, then $\delta \in F$. Therefore $F$ is an $f$-open filter on $X$.

$a(F) = \land \{ \mu : \mu \in F \}$.

$Y$ is fuzzy Hausdorff. Therefore for every distinct fuzzy singleton in $Y$, there exist $f$-open sets $\gamma$ and $\delta$ such that $\gamma \land \delta = 0$

Therefore $a(F) = 0$, which is a contradiction.

Hence $X$ is $f$-H closed.

ii)$\Rightarrow$iii) Suppose every $f$-open filter on $X$ has a non empty adherence.

Let $C = \{ \mu_i : i \in I \}$ be an open cover for $X$ such that for any finite subset $A$ of $C$, $\text{cl}(\bigvee \gamma_i : \gamma_i \in A) \neq 1$. 
Let $\mathcal{F} = \{ \delta : \delta \text{ open and } \delta \geq 1 - \text{cl}(\vee \gamma_i), \gamma_i \in A \}$

Then $0 \notin \mathcal{F}$.

$\mu_1, \mu_2 \in \mathcal{F} \Rightarrow \mu_1 \geq 1 - \text{cl}(\vee \gamma_i) \text{ and } \mu_2 \geq 1 - \text{cl}(\vee \gamma_i)$

$\therefore \mu_1 \land \mu_2 \geq 1 - \text{cl}(\vee \gamma_i)$

$\therefore \mu_1 \land \mu_2 \in \mathcal{F}$.

Let $\eta$ be an f-open set such that $\eta \geq \mu$ and $\mu \in \mathcal{F}$.

Then $\eta \in \mathcal{F}$. Therefore $\mathcal{F}$ is an f-open filter on $X$.

$a_\chi (\mathcal{F}) = \land \{ \bar{\delta} : \delta \in \mathcal{F} \}$

$\bar{\delta} \geq 1 - \text{cl}(\vee \mu_i)$. Therefore, $\land \bar{\delta} \leq \land \{ \text{cl}(1 - \text{cl}(\vee \mu_i)) \}$

$a_\chi (\mathcal{F}) = \land \{ \delta : \delta \in \mathcal{F} \}$

$\leq \land \{ \text{cl}(1 - \text{cl}(\vee \gamma_i)) : \gamma_i \in A \}$.

$\leq \land \{ \text{cl}(1 - \text{cl}(\vee \mu_i)) : \mu_i \in C \}$.

$= \land \{ 1 - \text{int cl}(\vee \mu_i) : \mu_i \in C \}$

$\leq \land \{ 1 - \vee \mu_i : \mu_i \in C \}$

$= 0$ Since $\vee \mu_i = 1$ for $\mu_i \in C$

Therefore $a_\chi (\mathcal{F}) \neq 0 \Rightarrow C$ has a finite sub family whose union is dense in $X$

iii)$\Rightarrow$ii) Let $\mathcal{F}$ be a fuzzy-open filter on $X$ such that $a (\mathcal{F}) = 0$. That is

$\land \{ \bar{\mu} : \mu \in \mathcal{F} \} = 0$. So $\{ 1 - \bar{\mu} : \mu \in \mathcal{F} \}$ is an f-open cover for $X$. Then for a finite subset $\mathcal{A}$ of $\mathcal{F}$, $\{ 1 - \bar{\gamma} : \gamma \in \mathcal{A} \}$ is a finite sub family of this open cover.

f-cl $\{ \vee \{ 1 - \bar{\gamma} : \gamma \in \mathcal{A} \} \} = \vee (f-cl(1 - \bar{\gamma}) : \gamma \in \mathcal{A})$. 

\[ = \vee \{ 1 - f\text{-int}(f\text{-cl} \gamma) : \gamma \in \mathcal{F} \} \]

\[ \leq 1 - (\wedge \gamma : \gamma \in \mathcal{F}) \neq 1 \text{ (since } \wedge \{ \gamma : \gamma \in \mathcal{F} \} \neq 0 \text{)} \]

which is not true by iii)

Therefore \( a_x(\mathcal{F}) \neq 0 \)

3.3 \( s \)-continuous mapping:-

3.3.1 Definitions: Let \( X \) be a fuzzy topological space and \( \{ p_i \} \) be a set of fuzzy points in \( X \). Then for any fuzzy set \( \gamma \) in \( X \), \( \{ p_i \} \) is said to be subordinate to \( \gamma \) denoted as \( \{ p_i \} \subseteq \gamma \) if and only if \( p_i \leq \gamma \) for every \( i \).

3.3.2 Definition: Let \( X \) be a topological space and \( Y \) be a fuzzy topological space. A mapping \( f \) from \( X \) to the set of fuzzy points in \( Y \) is said to be \( s \)-continuous at \( x_0 \in X \) if for every \( f \)-open set \( \gamma \) such that \( f(x_0) \in \gamma \) there is an open neighbourhood \( U \) of \( x_0 \) such that \( f(\text{cl}(U)) \subseteq \text{cl}(\gamma) \). If \( f \) is \( s \)-continuous at every \( x_0 \in X \), then \( f \) is \( s \)-continuous on \( X \).

3.3.3 Example: Let \( X = \mathbb{R} \) be the set of real numbers with usual topology and \( Y \) be the fuzzy topology generated by usual crisp topology in \( \mathbb{R} \) and \( \mu \) where \( \mu : \mathbb{R} \to [0,1] \) is defined as \( \mu(0) = \frac{1}{4} \) and \( \mu(x) = 0 \) \( \forall x \neq 0 \).

Define \( f \) from \( X \) to the set of fuzzy points in \( Y \) as \( f(x) = (x^2)^{1/2} \) (fuzzy point with support \( x^2 \) and value \( \frac{1}{2} \)). Let \( x \in \mathbb{R} \) and \( (a,b) \) be an open interval such that \( x^2 \in (a,b) \). Therefore \( \chi_{(a,b)} \) is an \( f \)-open set in \( Y \) containing \( x^2 \).
Correspondingly there is an open set \( \mathcal{U} = (\sqrt{a}, \sqrt{b}) \) in \( X \) such that \( f([\sqrt{a}, \sqrt{b}]) \) is subordinate to \( \chi_{[a,b]} \). i.e. \( f(\text{cl} (\mathcal{U})) \subset \text{cl} \chi_{(a,b)} \).

3.3.4 Definition(s-regular): A fuzzy topological space \( X \) is said to be strong regular (or s-regular) at \( x \in X \) if for each \( f \)-open set \( \gamma \) such that \( \gamma(x) = 1 \) there exists a crisp open set \( U \) containing \( x \) in \( X \) such that \( U \subset \text{cl} U \subset \gamma \).

3.3.5 Result: Let \( S(X,Y) \) denote the set of all \( s \)-continuous mappings from the topological space \( X \) to the fuzzy topological space \( Y \). If \( Y \) is s-regular then \( S(X,Y) = C(X,Y') \) where \( C(X,Y') \) is the set of all continuous functions from \( X \) to \( Y' \) - the background space of the fuzzy topological space \( Y \).

Proof: Let \( f \in S(X,Y) \). Let \( x_0 \in X \) and \( V \) be any open set in \( Y' \) such that \( f(x_0) \in V \). Then \( V \) is an \( f \)-open set. Since \( f \) is \( s \)-continuous there exists an open set \( U \) containing \( x_0 \) such that \( f(\text{cl}(U)) \subset \text{cl}(V) \). i.e. \( f(U) \subset V \). Therefore, \( f \) is \( s \)-continuous.

\[ : S(X,Y) \subset C(X,Y') \quad (1) \]

Now let \( f \in C(X,Y') \) and \( \gamma \) be an \( f \)-open set in \( Y' \) such that \( f(x_0) \in \gamma \). Since \( Y' \) is s-regular there exists an open set \( V \) in \( \gamma \) such that \( f(x_0) \in V \subset \text{cl} V \subset \gamma \). \( f \) is continuous. Therefore there exists an open set \( U \) in \( X \) such that \( x_0 \in U \) and \( f(U) \subset V \). Therefore \( f(\text{cl}(U)) \subset \text{cl} V \subset \gamma \).

i.e. \( f \) is \( s \)-continuous.

Therefore \( C(X,Y') \subset S(X,Y) \quad (2) \)

Hence \( S(X,Y) = C(X,Y') \).
### 3.3.6 Remark:

The result holds also when $Y$ has the associated topology instead of the background topology.

### 3.3.7 Result:

Let $f$ be an s-continuous mapping from the topological space $X$ to the fuzzy topological space $Y$. If $X$ is $H$-closed, then $Y$ is $f$-$H$ closed.

**Proof:** Let $C$ be an open cover for $Y$. For each $x \in X$, there is an open set $\mu \in C$ such that $f(x) \in \mu$. Since $f$ is s-continuous, there exists an open set say $U(x)$ in $X$ such that $f(\text{cl}(U(x))) \subseteq \text{cl}(\mu)$.

$X$ is $H$-closed. Therefore for every open cover of $X$, there is a finite family whose union is dense in $X$. Therefore there is a finite subset $F$ such that

$$X = \text{cl}(\bigcup \{U(x) : x \in F\})$$

$$= \bigcup \text{cl}\{U(x) : x \in F\}$$

Therefore $f(X) = \bigcup (f \{\text{cl}(U(x)) : x \in F\})$

$$\leq \bigvee (\text{cl}(\mu_i), \quad i = 1, 2, \ldots, n, \quad n = |F|$$

That is $Y \leq \bigvee \text{cl}(\mu_i), \quad \mu_i \in C$

$\therefore$ The open cover $C$ of $Y$ has a finite sub cover whose union is dense in $Y$. Therefore $Y$ is $f$-$H$ closed.

### 3.3.8 Theorem:

Let $\{X_i, i \in I\}$ be a set of topological spaces and $\{Y_i, i \in I\}$ be a set of fuzzy topological spaces. Also for each $i \in I$, let $f_i$ be s-continuous from $X_i$ onto the set of fuzzy singletons in $Y_i$. Then
a) $\pi f_i: \pi X_i \to \pi Y_i$ and

b) $\odot f_i: \odot X_i \to \odot Y_i$ are both s-continuous.

Proof:

a) For each $i$, $f_i$'s are s-continuous from $X_i$ to $Y_i$. Therefore if $x_i \in X_i$ and $y_i$'s are open sets in $Y_i$ such that $f_i(x_i) \in y_i$, then there exist open sets $U_i$ containing $x_i$ such that $f_i(\overline{U_i})$ is subordinate to $\overline{y_i}$.

i.e. $f_i(\overline{U_i}) \subseteq \overline{y_i}$.

Since $y_i$'s are open in $y_i$, $\gamma = \bigwedge_{i=1}^{n} \pi_i^{-1}(y_i)$ is open in $Y = \pi Y_i$.

Let $x \in X = \pi X_i$

$\pi f_i(x) = f(x) = (p_1, p_2, p_3 \ldots)$ where $p_i = f_i(x_i)$, $p_i \leq \gamma$ for every $i$.

Therefore $f(x) \in \gamma$ and $\gamma$ is open.

Let $U = U_1 \times U_2 \times \ldots \times U_n \times X \times X \times \ldots$

Then $U$ is an open set containing $x$.

$f(U) = f(\overline{U_1} \times \overline{U_2} \times \ldots \times \overline{U_n} \times X \times X \times \ldots)$

$= (f_1(\overline{U_1}), f_2(\overline{U_2}), \ldots)$

Each $f_i(\overline{U_1}), f_2(\overline{U_2})$ etc are subordinate to $\overline{y_1}, \overline{y_2}, \ldots$.

Therefore $f(U)$ is subordinate to $\overline{\gamma}$.

Hence $\pi f_i = f$ is s-continuous.

b) Now $f = \odot f_i$ defined from $\odot X_i$ to $\odot Y_i$ as $f(x) = \begin{cases} f_i(x) & \text{if } x \in X_i \\ 0 & \text{otherwise} \end{cases}$ is s-continuous since each $f_i$ is s-continuous.

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