CHAPTER 6

HEAT TRANSFER EFFECTS ON ACCELERATED ISOTHERMAL VERTICAL PLATE WITH MASS FLUX

6.1 INTRODUCTION

Processes involving coupled heat and mass transfer occur frequently in nature. It occurs not only due to temperature difference, but also due to concentration difference or the combination of these two. Quite often, there exist certain industrial processes involving continuous surfaces that move steadily through an otherwise quiescent ambient environment for which a correct assessment of the axial temperature and concentration variation of the material are given relevant importance.

Gupta et al. (1979) studied free convection on flow past a linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis et al. (1981). MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh (1981). Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar (1982). Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Basant Kumar Jha et al. (1991) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources. The skin-friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986).

It is proposed to study unsteady flow past a uniformly accelerated infinite vertical isothermal plate in the presence of uniform mass flux. The dimensionless governing equations are solved using the Laplace-transform technique. Such a study found useful in
hot extrusion of steel, the lamination and melt-spinning processes in the extrusion of polymers.

6.2 HEAT TRANSFER EFFECTS ON ISOTHERMAL VERTICAL PLATE WITH MASS FLUX

6.2.1 MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past a uniformly accelerated isothermal vertical infinite plate with uniform mass flux has been considered. The \( x \)-axis is taken along the plate in the vertically upward direction and the \( y \)-axis is taken normal to the plate. At time \( t' \leq 0 \), the plate and fluid are at the same temperature \( T_\infty \) and concentration \( C'_\infty \). At time \( t' > 0 \), the plate is accelerated with a velocity \( u = \frac{u_0 t'}{v} \) in its own plane against gravitational field. The temperature from the plate to the fluid is maintained uniformly and the concentration level is raised at an uniform rate. Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equations:

\[
\frac{\partial u}{\partial t'} = g \beta (T - T_\infty) + g \beta' (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \quad (6.2.1)
\]

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \quad (6.2.2)
\]

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (6.2.3)
\]
with the following initial and boundary conditions:

\[ u = 0, \quad T = T_x, \quad C' = C'_x \quad \text{for all} \quad y, t \leq 0 \]

\[ t' > 0: \quad u = \frac{u_0 t'}{v}, \quad T = T'_w, \quad \frac{\partial C}{\partial y} = -\frac{j''}{D} \quad \text{at} \quad y = 0 \]

\[ u \to 0, \quad T \to T'_x, \quad C' \to C'_x \quad \text{as} \quad y \to \infty \]

On introducing the following non-dimensional quantities:

\[ U = \frac{u}{u_0}, \quad t = \frac{u_0 t'}{v}, \quad Y = \frac{y u_0}{v}, \]

\[ \theta = \frac{T - T_x}{T'_w - T'_w}, \quad Gr = \frac{g\nu\beta(T'_w - T'_x)}{u_0^3}, \quad C = \frac{Du_0(C' - C'_x)}{j''v^2} \]

\[ Gc = \frac{g\beta^* j''v^2}{Du_0^2}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D} \]

in equations (6.2.1) to (6.2.4), lead to

\[ \frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} \]

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \]

\[ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \]

The initial and boundary conditions in non-dimensional quantities are

\[ U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0 \]

\[ t > 0: \quad U = t, \quad \theta = 1, \quad \frac{\partial C}{\partial Y} = -1 \quad \text{at} \quad Y = 0 \]

\[ U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty \]


6.2.2 METHOD OF SOLUTION

All the physical variables are defined in the nomenclature. The dimensionless governing equations (6.2.6) to (6.2.8), subject to the boundary conditions (6.2.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

\[ \theta = \text{erfc}(\eta\sqrt{Pr}) \]  
(6.2.10)

\[ C = 2\sqrt{t} \left[ \frac{\exp(-\eta^2 Sc)}{\sqrt{\pi} \sqrt{Sc}} - \eta \text{erfc}(\eta\sqrt{Sc}) \right] \]  
(6.2.11)

\[ U = t \left[ (1 + 2 \eta^2) \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] \]

\[ + \frac{Gr t}{(Pr - 1)} \left[ (1 + 2 \eta^2) \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) - (1 + 2 \eta^2 Pr) \text{erfc}(\eta\sqrt{Pr}) \right. \]

\[ \left. + \frac{2\eta\sqrt{Pr}}{\sqrt{\pi}} \exp(-\eta^2 Pr) \right] \]

\[ + \frac{Ge t \sqrt{t}}{3(Sc - 1) \sqrt{Sc}} \left[ \frac{4}{\sqrt{\pi}} (1 + \eta^2) \exp(-\eta^2) - \frac{4}{\sqrt{\pi}} (1 + \eta^2 Sc) \exp(-\eta^2 Sc) - \eta (6 + 4\eta^2) \text{erfc}(\eta) + \eta \sqrt{Sc} (6 + 4 \eta^2 Sc) \text{erfc}(\eta \sqrt{Sc}) \right] \]  
(6.2.12)

where, \( \eta = \frac{Y}{2\sqrt{t}} \).

Where, \text{erfc} is the complementary error function.
6.2.3 RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters $Gr, Gc, Sc, Pr$ and $t$ upon the nature of the flow and transport. The value of the Schmidt number $Sc$ is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number $Pr$ are chosen such that they represent air ($Pr = 0.71$) and water ($Pr = 7.0$). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity profiles for different values of the Schmidt numbers ($Sc = 0.16, 0.3, 0.6, 2.01$), $Gr = 5, Gc = 5$ and time $t = 0.2$ are shown in figure 6.1. in the presence of air. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.

The effect of velocity for different ($t = 0.2, 0.4, 0.6$), $Gr = 5, Pr = 0.71$ and $Gc = 5$ are studied and presented in figure 6.2. It is observed that the velocity increases with increasing values of $t$.

Figure 6.3. demonstrates the effects of different thermal Grashof number ($Gr = 2, 5$), mass Grashof number ($Gc = 2, 5$) and $Pr = 0.71$ on the velocity at time $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The temperature profiles are calculated for water and air from equation (6.2.10) and these are shown in figure 6.4. at time $t = 0.2$. The effect of the Prandtl number plays an important role in temperature field. It is observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.
Figure 6.5 represents the effect of concentration profiles at time \( t = 0.2 \) for different Schmidt numbers \( (Sc = 0.16, 0.3, 0.6, 2.01) \). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.

![Velocity profiles for different values of Sc](image-url)
Figure 6.2. Velocity profiles for different values of $t$

Figure 6.3. Velocity profiles for different values of $Gr$ and $Gc$
Figure 6.4. Temperature profiles for different values of Pr

Figure 6.5. Concentration profiles for different values of Sc
6.3 HEAT TRANSFER EFFECTS ON VARIABLE TEMPERATURE AND MASS FLUX

6.3.1 ANALYSIS

In the previous section, the heat transfer effects on linearly accelerated isothermal vertical plate with mass flux was considered. In this section, the heat transfer effects on linearly accelerated vertical plate with variable temperature and mass flux has been considered. At time \( t' \leq 0 \), the plate and fluid are at the same temperature \( T_\infty \) and concentration \( C'_\infty \). At time \( t' > 0 \), the plate is accelerated with a velocity \( u = \frac{u_0 t'}{\nu} \) in its own plane against gravitational field. The temperature of the plate as well as concentration level near the plate are raised at a uniform rate. It is assumed that the effects of viscous dissipation due to frictional heating is negligible in the energy equation. Then under the usual Boussinesq's approximation the unsteady flow is governed by the following equations:

\[
\frac{\partial u}{\partial t'} = g \beta(T - T_\infty) + g \beta'(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y'^2} \tag{6.3.1}
\]

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y'^2} \tag{6.3.2}
\]

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \tag{6.3.3}
\]

with the following initial and boundary conditions:

\[
u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, t' \leq 0
\]

\[
t' > 0: \quad u = \frac{u_0 t'}{\nu}, \quad \frac{\partial T'}{\partial y} = -\frac{q}{k}, \quad \frac{\partial C'}{\partial y} = -\frac{j''}{D} \quad \text{at} \quad y = 0 \tag{6.3.4}
\]

\[
u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty
\]
On introducing the following non-dimensional quantities:

\[
U = \frac{u}{u_0}, \quad t = \frac{u_0^2't'}{v}, \quad Y = \frac{yu_0}{v},
\]

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\nu \beta (T_w - T_\infty)}{u_0^3}, \quad C = C' - C_\infty, \quad \left(\frac{j''v}{Du_0}\right),
\]

\[
Gc = \frac{g\beta'' j''v^2}{Du_0^4}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D}
\]

in equations (6.3.1) to (6.3.4), lead to

\[
\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2},
\]

(6.3.6)

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2},
\]

(6.3.7)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2}
\]

(6.3.8)

The initial and boundary conditions in non-dimensional quantities are

\[
U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0
\]

\[
t > 0: \quad U = t, \quad \theta = t, \quad \frac{\partial C}{\partial Y} = -1 \quad \text{at} \quad Y = 0
\]

\[
U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty
\]

(6.3.9)

The solutions are in terms of exponential and complementary error function. All the physical variables are defined in the nomenclature. The dimensionless governing equations (6.3.6) to (6.3.8), subject to the boundary conditions (6.3.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:
\[
\theta = t \left[ (1 + 2 \eta^2 Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{Pr} \exp(-\eta^2 Pr) \right] 
\]

\[
C = 2\sqrt{t} \left[ \frac{\exp(-\eta^2 Sc)}{\sqrt{\pi} \sqrt{Sc}} - \eta \text{erfc}(\eta \sqrt{Sc}) \right] 
\]

\[
U = t \left[ (1 + 2 \eta^2) \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] 
\]

\[
+ \frac{Gr t^2}{6(Pr - 1)} \left[ (3 + 12\eta^2 + 4\eta^4) \text{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2) \right.
\]

\[
- (3 + 12\eta^2 Pr + 4\eta^4(Pr)^2) \text{erfc}(\eta \sqrt{Pr}) + \frac{\eta \sqrt{Pr}}{\sqrt{\pi}} (10 + 4\eta^2 Pr) \exp(-\eta^2 Pr) \left. \right] 
\]

\[
+ \frac{Gc t \sqrt{t}}{3(Sc - 1) \sqrt{Sc}} \left[ \frac{4}{\sqrt{\pi}} (1 + \eta^2) \exp(-\eta^2) - \frac{4}{\sqrt{\pi}} (1 + \eta^2 Sc) \exp(-\eta^2 Sc) \right.
\]

\[
- \eta(6 + 4\eta^2) \text{erfc}(\eta) + \eta \sqrt{Sc} (6 + 4\eta^2 Sc) \text{erfc}(\eta \sqrt{Sc}) \left. \right] 
\]

(6.3.12)

where, \( \eta = \frac{Y}{2\sqrt{t}} \).

### 6.3.2 RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters \( Gr, Gc, Sc, Pr \) and \( t \) upon the nature of the flow and transport. The value of the Schmidt number \( Sc \) is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number \( Pr \) are chosen such that they represent air \( (Pr = 0.71) \) and water \( (Pr = 7.0) \). The numerical values of the velocity, temperature
and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The velocity for different Schmidt numbers \((Sc = 0.16, 0.3, 0.6, 2.01)\), \(Gr = 5, Gc = 5\) and \(Pr = 0.71\) time \(t = 0.4\) are shown in figure 6.6. in the presence of air. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.

The effect of velocity for different \((t = 0.2, 0.4, 0.6)\), \(Gr = 5, Pr = 0.71\) and \(Gc = 5\) are studied and presented in figure 6.7. It is observed that the velocity increases with increasing values of \(t\).

Figure 6.8. demonstrates the effects of different thermal Grashof number \((Gr = 2, 5)\), mass Grashof number \((Gc = 2, 5)\) and \(Pr = 0.71\) on the velocity at time \(t = 0.4\). It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The temperature profiles are calculated for water and air from equation (6.3.10) and these are shown in figure 6.9. at time \(t = 0.4\). The effect of the Prandtl number plays an important role in temperature field. It is observed that the temperature increases with decreasing Prandtl number. This shows that the heat transfer is more in air than in water.

Figure 6.10. represents the effect of concentration profiles at time \(t = 0.4\) for different Schmidt numbers \((Sc = 0.16, 0.3, 0.6, 2.01)\). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.
Figure 6.6. Velocity profiles for different values of Sc

Figure 6.7. Velocity profiles for different values of t
Figure 6.8. Velocity profiles for different values of Gr and Gc

Figure 6.9. Temperature profiles for different values of Pr
6.4 CONCLUDING REMARKS

Here the first and second part of this chapter deal with the theoretical study of heat transfers on accelerated isothermal vertical plate with mass flux and variable temperature and mass flux have been analyzed using Laplace-transform technique. The effect of different physical parameters like such as the Schmidt number, thermal Grashof number, mass Grashof number and \( t \) are studied. The conclusion of the study are as follows: The velocity increases with increasing values of \( Gr, Gc \) and \( t \). But, the trend is just reversed with respect to the Schmidt number \( Sc \). The effect of heat transfer is more in air than in water. The wall concentration increases with decreasing the Schmidt number.