CHAPTER 2
SOLID TRANSPORTATION PROBLEM (STP):
A REVIEW

2.1 INTRODUCTION

The classical Transportation Problem (TP) is a special type of linear programming problem and it was originally developed by Hitchcock [1]. The purpose of TP is to transport the goods from sources to destinations. TP is also used in inventory control, manpower planning, personnel allocation, allocation models etc.

The Solid Transportation Problem (STP) is a generalization of the well known classical TP. The necessity of considering this special type of TP arises when there are different types of products are to transported using heterogeneous transportation modes called conveyances. Thus, three item properties are taken into account in the constraints set of STP instead of two constraints (source and destination). The STP was stated by Shell [2] who discussed four different cases based on the given data on the item properties such as three planar sums, two planar sums, one planar and one axial sum, and three axial sums. An STP in typical form is defined as follows. Assume that a homogeneous product is to be transported from each of m sources to n destinations. The sources are production facilities, warehouses, or supply points that are characterized by the available capacities \( a_i, i = 1, 2, 3, \ldots, m \). The destinations are consumption points, warehouses, or demand points that are characterized by required levels of demands \( b_j, j = 1, 2, 3, \ldots, n \). Let \( e_{kj} \) be the number of units transported by \( k \)th type of the conveyance \( k = 1, 2, 3, \ldots, l \) from sources to destinations. The conveyances may be trucks, air freights, freight trains, and ships. A penalty or cost \( c_{ijk} \geq 0 \) and a cost function \( f_{ijk} \) are associated with transportation of a unit of the product from
source $i$ to the destination $j$ by means of the conveyance $k$. The problem is to determine the unknown quantities $x_{ijk}$ of the product to be transported from the each of the sources $i$ to each of the destination $j$ by each of the conveyances $k$ so that the total transportation cost is minimized. The STP can be defined as a minimization problem with linear constraints in the following form.

Minimize $Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} f_{ijk}(x_{ijk})$, $p = 1, 2, 3, ..., P.$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = a_i, \quad i = 1, 2, 3, ..., m. \quad (1)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} = b_j, \quad j = 1, 2, 3, ..., n.$$  

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k, \quad k = 1, 2, 3, ..., l.$$  

where $f_{ijk}(x_{ijk})$ is a linear function, $x_{ijk} \geq 0$, $a_i \geq 0$, $b_j \geq 0$, $e_k \geq 0$, and $c_{ijk} \geq 0$ for all $i, j$ and $k$. Also $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} e_k$ for balanced condition. Here (1) generalizes the classical TP, where there is only one conveyance.

The most studied STP has been the linear STP, i.e. $f_{ijk}(x_{ijk}) = c_{ijk}x_{ijk}$ for all $i, j, k$. The existence of a feasible solution to this problem is guaranteed by Shell [2], and $m+n+l-2$ nonzero values of the decision variables becomes the nondegenerate basic feasible solution. The necessary definitions to reformulate the three axial sums problem (1) as a three planar sums have been showed by Haley[3, 4] and also he described the procedure for the three planar sums, which is an extension of the modified distribution method [5] for the classical TP. Haley [6] addressed the theorems for justifying the use of algorithms and gave a necessary condition for the
existence of a solution. Recently P.Pandian and D.Anuradha [7, 8] proposed a new method called zero point method to find the optimal solution to STP. In this method no need to find an initial basic feasible solution by using any existing method and it is not necessary to use modified distribution method for checking of optimality and improving the basic feasible solution to the STP. But this method directly gives an optimal solution to the STP.

2.2 MULTI-OBJECTIVE STP

In most of the real-life situations, it is necessary to take into account more than one criterion or objective to reflect the problem more realistically to satisfy the given set of constraints. The objectives may be transportation cost, quantity of goods delivered, unfulfilled demand, average delivery time of the commodities, reliability of transportation, accessibility to the users, product deterioration. The posing of Multi-Objective Solid Transportation Problem (MOSTP) in addition to the assumption made in (1) the STP is as follows.

A penalty or cost $c_{ijk} \geq 0$ is associated with transportation of a unit product from source ‘i’ to destination ‘j’ by means of the conveyance ‘k’ for the $p^{th}$ decision criterion. The penalty cost could represent transportation cost, delivery time, quantity of goods delivered, duty paid, underused capacity, etc.,. One must determine the amount of product (unknown quantity) $x_{ijk}$ to be transported from each source ‘i’ to each destination ‘j’ by means of each conveyance ‘k’ such that the total transportation cost is minimized.

Thus the Multi-Objective Solid Transportation Problem (MOSTP) is the problem of minimizing $P$ objective functions which can be formulated as a linear programming problem as follows:
Minimize $Z^p = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} f_{ijk}^p (x_{ijk})$, $p = 1, 2, 3, \ldots, P.$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = a_i, \quad i = 1, 2, 3, \ldots, m.$$  

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = b_j, \quad j = 1, 2, 3, \ldots, n.$$  \hspace{1cm} (2)

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = e_k, \quad k = 1, 2, 3, \ldots, l.$$  

$x_{ijk} \geq 0$, $a_i \geq 0$, $b_j \geq 0$, $c_k \geq 0$, $c_{ijk} \geq 0$ for all $i, j, k$. where the superscripts on $Z^p$ and on $f_{ijk}^p$ denote the $p$th penalty criterion for $p = 1, 2, 3, \ldots, P$, and $\sum_{j=1}^{n} a_i = \sum_{j=1}^{n} b_j$.

To enhance the multiple decision criteria, many researchers are contributing more for multi-objective transportation problems [14, 15, 16, 17, 18]. Many authors for example, H.J. Zimmerman [19], M.K. Lukandjula [20], E.S. Lee and R.J. Li [21] and Y.K.Wu and S.M.Guu [22] developed the solution methodology by using fuzzy logic. Different types of approaches has been addressed and developed by many authors for the multi-objective TP, STP and MOSTP. For example, Ahlatcioglu and Sivri [9] proposed an efficient solution method to MOSTP by reducing the dimension using decomposing techniques. Tzeng et al. [10] formulated fuzzy bi-criteria multi-index TP concerning multiple sources, multiple destinations, multiple commodity and different shipping vessels for coal allocation planning. Bit et al. [11] presented and elaborated application of fuzzy linear programming to the linear MOSTP. Cadenas and Jimenez [12] proposed a genetic algorithm based solution method to the case in
which fuzzy goals are assumed in the MOSTP. Li et al. [13] proposed a neural network approach for MOSTP.

2.3 INTERVAL STP

The interval solid transportation (ISTP) is a generalization of the STP in which input parameters are considered as intervals instead of point values. These type of problems are normally arise when uncertainty occurs in data and decision makers are more convenient to express them as intervals. Here we introduce the STP in which the interval appears in both objective function and in the equality constraints.

A typical ISTP can be defined as

\[
\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} [c_{Lijk}, c_{Rijk}] x_{ijk}
\]

subject to

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = [a_{Li}, a_{Ri}] , \quad i = 1, 2, 3, \ldots, m.
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} = [b_{Lj}, b_{Rj}] , \quad j = 1, 2, 3, \ldots, n. \tag{3}
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = [e_{Lk}, e_{Rk}] , \quad k = 1, 2, 3, \ldots, l.
\]

where \( x_{ijk} \geq 0 \), \( c_{Lijk} \geq 0 \), \( c_{Rijk} \geq 0 \), \( a_{Li} \geq 0 \), \( a_{Ri} \geq 0 \), \( b_{Lj} \geq 0 \), \( b_{Rj} \geq 0 \), \( e_{Lk} \geq 0 \), \( e_{Rk} \geq 0 \)

for all \( i, j \) and \( k \) with \( \sum_{i=1}^{m} a_{Li} = \sum_{j=1}^{n} b_{Lj} = \sum_{k=1}^{l} e_{Lk} \), \( \sum_{i=1}^{m} a_{Ri} = \sum_{j=1}^{n} b_{Rj} = \sum_{k=1}^{l} e_{Rk} \)

(balanced condition). Where \([c_{Lijk}, c_{Rijk}]\) in (3) represents the uncertain cost for the transportation problem; it can represent delivery time, quantity of goods delivered, under used capacity, etc. The source parameter lies between left limit \( a_{Li} \) and right
limit \( a_{R_i} \), similarly, destination parameter lies between left limit \( b_{L_j} \) and right limit \( b_{R_j} \) and conveyance parameter lies between left limit \( e_{L_k} \) and right limit \( e_{R_k} \).

Chanas et al. [23] developed an approach to solve the interval TP. In this approach, interval problem is transformed into a classical TP by adding sources and destinations with suitable supplies and levels of demand in order to make use of the solution procedure exists for the standard TP. Steuer [24] and Tong [25] have proposed linear programming models with interval objective functions. The interval objective in linear programming problem was developed by Inuiguchi and Kume [26] by introducing the minimax regret criterion as used in the decision theory. Chanas and Kuchta [27] have generalized the concept of the solution of the linear programming problem with interval coefficients in the objective function based on preference relations between intervals. S.K. Das, A. Goswami and S.S.Alam [28] have developed theory and methodology for multi-objective transportation problem with interval cost, source and destination parameters. Ishibuchi and Tanaka [29] developed a concept for optimization of multi-objective programming problems with interval objective functions.

2.4 MULTI-OBJECTIVE INTERVAL STP

In most of the cases we need to solve the interval STP with more than one decision criterion, that leads to multi-objective interval solid transportation problem (MOISTP). The MOISTP is defined as

\[
\text{Minimize } Z^p = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} [c^p_{L_{ijk}}, c^p_{R_{ijk}}] x_{ijk}, \quad p = 1, 2, 3,\ldots, P
\]

subject to

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} = [a_{L_i}, a_{R_i}], \quad i = 1, 2, 3,\ldots, m.
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{l} x_{ijk} = [b_{Lj}, b_{Rj}], \quad j = 1, 2, ..., n. \quad (4)
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = [e_{Lk}, e_{Rk}], \quad k = 1, 2, ..., l.
\]

where \(x_{ijk} \geq 0\), \(c_{pLijk} \geq 0\), \(c_{pRijk} \geq 0\), \(a_{Li} \geq 0\), \(a_{Ri} \geq 0\), \(b_{Lj} \geq 0\), \(b_{Rj} \geq 0\), \(e_{Lk} \geq 0\), \(e_{Rk} \geq 0\) for all \(i, j\) and \(k\) with \(a_{Li} = \sum_{j=1}^{n} b_{Lj} = \sum_{k=1}^{l} e_{Lk} = \sum_{j=1}^{n} a_{Ri} = \sum_{j=1}^{n} b_{Rj} = \sum_{k=1}^{l} e_{Rk}\) (balanced condition). The superscript \(p\) of \(Z^p\) and of \([c_{pLijk}, c_{pRijk}]\) denote the \(p\)-th decision criterion and \([c_{pLijk}, c_{pRijk}]\) in (4) for \(p = 1, 2, 3, ..., P\) are intervals representing the uncertain cost for the transportation problem. The rest of the parameters have the same meaning as in the previous problem (3).

Jimenez and Verdegay [30] have developed a nonstandard genetic algorithm for solving MOISTP in which intervals are defined in point values, thereafter no researchers have been focused in the area of MOISTP.

### 2.5 MOISTP UNDER STOCHASTIC ENVIRONMENT

In traditional MOSTP, the sum of supplies, the sum of demands and the sum of conveyances are equal. But in the real world problems it is not guaranteed to ensure this balanced condition. Suppose that there are enough products in ‘m’ sources to satisfy the demands in ‘n’ destinations, also the ‘l’ conveyances have ability to transport products to satisfy the demand of each destination. In this situation the constraints are non-balanced. Thus, for \(a_i, b_j\) and \(e_k\), it is required that \(\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j, \sum_{k=1}^{l} e_k \geq \sum_{j=1}^{n} b_j\). Hence MOISTP is defined as

\[
\text{Minimize } Z^p = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk}^p x_{ijk}, \quad p = 1, 2, 3, ..., P. \quad (5)
\]
subject to

\[ \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i, \quad i = 1, 2, 3, \ldots, m. \quad \text{(I)} \]

\[ \sum_{j=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j, \quad j = 1, 2, 3, \ldots, n. \quad \text{(II)} \]

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, 2, 3, \ldots, l. \quad \text{(III)} \]

where \( x_{ijk} \geq 0, \ c_{ijk}^p \geq 0, \ a_i \geq 0, \ b_j \geq 0, \ e_k \geq 0, \) for all \( i, j \) and \( k \). The superscript \( p \) of \( Z^p \) and \( c_{ijk}^p \) denote the \( p^{th} \) penalty criterion.

In the above mathematical model, the constraints

(I) mean that the total number of units of the product transported from the source ‘i’ cannot exceed its production capacity.

(II) mean that the requirement of each of the destination is atleast satisfied.

(III) mean that the total units of products transported by each conveyance cannot exceed its transportation capacity.

The existence of a feasible solution to (5) is guaranteed [31], and a non-degenerate basic feasible solution contains “\( m + n + l – 2 \)” nonzero values of the variables.

For the problem defined above, the value of the parameters \( a_i, b_j \) and \( e_k \) cannot be obtained in advance, hence they treated as random variables. Then the MOSTP in certain environment becomes a stochastic MOSTP and in this entire thesis only non-balanced interval solid transportation problem in stochastic nature is considered.

In the earlier works [27, 28, 29, 30] concerning the interval solid transportation problem was solved by using fuzzy programming approach and genetic algorithm approach when the intervals are introduced in both objective function and in the constraints, instead point values. No one has discussed the treatment of the MOISTP
in stochastic nature. The first purpose of this thesis is to introduce the nature of the MOSTP with stochastic intervals, its variants and a systematic way to convert the problem with stochastic intervals into traditional mathematical programming problem. Also this technique is generalized to handle the problems with different types of constraints.

In most of the works MOISTP are solved by using modified distribution method or stepping stone method or zero point method or fuzzy method or genetic algorithm or neural network method and finally the solutions are obtained in prescribed nature. In this thesis, the problem designed is of stochastic nature, the total transportation cost becomes stochastic. Therefore it is difficult to handle the problem by certain known methods and hence the probability theory has been extensively used in this thesis to solve the problems with randomness. With the requirement of considering randomness, different types of stochastic programming models have been developed to suit the different needs.

The second purpose of this thesis is to use the first type of stochastic programming called the expected value programming, which optimizes the expected value of objective functions subject to some expected constraints. The third purpose of this thesis is to use the second type of stochastic programming called the chance constrained programming developed by Charnes and Cooper [46] as a means of handling uncertainty by specifying a confidence level at which the stochastic constraints are desirable. Moreover, this approach is extended to solve the MOISTP in stochastic nature with mixed type of constraints. Some of the advantages of this approach are: (i) it simplifies transformation procedure (ii) it gives explicit formula to approximate the performance (iii) it minimize the size of the mathematical programming models.
The fourth purpose of this thesis is to use fuzzy programming approach to solve the crisp equivalent model obtained from one of the previous mentioned stochastic programming methods.

In short it can be said that this thesis answers the unanswered questions in the earlier works, completes and extends earlier works, and develops new direction of research concerned with modeling and solution of MOISTP with stochastic nature. Hence, the MOISTP with stochastic nature is reduced to a standard linear programming problems. In this thesis, all aspects of the models are provided with suitable example.

Statement of MOISTP under stochastic environment, theory of interval arithmetic, stochastic programming, fuzzy programming approach and some numerical examples to illustrate the model is proposed in the next chapter.