CHAPTER 4

MHD CONVECTIVE FLOW PAST AN ACCELERATED INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

4.1 INTRODUCTION

The problem of hydromagnetic convection plays an important role in petroleum industries, geophysics and in astrophysics. It also finds important applications in many engineering problems such as magnetohydrodynamic generator, plasma studies, the study of geological formations, in exploration and thermal recovery of oil and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has applications in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics.

Gupta et al. (1979) studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis et al. (1981). MHD effects on flow past an infinite vertical plate for both the classes of impulse as well as accelerated motion of the plate was studied by Raptis and Singh (1983). Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar (1982). Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Basant Kumar Jha and Ravindra Prasad (1990) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.
The skin-friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Exact solution of flow past an accelerated infinite vertical plate with uniform heat and mass flux analyzed by Muthucumaraswamy et al. (2009).

However the study of flow past a uniformly accelerated infinite vertical plate with variable temperature and mass diffusion has not received attention of any researcher. Hence it is proposed to study the effect of flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and mass diffusion and also study the hydromagnetic effects on flow past an accelerated vertical plate with variable temperature and mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

### 4.2 INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION

#### 4.2.1 BASIC EQUATIONS AND ANALYSIS

In this section, the unsteady flow of a viscous incompressible fluid past an uniformly accelerated vertical infinite plate with variable temperature and uniform mass diffusion has been considered. The $x$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature $T_\infty$ and concentration $C_\infty'$. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_\infty t'}{v}$ in its own plane and the temperature from the plate is raised linearly with respect to time and the concentration level near the plate is raised linearly with time $t$. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:
\[ \frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta'(C' - C'_\infty) + v \frac{\partial^2 u}{\partial y^2} \] (4.2.1)

\[ \rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \] (4.2.2)

\[ \frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \] (4.2.3)

with the following initial and boundary conditions:

\[ u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, t' \leq 0 \]

\[ t' > 0: \quad u = \frac{u_0^2}{v} t', \quad T = T_\infty + (T_w - T_\infty) A t', \quad C' = C'_\infty + (C'_w - C'_\infty) A t' \quad \text{at} \quad y = 0 \]

\[ u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty \] (4.2.4)

where, \( A = \frac{u_0^2}{v} \).

On introducing the following non-dimensional quantities:

\[ U = \frac{u}{u_0}, \quad t = \frac{u_0^2}{v} t', \quad Y = \frac{y u_0}{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g v \beta (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \] (4.2.5)

\[ Gc = \frac{g v \beta' (C'_w - C'_\infty)}{u_0^3}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D} \]

in equations (4.2.1) to (4.2.4), lead to

\[ \frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} \] (4.2.6)
\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}
\]  
(4.2.7)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2}
\]  
(4.2.8)

The initial and boundary conditions in non-dimensional quantities are

\[
U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0
\]

\[
t > 0: \quad U = t, \quad \theta = t, \quad C = t \quad \text{at} \quad Y = 0
\]

\[
U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty
\]  
(4.2.9)

All the physical variables are defined in the nomenclature. The solutions are obtained for flow past an uniformly accelerated infinite isothermal vertical plate in the presence of variable mass diffusion.

### 4.2.2 SOLUTION PROCEDURE

The dimensionless governing equations (4.2.6) to (4.2.8), subject to the initial and boundary conditions (4.2.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

\[
\theta = t \left[ (1 + 2 \eta^2 Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{Pr} \exp(-\eta^2 Pr) \right]
\]  
(4.2.10)

\[
C = t \left[ (1 + 2 \eta^2 Sc) \text{erfc}(\eta \sqrt{Sc}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{Sc} \exp(-\eta^2 Sc) \right]
\]  
(4.2.11)
\[ U = t \left[ (1 + 2 \eta^2) \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] \]

\[ + \frac{Gr t^2}{6(Pr - 1)} \left[ (3 + 12\eta^2 + 4 \eta^4) \text{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4 \eta^2 \exp(-\eta^2)) \right. \]

\[ \left. - (3 + 12\eta^2 \text{Pr} + 4 \eta^4 (\text{Pr})^2) \text{erfc}(\eta \sqrt{\text{Pr}}) - \frac{\eta \sqrt{\text{Pr}}}{\sqrt{\pi}} (10 + 4 \eta^2 \text{Pr} \exp(-\eta^2 \text{Pr}) \right] \]

\[ + \frac{Gc t^2}{6(Sc - 1)} \left[ (3 + 12\eta^2 + 4 \eta^4) \text{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4 \eta^2 \exp(-\eta^2)) \right. \]

\[ \left. - (3 + 12\eta^2 \text{Sc} + 4 \eta^4 (\text{Sc})^2) \text{erfc}(\eta \sqrt{\text{Sc}}) + \frac{\eta \sqrt{\text{Sc}}}{\sqrt{\pi}} (10 + 4 \eta^2 \text{Sc} \exp(-\eta^2 \text{Sc}) \right] \]

(4.2.12)

Where, \( \eta = \frac{Y}{2 \sqrt{t}} \).

Where, \( \text{erfc} \) is the complementary error function.

### 4.2.3 RESULTS AND DISCUSSION

For the purpose of physical understanding of the problem, numerical computations were carried out for different physical parameters \( Gr, Gc, Sc, Pr \) and \( t \) upon the nature of the flow and transport. Then the value of the Prandtl number \( Pr \) is chosen such that it represents air (\( Pr = 0.71 \)). The numerical values of the velocity, the temperature and the concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The effect of velocity for different values of the Schmidt numbers (\( Sc = 0.16, 0.6, 2.01 \)), \( Gr = Gc = 5 \) and time \( t = 0.2 \) are shown in figure 4.1. The trend
shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.

The velocity profiles for different \((t = 0.2, 0.4, 0.6, 0.8)\), \(Gr = 5\) and \(Gc = 5\) are studied and presented in figure 4.2. It is observed that the velocity increases with increasing values of \(t\). Figure 4.3. demonstrates the effects of different thermal Grashof number \((Gr = 2, 5)\) and mass Grashof number \((Gc = 2, 5)\) on the velocity at time \(t = 0.6\). It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

The temperature profiles for different time are calculated for air from equation (4.2.10) and these are shown in figure 4.4. It is observed that the temperature increased with increasing values of time \(t\).

Figure 4.5. represents the effect of concentration profiles for different Schmidt numbers \((Sc = 0.16, 0.6, 2.01)\) and time \((t = 0.2, 0.4)\). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.
Figure 4.1. Velocity profiles for different values of $Sc$

Figure 4.2. Velocity profiles for different values of $t$
Figure 4.3. Velocity profiles for different values of Gr and Gc

Figure 4.4. Temperature profiles for different values of t
From the velocity field, the effect of heat and mass transfer on the skin-friction is studied and is given in dimensionless form as

\[
\tau = -\left(\frac{du}{dy}\right)_{\eta=0} = -\frac{1}{2\sqrt{t}} \left(\frac{du}{d\eta}\right)_{\eta=0}
\]  \hspace{1cm} (4.2.13)

Hence, from equations (4.2.12) and (4.2.13),

\[
\tau = \frac{1}{\sqrt{\pi t}} \left[ 2t - 4 \frac{Gr^2}{3(Pr+1)} - 4 \frac{Gr^2}{3(Sc+1)} \right]
\]  \hspace{1cm} (4.2.14)

The numerical values of \(\tau\) are computed and listed in table 1 for different physical parameters. It is observed from this table, that an increase in the Schmidt number or time leads to fall in skin-friction. The trend is same for increasing thermal
Grashof number or mass Grashof number. It is also observed that the skin-friction is more in water as compared to that in air.

<table>
<thead>
<tr>
<th>t</th>
<th>Pr</th>
<th>Gr</th>
<th>Gc</th>
<th>Sc</th>
<th>τ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.71</td>
<td>2</td>
<td>2</td>
<td>0.6</td>
<td>0.0179</td>
</tr>
<tr>
<td>0.2</td>
<td>0.71</td>
<td>2</td>
<td>5</td>
<td>0.6</td>
<td>-0.3403</td>
</tr>
<tr>
<td>0.2</td>
<td>0.71</td>
<td>5</td>
<td>5</td>
<td>0.6</td>
<td>-0.7123</td>
</tr>
<tr>
<td>0.2</td>
<td>0.71</td>
<td>2</td>
<td>2</td>
<td>0.16</td>
<td>0.0683</td>
</tr>
<tr>
<td>0.2</td>
<td>0.71</td>
<td>2</td>
<td>2</td>
<td>2.01</td>
<td>-0.0687</td>
</tr>
<tr>
<td>0.4</td>
<td>0.71</td>
<td>2</td>
<td>2</td>
<td>0.6</td>
<td>-0.6631</td>
</tr>
<tr>
<td>0.2</td>
<td>7.0</td>
<td>2</td>
<td>2</td>
<td>0.6</td>
<td>-0.2248</td>
</tr>
</tbody>
</table>

From the temperature field, the rate of heat transfer is studied and is given in non-dimensional form as

\[ Nu = -\left( \frac{d\theta}{dy} \right)_{\eta=0} = \frac{2\sqrt{l} \sqrt{Pr}}{\sqrt{\pi}} \]  \hspace{1cm} (4.2.15)

The numerical values are computed and presented in table 2 for different Prandtl number and time.

<table>
<thead>
<tr>
<th>t</th>
<th>Pr</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.71</td>
<td>0.4252</td>
</tr>
<tr>
<td>0.4</td>
<td>0.71</td>
<td>0.6013</td>
</tr>
<tr>
<td>0.2</td>
<td>7.0</td>
<td>1.3351</td>
</tr>
</tbody>
</table>

It is observed that the rate of heat transfer is more in water than in air. This shows that the Nusselt number increases with increasing the Prandtl number.

From the concentration field, the rate of concentration is analyzed, which is expressed in terms of the Sherwood number as
\[ Sh = - \left( \frac{dC}{dy} \right)_{y=0} = \frac{2 \sqrt{t} \sqrt{Sc}}{\sqrt{\pi}} \] (4.2.16)

The numerical values of the Sherwood number is given in table 3 for different Schmidt number and time.

<table>
<thead>
<tr>
<th>t</th>
<th>Sc</th>
<th>Sh</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.16</td>
<td>0.2019</td>
</tr>
<tr>
<td>0.2</td>
<td>0.3</td>
<td>0.2765</td>
</tr>
<tr>
<td>0.2</td>
<td>0.6</td>
<td>0.3910</td>
</tr>
<tr>
<td>0.2</td>
<td>2.01</td>
<td>0.7156</td>
</tr>
<tr>
<td>0.4</td>
<td>0.6</td>
<td>0.5528</td>
</tr>
</tbody>
</table>

It is observed that the Sherwood number increases with increasing values of the Schmidt number.

### 4.3 HYDROMAGNETIC EFFECTS ON ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND MASS DIFFUSION

#### 4.3.1 ANALYSIS

In the previous section, the flow past an accelerated infinite vertical plate with variable temperature and mass diffusion has been considered. In this section the hydromagnetic flow past an accelerated vertical plate with variable temperature and mass diffusion has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature \( T_\infty \) and concentration \( C_\infty' \). At time \( t' > 0 \), the plate is accelerated with a velocity \( u = \frac{u_0 t'}{v} \) in its own plane and the temperature from the plate is raised to \( T_w' \) and the mass is diffused from the plate to the fluid linearly with time. A transverse magnetic field of uniform
strength $B_0$ is assumed to be applied normal to the plate. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

\[
\frac{\partial u}{\partial t'} = g\beta (T - T_\infty) + g\beta' (C' - C'_\infty) + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u
\]  
(4.3.1)

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2}
\]  
(4.3.2)

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2}
\]  
(4.3.3)

With the following initial and boundary conditions:

\[
u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y,t' \leq 0
\]

\[
u > 0: \quad u = \frac{u_0 t'}{v}, \quad T = T_\infty + (T_w - T_\infty) A t', \quad C' = C'_\infty + (C'_w - C'_\infty) A t' \quad \text{at} \quad y = 0
\]

\[
u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty
\]  
(4.3.4)

On introducing the following non-dimensional quantities:

\[
U = \frac{u}{u_0}, \quad \tau = \frac{u_0^2 t'}{v}, \quad Y = \frac{yu_0}{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},
\]

\[
Gr = \frac{g\nu \beta (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \frac{Gc}{u_0^3} = \frac{g\nu \beta' (C'_w - C'_\infty)}{u_0^3}, \quad M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D}
\]  
(4.3.5)
in equations (4.3.1) to (4.3.4), lead to

\[
\frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U
\]  
(4.3.6)

\[
\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2}
\]  
(4.3.7)

\[
\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2}
\]  
(4.3.8)

The initial and boundary conditions in non-dimensional quantities are

\[
\begin{align*}
U &= 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0 \\
t > 0: \quad U = t, \quad \theta = t, \quad C = t \quad \text{at} \quad Y = 0 \\
U &\to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty
\end{align*}
\]  
(4.3.9)

### 4.3.2 METHOD OF SOLUTION

The dimensionless governing equations (4.3.6) to (4.3.8), subject to the initial and boundary conditions (4.3.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

\[
\theta = t \left[ (1 + 2 \eta^2 \Pr) \text{erfc}(\eta \sqrt{\Pr}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{\Pr} \exp(-\eta^2 \Pr) \right]
\]  
(4.3.10)

\[
C = t \left[ (1 + 2 \eta^2 \Sc) \text{erfc}(\eta \sqrt{\Sc}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{\Sc} \exp(-\eta^2 \Sc) \right]
\]  
(4.3.11)
\[ U = \left( \frac{t}{2} (1 + 2ac + c + d) \right) \left[ \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) \right] \]

\[-\frac{\eta \sqrt{t}}{2\sqrt{M}} (1 + 2ac + 2bd) \left[ \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) \right] \]

\[-2c \text{erfc}(\eta \sqrt{Pr}) - c \exp(at) \left[ \exp(2\eta \sqrt{(M + a)t}) \text{erfc}(\eta + \sqrt{(M + a)t}) + \exp(-2\eta \sqrt{(M + a)t}) \text{erfc}(\eta - \sqrt{(M + a)t}) \right] \]

\[-2d \text{erfc}(\eta \sqrt{Sc}) - d \exp(bt) \left[ \exp(2\eta \sqrt{(M + b)t}) \text{erfc}(\eta + \sqrt{(M + b)t}) + \exp(-2\eta \sqrt{(M + b)t}) \text{erfc}(\eta - \sqrt{(M + b)t}) \right] \]

\[+ c \exp(at) \left[ \exp(2\eta \sqrt{a Pr t}) \text{erfc}(\eta \sqrt{Pr} + \sqrt{at}) + \exp(-2\eta \sqrt{a Pr t}) \text{erfc}(\eta \sqrt{Pr} - \sqrt{at}) \right] \]

\[+ d \exp(bt) \left[ \exp(2\eta \sqrt{Sc bt}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{bt}) + \exp(-2\eta \sqrt{Sc bt}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{bt}) \right] \]

\[-2act \left[ (1 + 2\eta^2 \Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{Pr} \exp(-\eta^2 \Pr) \right] \]

\[-2bdt \left[ (1 + 2\eta^2 Sc) \text{erfc}(\eta \sqrt{Sc}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{Sc} \exp(-\eta^2 Sc) \right] \]

(4.3.12)

where, \( a = \frac{M}{Pr - 1}, b = \frac{M}{Sc - 1}, c = \frac{Gr}{2a^2 (1 - Pr)}, d = \frac{Gc}{2b^2 (1 - Sc)} \) and \( \eta = \frac{Y}{2\sqrt{t}} \).
4.3.3 DISCUSSION OF RESULTS

For physical understanding of the problem, numerical computations were carried out for different physical parameters $Gr, Gc, Sc, Pr, M$ and $t$ upon the nature of the flow and transport. The value of the Schmidt number $Sc$ is taken to be 2.01 which corresponds to water-vapor. Also, the values of Prandtl number $Pr$ is taken to be water ($Pr = 7.0$). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like magnetic field parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 4.6. illustrates the effects of the magnetic field parameter on the velocity when ($M = 2, 5, 10$), $Gr = Gc = 5$, $Pr = 7$ and $t = 0.2$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different ($t = 0.2, 0.4, 0.6$), $M = 2$, $Gr = Gc = 5$, $Pr = 7$ are studied and presented in figure 4.7. It is observed that the velocity increases with increasing values of $t$.

Figure 4.8. illustrates the effects of different thermal Grashof number ($Gr = 2, 5$), mass Grashof number ($Gc = 2, 5$) and $M = 2$ on the velocity at time $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.
Figure 4.6. Velocity profiles for different values of $M$

Figure 4.7. Velocity profiles for different values of $t$
4.4 CONCLUDING REMARKS

Here the first and second part of this chapter deal with an exact solution of unsteady flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion. The dimensionless governing equations are tackled using the usual Laplace-transform technique. The effect of different parameters like magnetic field parameter, thermal Grashof number, mass Grashof number and $t$ are studied graphically. The conclusions of the study are (i) The velocity increases with increasing values of thermal Grashof number, mass Grashof number and $t$ (ii) The wall concentration increases with decreasing values of Schmidt number and Magnetic field parameter.