CHAPTER 3

UNSTEADY MHD FLOW PAST AN ACCELERATED INFINITE VERTICAL PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION

3.1 INTRODUCTION

Heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, spacecraft design, solar energy collectors, design of chemical processing equipment, satellites and space vehicles.

Hydromagnetic convection plays an important role in petroleum industries, geophysics and in astrophysics. It also finds important applications in many engineering problems such as magnetohydrodynamic (MHD) generator, plasma studies, the study of geological formations, in exploration and thermal recovery of oil and in the assessment of aquifers, geothermal reservoirs and underground nuclear waste storage sites. MHD flow has application in metrology, solar physics and in motion of earths core. Also it has applications in the field of stellar and planetary magneto spheres, aeronautics, chemical engineering and electronics.

Gupta et al. (1979) studied free convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method. Kafousias and Raptis (1981) extended the above problem to include mass transfer effects subjected to variable suction or injection. Free convection effects on flow past an accelerated vertical plate with variable suction and uniform heat flux in the presence of magnetic field was studied by Raptis et al. (1981). Mass transfer effects on flow past an uniformly accelerated vertical plate was studied by Soundalgekar (1982). Again, mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was
analyzed by Singh and Singh (1983). Basant Kumar Jha and Ravindra Prasad (1990) analyzed mass transfer effects on the flow past an accelerated infinite vertical plate with heat sources.

The objective of the present investigation is to study the effects on flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion and also to study the MHD effects on flow past a uniformly accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function. Such a study is found useful in magnetic control of molten iron flow in the steel industry, liquid metal cooling in nuclear reactors and magnetic suppression of molten semi-conducting materials.

3.2 VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION

3.2.1 MATHEMATICAL ANALYSIS

In this section, the unsteady flow of a viscous incompressible fluid past an uniformly accelerated vertical infinite plate with variable temperature and uniform mass diffusion has been considered. The $x$-axis is taken along the plate in the vertically upward direction and the $y$-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature $T_{\infty}$. At time $t' > 0$, the plate is accelerated with a velocity $u = \frac{u_0 t'}{\nu}$ in its own plane and the temperature from the plate is raised linearly with respect to time and the concentration level near the plate is raised to $C'_w$. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:
\[
\frac{\partial u}{\partial t'\prime} = g\beta(T - T_\infty) + g\beta'(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2}
\]  \hspace{1cm} (3.2.1)

\[
\rho C_p \frac{\partial T}{\partial t'\prime} = k \frac{\partial^2 T}{\partial y^2}
\]  \hspace{1cm} (3.2.2)

\[
\frac{\partial C'}{\partial t'\prime} = D \frac{\partial^2 C'}{\partial y^2}
\]  \hspace{1cm} (3.2.3)

with the following initial and boundary conditions:

\[
u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, t' \le 0
\]

\[
u > 0: \quad u = \frac{u_0^3 t'}{\nu}, \quad T = T_\infty + (T_w - T_\infty) A t', \quad C' = C'_w \quad \text{at} \quad y = 0
\]  \hspace{1cm} (3.2.4)

\[
u \to 0, \quad T \to T_\infty, \quad C' \to C_\infty \quad \text{as} \quad y \to \infty
\]

where, \(A = \frac{u_0^2}{\nu}\).

On introducing the following non-dimensional quantities:

\[
U = \frac{u}{u_0}, \quad t = \frac{u_0^2 t'}{\nu}, \quad Y = \frac{\nu u_0}{\nu},
\]

\[
\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\nu\beta(T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \hspace{1cm} (3.2.5)
\]

\[
Gc = \frac{g\nu\beta^* (C_w' - C'_\infty)}{u_0^3}, \quad Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{\nu}{D}
\]
in equations (3.2.1) to (3.2.4), lead to

\[ \frac{\partial U}{\partial t} = Gr \, \theta + Gc \, C + \frac{\partial^2 U}{\partial Y^2} \]  \hspace{1cm} (3.2.6)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^3 \theta}{\partial Y^2} \]  \hspace{1cm} (3.2.7)

\[ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^3 C}{\partial Y^2} \]  \hspace{1cm} (3.2.8)

The initial and boundary conditions in non-dimensional quantities are

\[ U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0 \]

\[ t > 0: \quad U = t, \quad \theta = t, \quad C = 1 \quad \text{at} \quad Y = 0 \]

\[ U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty \]  \hspace{1cm} (3.2.9)

All the physical variables are defined in the nomenclature. The solutions are obtained for flow past an uniformly accelerated infinite isothermal vertical plate in the presence of variable mass diffusion.

### 3.2.2 METHOD OF SOLUTION

The dimensionless governing equations (3.2.6) to (3.2.8), subject to the initial and boundary conditions (3.2.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

\[ C = \text{erfc}(\eta \sqrt{Sc}) \]  \hspace{1cm} (3.2.10)

\[ \theta = t \left[ (1 + 2\eta^2 Pr) \text{erfc}(\eta \sqrt{Pr}) - \frac{2\eta}{\sqrt{\pi Pr}} \exp(-\eta^2 Pr) \right] \]  \hspace{1cm} (3.2.11)
\[ U = t \left[ (1 + 2\eta^2) \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) \right] + \frac{Gr t^2}{6(Pr - 1)} \left[ (3 + 12\eta^2 + 4\eta^4) \text{erfc}(\eta) - \frac{\eta}{\sqrt{\pi}} (10 + 4\eta^2) \exp(-\eta^2) 
- (3 + 12\eta^2 Pr + 4\eta^4 (Pr)^2) \text{erfc}(\eta \sqrt{Pr}) + \frac{\eta \sqrt{Pr}}{\sqrt{\pi}} (10 + 4\eta^2 \Pr) \exp(-\eta^2 \Pr) \right] \]

\[ + \frac{Gct}{Sc - 1} \left[ (1 + 2\eta^2) \text{erfc}(\eta) - \frac{2\eta}{\sqrt{\pi}} \exp(-\eta^2) - (1 + 2\eta^2 Sc) \text{erfc}(\eta \sqrt{Sc}) \right] \]

\[ + \frac{2\eta \sqrt{Sc}}{\sqrt{\pi}} \exp(-\eta^2 \sqrt{Sc}) \]  
(3.2.12)

where,  \( \eta = \frac{Y}{2\sqrt{t}} \).

Where, \( \text{erfc} \) is the complementary error function.

### 3.2.3 RESULTS AND DISCUSSION

For physical understanding of the problem, numerical computations are carried out for different physical parameters \( Gr, Gc, Sc, Pr \) and \( t \) upon the nature of the flow and transport. The value of the Schmidt number \( Sc \) is taken to be 0.6 which corresponds to water-vapor. Also, the values of Prandtl number \( Pr \) are chosen such that they represent air (\( Pr = 0.71 \)). The numerical values of the velocity and concentration are computed for different physical parameters like Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

The effect of velocity for different values of the Schmidt numbers \( (Sc = 0.16, 0.3, 0.6, 2.01) \), \( Gr = Gc = 5 \) and time \( t = 0.4 \) are shown in figure 3.1. The trend shows that the velocity increases with decreasing Schmidt number. It is observed that the relative variation of the velocity with the magnitude of the Schmidt number.
The velocity profiles for different \( (t = 0.2, 0.4, 0.6), Gr = 5 \) and \( Gc = 5 \) are studied and presented in figure 3.2. It is observed that the velocity increases with increasing values of \( t \).

Figure 3.3 demonstrates the effects of different thermal Grashof number \( (Gr = 2, 5) \) and mass Grashof number \( (Gc = 2, 5) \) on the velocity at time \( t = 0.4 \). It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.

Figure 3.4 represents the effect of concentration profiles for different Schmidt numbers \( (Sc = 0.16, 0.3, 0.6, 2.01) \) and time \( t = 0.4 \). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number.
Figure 3.1. Velocity profiles for different values of $Sc$

Figure 3.2. Velocity profiles for different values of $t$
Figure 3.3. Velocity profiles for different values of $\text{Gr}$ and $\text{Gc}$

Figure 3.4. Concentration profiles for different values of $\text{Sc}$
3.3 MHD EFFECTS ON VARIABLE TEMPERATURE AND MASS DIFFUSION

3.3.1 GOVERNING EQUATIONS

In the previous section, the unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion has been considered. In this section the unsteady flow of a viscous incompressible fluid past an uniformly accelerated isothermal vertical infinite plate with variable mass diffusion in the presence of magnetic field has been considered. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature \( T_\infty \) and concentration \( C'_\infty \). At time \( t' > 0 \), the plate is accelerated with a velocity \( u = \frac{u_0 t'}{v} \) in its own plane and the temperature from the plate is raised to \( T_w \) and the mass is diffused from the plate to the fluid linearly with time. A transverse magnetic field of uniform strength \( B_0 \) is assumed to be applied normal to the plate. Then under usual Boussinesq's approximation the unsteady flow is governed by the following equations:

\[
\frac{\partial u}{\partial t'} = g \beta (T - T_\infty) + g \beta' (C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u \quad (3.3.1)
\]

\[
\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} \quad (3.3.2)
\]

\[
\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3.3.3)
\]

With the following initial and boundary conditions:

\[
u = 0, \quad T = T_\infty, \quad C' = C'_\infty \quad \text{for all} \quad y, t' \leq 0
\]

\[
t' > 0: \quad u = \frac{u_0 t'}{v}, \quad T = T_w + (T_w - T_\infty) A t', \quad C' = C'_w \quad \text{at} \quad y = 0 \quad (3.3.4)
\]

\[
u \to 0, \quad T \to T_\infty, \quad C' \to C'_\infty \quad \text{as} \quad y \to \infty
\]
On introducing the following non-dimensional quantities:

\[ U = \frac{u}{u_0}, \quad t = \frac{u_0^2}{\nu} t', \quad Y = \frac{y u_0}{\nu}, \]

\[ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g \nu \beta (T_w - T_\infty)}{u_0^3}, \quad C = \frac{C' - C''}{C_w - C''_\infty}, \]

\[ Gc = \frac{g \nu \beta' (C'_w - C''_\infty)}{u_0^3}, \quad M = \frac{\sigma B_0^2}{\rho} \left( \frac{\nu}{u_0^2} \right), \quad Pr = \frac{\mu C_p}{k}, \]

\[ Sc = \frac{\nu}{D} \]

in equations (3.3.1) to (3.3.4), lead to

\[ \frac{\partial U}{\partial t} = Gr \theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - M U \]  

(3.3.6)

\[ \frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \]  

(3.3.7)

\[ \frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \]  

(3.3.8)

The initial and boundary conditions in non-dimensional quantities are

\[ U = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad Y, t \leq 0 \]

\[ t > 0: \quad U = t, \quad \theta = t, \quad C = 1 \quad \text{at} \quad Y = 0 \]

\[ U \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad Y \to \infty \]  

(3.3.9)

### 3.3.2 SOLUTION PROCEDURE

The dimensionless governing equations (3.3.6) to (3.3.8), subject to the initial and boundary conditions (3.3.9), are solved by the usual Laplace-transform technique and the solutions are derived as follows:
\[ C = \text{erfc}(\eta \sqrt{Sc}) \]  
\( \text{(3.3.10)} \)

\[ \theta = t \left[ (1 + 2 \eta^2 \text{Pr}) \text{erfc}(\eta \sqrt{\text{Pr}}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{\text{Pr}} \exp(-\eta^2 \text{Pr}) \right] \]  
\( \text{(3.3.11)} \)

\[ U = \left( \frac{t}{2} (1 + 2ac) + c + d \right) \left[ \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) + \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) \right] \]

\[-\frac{\eta \sqrt{t}}{2\sqrt{M}} (1 + 2ac) \sqrt{Mt} \left[ \exp(-2\eta \sqrt{Mt}) \text{erfc}(\eta - \sqrt{Mt}) - \exp(2\eta \sqrt{Mt}) \text{erfc}(\eta + \sqrt{Mt}) \right] \]

\[-2c \text{erfc}(\eta \sqrt{\text{Pr}}) - c \exp(at) \left[ \exp(2\eta \sqrt{(M + a)t}) \text{erfc}(\eta + \sqrt{(M + a)t}) \right.\]
\[\left. + \exp(-2\eta \sqrt{(M + a)t}) \text{erfc}(\eta - \sqrt{(M + a)t}) \right] \]

\[-2d \text{erfc}(\eta \sqrt{Sc}) - d \exp(bt) \left[ \exp(2\eta \sqrt{(M + b)t}) \text{erfc}(\eta + \sqrt{(M + b)t}) \right.\]
\[\left. + \exp(-2\eta \sqrt{(M + b)t}) \text{erfc}(\eta - \sqrt{(M + b)t}) \right] \]
\[+ c \exp(at) \left[ \exp(2\eta \sqrt{a \text{Pr}t}) \text{erfc}(\eta \sqrt{\text{Pr} + \sqrt{at}}) + \exp(-2\eta \sqrt{a \text{Pr}t}) \text{erfc}(\eta \sqrt{\text{Pr} - \sqrt{at}}) \right] \]

\[+ d \exp(bt) \left[ \exp(2\eta \sqrt{Sc \sqrt{bt}) \text{erfc}(\eta \sqrt{Sc} + \sqrt{bt}) + \exp(-2\eta \sqrt{Sc \sqrt{bt}) \text{erfc}(\eta \sqrt{Sc} - \sqrt{bt}) \right] \]

\[-2act \left[ (1 + 2\eta^2 \text{Pr}) \text{erfc}(\eta \sqrt{\text{Pr}}) - \frac{2\eta}{\sqrt{\pi}} \sqrt{\text{Pr}} \exp(-\eta^2 \text{Pr}) \right] \]
\( \text{(3.3.12)} \)

where, \( a = \frac{M}{\text{Pr} - 1}, b = \frac{M}{\text{Sc} - 1}, c = \frac{\text{Gr}}{2a^2(1 - \text{Pr})}, d = \frac{\text{Gc}}{2b(1 - \text{Sc})} \) and \( \eta = \frac{Y}{2\sqrt{t}} \).
3.3.3 DISCUSSION OF RESULTS

The numerical computations are carried out for different physical parameters $Gr, Gc, Sc, Pr, M$ and $t$ upon the nature of the flow and transport. The value of the Schmidt number $Sc$ is taken to be 2.01 which corresponds to water-vapor. Also, the values of Prandtl number $Pr$ is taken to be water ($Pr = 7.0$). The numerical values of the velocity, temperature and concentration are computed for different physical parameters like magnetic field parameter, Prandtl number, thermal Grashof number, mass Grashof number, Schmidt number and time.

Figure 3.5. demonstrates the effects of the Magnetic field parameter on the velocity when $(M = 2, 5, 10)$, $Gr = Gc = 5$, $Pr = 7$ and $t = 0.4$. It is observed that the velocity increases with decreasing values of the magnetic field parameter. This shows that the increase in the magnetic field parameter leads to a fall in the velocity. This agrees with the expectations, since the magnetic field exerts a retarding force on the free convective flow.

The velocity profiles for different $(t = 0.2, 0.4, 0.6)$, $M = 2$, $Gr = Gc = 5$, $Pr = 7$ are studied and presented in figure 3.6. It is observed that the velocity increases with increasing values of $t$.

Figure 3.7. illustrates the effects of different thermal Grashof number ($Gr = 2, 5$), mass Grashof number ($Gc = 2, 5$) and $M = 2$ on the velocity at time $t = 0.4$. It is observed that the velocity increases with increasing values of the thermal Grashof number or mass Grashof number.
Figure 3.5. Velocity profiles for different values of $M$

Figure 3.6. Velocity profiles for different values of $t$
3.4 SUMMARY AND CONCLUSION

In the first and second part of this chapter deals with an exact solution of unsteady flow past an uniformly accelerated infinite vertical plate in the presence of variable temperature and uniform mass diffusion. The dimensionless governing equations are tackled using the usual Laplace-transform technique. The effect of different parameters like magnetic field parameter, thermal Grashof number, mass Grashof number and $t$ are studied graphically. The conclusions of the study are as follows:

- The velocity increases with increasing values of thermal Grashof number, mass Grashof number and $t$.
- The velocity increases with decreasing values of Schmidt number and magnetic field parameter.