Chapter 1

Soliton communication in optical fiber

1.1 Introduction

Development of optical fibers and devices for optical communications began in the early 1960s and continues to grow today. But the real turn around in the field of fiber optics emerge in the late 1980s. During this decade optical communication in communication networks developed with a curiosity of blossoming as the technology of choice for modern communication system. However, there were evolving international telecommunications standards that were predicting very high data rate requirements. Although transmission capacity could be obtained from conventional cable, microwave and satellite technologies, there was a definite shortage of transmission capacity for the term data transfer requirements. Fiber optic transmission systems have provided enormous capacity required to overcome short falls due the various catastrophic effects like noise, attenuation, dispersion and nonlinear effects. Fiber optic transmission systems have many advantages over most conventional transmission systems. They are less affected by noise, do not conduct electricity and therefore provide electrical isolation, carry extremely high data transmission rates and carry data over very long distances.
In 1880, pioneer of communication system, Alexander Graham Bell invented a device called photophone, which contained a membrane made of reflective material [1]. When sound caused photophone to vibrate, it would modulate a light beam that was shining on it and reflect this light to a distant location. The reflected light could then be demodulated using another photophone. Applying this method, Bell was able to communicate to a maximum distance of 213 meters. This was the first experiment on optical communication. After this experiment by Alexander Graham Bell, there were hardly many works in the field of optical communications. This is purely because of the lack of suitable light source that could reliably be used as the information carrier. The advent of lasers in 1960 had revolutionized the field of optics and development of several optics based communication systems [2]. The first such modern optical communication experiments involved laser-beam transmission through the atmosphere. However, it was soon realized that laser beams could not be sent in the open atmosphere for reasonably long distances to carry signals unlike, for example, microwave or radio systems operating at longer wavelengths. This is due to the fact that a laser beam is severely attenuated and distorted owing to scattering and absorption by the atmosphere. Thus for reliable long-distance light-wave communication in terrestrial environments it would be necessary to provide a transmission medium that could protect the signal-carrying light beam from the vagaries of the terrestrial atmosphere. In 1966, Kao and Hockham made an extremely important suggestion; they noted that optical fibers based on silica glass could provide the necessary transmission medium, provided the medium is free from metallic and other impurities [3]. A remarkable work of Kao and Hockham was reported in 1966, the most transparent glass available had extremely high losses; this high loss was primarily due to the presence of some amounts of impurities in the glass [3]. They triggered the beginning of ultra-purification of glasses, which resulted in the realization of low loss optical
fibers. They had also arrived into a conclusion that losses are not an intrinsic property of the glass itself, but rather were due to impurities. For this remarkable insight, Kao was honored by the Nobel prize in physics in 2009. In 1970, Kapron et al were successful in producing silica fibers with a loss of about 17 dB/km at the Helium-Neon laser wavelength of 632.8 nm [4]. Since then, technology has advanced with tremendous rapidity. By 1986 glass fibers were being produced with extremely low loss of 0.2 dB/km [5]. Because of such low losses, the distance between two consecutive repeaters (used for amplifying and restoring the attenuated signals) could be as large as 250 km. In 1986, erbium doped fiber amplifier (EDFA) was invented and made tremendous revolution in long distance optical communication systems [6]. The optical amplifier has replaced the rather involved process of the fiber optic repeater station. In 2001, using glass fibers as the transmission medium and light waves as carrier waves, information was transmitted at a rate of more than 1 Tb/s through one hair-thin optical fiber [7]. Experimental demonstration of transmission at the rate of 14 Tb/s over a 160 km-long single fibers was demonstrated in 2006, which is equivalent to sending 140 digital high definition movies in one second [8]. This can be considered to be one of the most remarkable achievement ever in the history of optical communications. In the following section, we have discussed the propagation characteristics of optical fibers with special applications to optical communication systems.

1.2 Optical fiber

The heart of an optical communication system is the optical fiber that acts as the transmission channel carrying the light beam loaded with information. The light beam is guided through the optical fiber due to the phenomenon of total internal reflection. An optical fiber is a very narrow and long cylindrical thin strand of silica glass in geometry quite like a human hair with special characteristics. When light
injected at one end of the fiber remain confined within the fiber and it leaves the fiber at the other end. The light is guided down the center of the fiber called the core. The core is surrounded by a optical material called the cladding. The core and cladding are usually made of ultra-pure glass. The fiber is coated with a protective plastic covering called the primary buffer coating that protects it from moisture and other damage. More protection is provided by the cable, which has the fibers and strength members inside an outer covering called a jacket. Figure 1.1 shows the schematic sketch of the optical fiber. The refractive index of the core, $n_{cr}$, is higher than that of the cladding, $n_{cl}$. Therefore the light beam that is coupled to the end face of the waveguide is confined in the core by total internal reflection. The condition for total internal reflection at the core-cladding interface is given by $n_{cr} \sin(\pi/2 - \phi) \geq n_{cl}$. Since the angle $\phi$ is related with the incident angle $\theta$ by $\sin\theta = n_{cr} \sin\phi \leq \sqrt{n_{cr}^2 - n_{cl}^2}$, the critical condition for the total internal reflection can be written as

$$\theta \leq \sin^{-1}\sqrt{n_{cr}^2 - n_{cl}^2} \equiv \theta_{\text{max}}. \quad (1.1)$$

The refractive index difference between the core and cladding is of the order of $n_{cr} - n_{cl} = 0.01$. Then $\theta_{\text{max}}$ in equation (1.1) can be approximated by

$$\theta_{\text{max}} \approx \sqrt{n_{cr}^2 - n_{cl}^2}. \quad (1.2)$$
θ_{max} denotes the maximum light acceptance angle of the waveguide and is known as the numerical aperture (NA). The relative refractive index difference between \( n_{cr} \) and \( n_{cl} \) is defined as
\[
\Delta = \frac{n_{cr}^2 - n_{cl}^2}{2n_{cr}^2} \approx \frac{n_{cr} - n_{cl}}{n_{cr}}. \tag{1.3}
\]
\( \Delta \) is commonly expressed as a percentage. NA is related to the relative refractive index difference \( \Delta \) by
\[
NA = \theta_{max} \approx n_{cr} \sqrt{2 \Delta}. \tag{1.4}
\]
The maximum angle for the propagating light within the core is given by
\[
\phi_{max} = \frac{\theta_{max}}{n_{cr}} \approx \sqrt{2 \Delta}. \tag{1.5}
\]
For typical communication optical fibers, \( NA = 0.21 \), \( \theta_{max} = 12^\circ \) and \( \phi_{max} = 18^\circ \) when \( n_{cr} = 1.47 \), \( n_{cl} = 1.455 \) and \( \Delta = 1\% \).

Optical fibers are classified into single mode and multi mode fibers depends upon the number of rays of light that can be carried down the fiber at an instant. This is referred to as the mode of operation of the fiber, a mode in simply means a ray of light in the context of the optical fiber. The higher the mode of operation of an optical fiber, more rays of light that can be guided through the core. The number of discrete modes in optical fiber is determined by the normalized frequency (V number) is given by
\[
V = \left(\frac{2\pi a}{\lambda}\right) \sqrt{n_{cr}^2 - n_{cl}^2}, \tag{1.6}
\]
where \( a \) is the core radius and \( \lambda \) is the wavelength of the light. It can be seen from this formula that as the diameter and the NA (equation (1.2)) decrease, so do the number of modes that can propagate down the fiber. The decrease in modes is more significant with a reduction in diameter than the reduction in NA. The NA tends to remain relatively constant for a wide range of diameters. Its noteworthy that as the diameter starts to approximate the wavelength of light, then only one mode
(\(V < 2.405\)) can travel down the fiber. This state of art is referred to as single mode propagation. Multimode (\(V > 2.405\)) fibers are characterized by a relatively large diameter of the light guiding core, which is much larger than the wavelength of the light. In general, for the multimode fiber, the core diameter is \(50 \mu m\) and the cladding diameter is \(125 \mu m\). In contrast, single-mode fibers have a core diameter larger than the wavelength only by a small factor; typical values range between \(8 \mu m\) and \(10 \mu m\) which is the same as for a multimode fiber; indeed, the cladding diameter of \(125 \mu m\) is the standard value for both fiber types. Based on the refractive index profile, multimode fibers can be distinguished in two major types such as step index and graded index optical fibers. When the core and cladding of each have a constant refractive index across their cross sectional area but the core refractive index being different to the cladding refraction index, then they behave like an optical waveguide. This is referred to as step index optical fiber. A graded index multimode fiber has a core where the refractive index gradually varies across the cross-section. The refractive index is maximum at its center and decreases gradually moving from the center to the edge. Because of this smooth changing refractive index, the light ray refracts as it moves through the core, and sets up a set of sinusoidal light wave patterns in the fiber. If an optical signal is sent down a narrow fiber, it will be degraded by its passage down the fiber. It will emerge (depending on the distance) much weaker, lengthened in time, and distorted in many ways. The reasons for this are as follow:

### 1.3 Attenuation

The optical signal will be weaker as it propagates down the fiber because all glass absorbs light. More accurately, impurities in the glass can absorb light but the glass itself does not absorb light at the wavelengths of interest. In addition to the loss due to material absorption, variations in the uniformity of the glass cause scattering of the light. Both the rate of light absorption and the amount of scattering are dependent on
the wavelength of the light and the characteristics of the particular glass. Most of the optical loss in a modern fiber is caused by Rayleigh scattering. Typical attenuation characteristics of fiber for varying wavelengths of light are illustrated in figure 1.2.

![Figure 1.2: Spectral dependence of the contributions to energy loss in a fiber.](image)

There are a wide range of glasses available and their characteristics vary depending on their chemical composition. Over the past three decades the transmission properties of glass have been improved considerably [3, 5, 9]. As the figure 1.2 shows, absorption varies considerably with wavelength and the two curves show just how different the characteristics of different glasses can be. Most of the attenuation in fiber is caused by light being scattered by finite variations in the density fluctuation of the glass. This is called Rayleigh scattering. From the definition, Rayleigh scattering is inversely proportional to the fourth power of the wavelength. In principle, it is clear from the definition that the lower wavelength components are scattered more than that of the higher wavelength which is illustrated in figure 1.2. The absorption peak shown in figure 1.2 is centered at 1.385 μm but it is broadened by several factors including the action of ambient heat. This absorption is caused by the presence of the $OH^-$ atomic bond, that is, the presence of water. Water is extremely hard to eliminate from the fiber during manufacturing and the small residual peak shown in
the diagram is typical of good quality fibers. In the early days of optical fiber communications impurities in the glass were the chief source of attenuation. Iron (Fe), chromium (Cr) and nickel (Ni) can cause significant absorption even in quantities as low as one part per billion. Nowadays, techniques of purifying silica have improved to the point, where impurities are no longer a significant concern from communication point of view [9].

The attenuation, which represents a decrease in optical power per unit length, is the most fundamental property of optical fibers. It is directly related to the power of optical signals that reach at the end of optical fiber after transmission is defined as [10]

\[
\alpha_{dB} = -\frac{10}{L} \log \left( \frac{P_T}{P_0} \right),
\]

where \( \alpha \) is the attenuation of the optical fiber in the unit of \( \text{dB/km} \), \( L \) is the length of the fiber, \( P_T \) is the transmitted power and \( P_0 \) is the power launched at the input of a fiber.

### 1.4 Dispersion

Dispersion is one of the most important distortion effect in the optical fibers. In the optical fibers, dispersion are classified into two types such as modal dispersion and chromatic dispersion. The major dispersion effect in multi mode fibers is modal dispersion. In the single mode fibers, there are no modal dispersion effects occur. Here, we will restrict our discussion to single mode fibers, which are the principal types of fibers used in today’s long haul communication systems. In the single mode fiber, the wavelength dependence of the material refractive index is behind two different type of dispersion as material dispersion and waveguide dispersion. They are collectively called as chromatic dispersion.

Material dispersion is a phenomenon that occurs because optical pulse put out a
signal, which contains a number of different wavelengths. As the different wavelengths travel through the optical fiber, they will effectively encounter different refractive indices. This means different spectral components of optical pulse will be traveling at different speeds. Thus, some wavelengths arrive before others and a optical pulse disperses. The effects of material dispersion become more pronounced in single mode fibers because of the higher bandwidths that are expected of them.

![Figure 1.3: Typical chirping caused in (a) the anomalous dispersion regime and (b) the normal dispersion regime when an unchirped pulse propagates through the fiber. The chirp is of opposite sign in the two cases.](image)

The second form of dispersion that makes up chromatic dispersion is waveguide dispersion. Waveguide dispersion occurs in single mode fibers where a certain amount of the light travels in the cladding. The dispersion occurs because the light moves faster in the low refractive index cladding than in the higher refractive index core.
The degree of waveguide dispersion depends on the proportion of light that travels in the cladding.

Chromatic dispersion is a measure of the change in the refractive index with wavelength. For this reason dispersion measurement can either take positive or negative values. That is, the change in refractive index with wavelength can be an increase or a decrease. The positive and the negative sign of the chromatic dispersion is called anomalous dispersion and normal dispersion regimes respectively. In the anomalous dispersion regime, frequency within the dispersed pulse decreases with time. This leads to a leading edge which is blue shifted and a trailing edge which is red shifted. In the normal dispersion regime, frequency within the dispersed pulse increases with time. This leads to a leading edge which is red shifted and a trailing edge which is blue shifted. These frequency shifts induce chirp over the whole duration of light pulse. Figure 1.3 shows the typical chirping in the anomalous dispersion and the normal dispersion regimes of propagation.

![Figure 1.3: Typical chirping in the anomalous dispersion and the normal dispersion regimes of propagation.](image)

In figure 1.4, the positive slope and the negative slope represent the variation of material dispersion and the waveguide dispersion, respectively [10]. At roughly
1.31 \mu m$, the two dispersion types tend to cancel out each other. This is referred to as the zero dispersion wavelength which is depicted in figure 1.4. In a physical sense, it can be perceived that material dispersion is causing the pulse to travel faster (relative to other wavelengths) and waveguide dispersion is causing the pulse to travel slower, and therefore the overall effect is the cancelation of some of this movement.

For the case of long haul-high speed communications, 1.55 \mu m is considered to be the operating wavelength of choice, because of the lower signal attenuation at this wavelength compared to the 1.31 \mu m operation. Another method is the so-called dispersion shifted fibers, that find its application in the optical fibers because of its lower chromatic dispersion at 1.55 \mu m [10,12]. It is not possible to change the overall effect of material dispersion, as this is a function of glass material itself. But as waveguide dispersion is caused by a certain amount of light traveling in the cladding, it is possible to change the construction of the core and cladding (and also to add further layers of cladding) so that the waveguide dispersion is shifted downward and therefore makes the zero dispersion wavelength move toward 1.55 \mu m [12].

### 1.5 Nonlinear optics

Nonlinear optics is the branch of optics that deals the interaction of light with matter in the regime where the response of the material system to the applied electromagnetic field is nonlinear in the amplitude of this field. At low light intensities, typical of non-laser sources, the properties of materials remain independent of the intensity of illumination. The superposition principle holds true in this regime, and light waves with matters can pass through materials or gets reflected from boundaries and interfaces without interacting with each other. Laser sources, on the other hand, can provide sufficiently high optical intensities to modify the optical properties of materials. Optical waves can then interact with each other, exchanging momentum and energy, and the superposition principle is no longer valid. This interaction of light
waves can result in the generation of optical fields at new frequencies, including optical harmonics of incident radiation or sum- or difference-frequency signals. Nonlinear optical phenomena are characterized by the ability of laser pulse to influence the optical properties in a material, which would affect not only its own propagation, but also the dynamics of other pulses as well. For very intense electromagnetic fields, all dielectrics behave nonlinearly; i.e., the properties of materials also depend on the pulse intensity. This can be very easily seen from the relation connecting the induced polarization $P$ and the electric field $E$ as [10]

$$P(r,t) = \varepsilon_0 \left( \chi^{(1)} \cdot E(r,t) + \chi^{(2)} : E(r,t)E(r,t) + \chi^{(3)} : E(r,t)E(r,t)E(r,t) + \cdots \right), \quad (1.8)$$

where $\varepsilon_0$ is the vacuum permittivity, and the $l^{th}$-order susceptibility represents the material-mediated interaction between $n$ fields. As discussed in the previous section, the linear effects are due to $\chi^{(1)}$, $\chi^{(2)}$ and $\chi^{(3)}$ are the second- and third-order nonlinear susceptibility, respectively. The second-order susceptibility is the source of the second-order nonlinearities, such as the second harmonic generation and the linear electrooptic effect [10, 11]. Second harmonic generation is a process by which the frequency of an incident beam of light propagating in a nonlinear medium is doubled. Similarly, in third harmonic generation, the frequency is tripled by a nonlinear medium. The third-order susceptibility, in turn, is the source of third-order effects, such as third harmonic generation, the quadratic electrooptic effect also known as the Kerr effect, and electrochromism [10,11]. Intensity of output light resulting through a higher-order harmonic generation is lesser than the lower-order harmonic generation. All the electrooptic effects result in the change of the material’s refractive index in response to the applied fields. Electrochromism is a process by which the absorption spectrum of a medium is shifted after the application of an external field.

The second-order susceptibility $\chi^{(2)}$ vanishes in the case of silica fiber since $SiO_2$ is a centro-symmetric molecule [10, 11]. Hence, the lowest-order nonlinear effects
in fiber originate from the third-order susceptibility $\chi^{(3)}$. Perhaps, the third-order nonlinear effect so called Kerr-effect is considered to be significant applications in optical fiber communications. Though the importance of nonlinear optics has been realized in many fields, its role in the field of communication is more noticeable as this present era is decidedly known to be the information era. Hence, nonlinearity is an essential contender to be discussed elaborately in the context of fiber optics. In the succeeding section, some of the nonlinear effects that play a vital role in long distance communication are briefly discussed.

1.5.1 Nonlinear effects

In general, phenomena whereby the material properties of a given medium change when high intensity light is transmitted through it are called nonlinear effects (optical Kerr effect). The nonlinear refractive index ($10^{-20}m^2/W$) of silica is significantly small compared with other various glasses [10]. It has, however, become important recently, because optical fiber amplifiers increase the optical signal power and the number of multiplexed optical signals in the optical fiber, resulting in noticeable nonlinear effects even in silica fibers [10].

The nonlinear effects that are of major concern for optical fibers are elastic and inelastic effects. Elastic effects where although the optical pulse interacts with and affected by the presence of medium there is no energy exchange between the two. This may cause various detrimental effects in the optical signal transmission, such as self-phase modulation (SPM), cross-phase modulation (XPM) and four wave mixing (FWM).

i) **Self-phase modulation:** As a result of Kerr effects, the refractive index of the material of the fiber experienced by an optical pulse varies depending on the point within the pulse. At different points within a single pulse of light in the fiber the refractive index of the fiber is different. So there is a difference
between the refractive indices at the leading edge, at the trailing edge and in the middle (figure 1.5). This changes the phase of the light waves that make up the pulse. Changes in phase amount to changes in frequency. Therefore the frequency spectrum of the pulse is broadened. In many situations the pulse can also be spread out and distorted. SPM creates a chirp over the whole duration of a pulse [10,11]. This chirp is conceptually like the chirp created by chromatic dispersion in the normal dispersion regime. That is, the chirped pulse has the long wavelengths at the beginning and the short ones at the end. However, unlike chromatic dispersion the chirp produced by SPM is radically different depending on the pulse shape and the instantaneous levels of power within the pulse. Over the duration of a Gaussian shaped pulse the chirp is reasonably even and gradual. If the pulse involves an abrupt change in power level, such as in the case of a square pulse, then the amount of chirp is much greater than in the case with a Gaussian pulse. However, in a square pulse the chirp is confined to the beginning and end of the pulse with little or no effect in the middle. The third-order nonlinearity gives rise to an intensity dependent additive to the refractive index [10]:

\[ n = n_0 + n_2 I(\tau), \]

(1.9)

where \( \tau = t - z/v_g \), \( v_g \) is the group velocity, \( n_0 \) is the linear refractive index of the medium, \( n_2 = (3\chi^{(3)}(\omega;\omega,\omega,-\omega)/(8n_0)) \) is the nonlinear refractive index and \( I(\tau) \) is the intensity of laser radiation. Then, the nonlinear phase shift \( \phi(\tau) \) at distance \( L \) is given by

\[ \phi(\tau) = \left( \frac{\omega}{c} \right) n_2 I(\tau)L. \]

(1.10)

Due to the time dependence of the radiation intensity within the light pulse,
the nonlinear phase shift is also time-dependent, giving rise to a generally time-dependent frequency deviation as follows

\[
\Delta \omega(\tau) = \left( \frac{\omega n_2 L}{c} \right) \frac{\partial I(\tau)}{\partial \tau}.
\]  

(1.11)

The resulting spectral broadening of the pulse can be estimated in the following form:

\[
\Delta \omega(\tau) = \left( \frac{\omega n_2 L}{c \tau_0} \right) I_0,
\]  

(1.12)

where \( I_0 \) and \( \tau_0 \) represent the initial intensity and initial width of the optical pulse respectively and \( \tau \) is the retarded time.

Figure 1.5: Typical chirping caused in nonlinear medium due to the Kerr nonlinear effect.

ii) **Cross-phase modulation:** When there are multiple signals at different wavelengths in the same fiber, Kerr effect caused by one signal can result in phase modulation of the other signal(s). This is called XPM because it acts between multiple signals rather than within a single signal. In contrast to other nonlinear effects XPM effect involves no power transfer between signals. The result can be asymmetric spectral broadening and distortion of the pulse shape. The
cross-action of a one (pump) pulse with a frequency \( \omega_1 \) on another (probe) pulse with a frequency \( \omega_2 \) gives rise to a phase shift of the pump pulse, which can be written as [10]

\[
\Phi_c(\tau) = \left( \frac{\omega n_{2c} L}{c} \right) I_2(\tau),
\]

(1.13)

where \( n_{2c} = (3\chi^{(3)}(\omega_1; \omega_1, \omega_2, -\omega_2)/(4n_0)) \) is the nonlinear refractive index that depends upon the intensity \( I_2 \) of the probe pulse. Taking the time derivative of the nonlinear phase shift, we arrive at the following expression for the frequency \( (\Delta\omega_c) \) deviation of the probe pulse

\[
\Delta\omega_c(\tau) = \left( \frac{\omega n_{2c} L}{c} \right) \frac{\partial I_2(\tau)}{\partial \tau}.
\]

(1.14)

Similar to SPM, XPM can also be employed for pulse compression. The dependence of the chirp of the probe pulse on the pump pulse intensity can be used to control the parameters of ultra-short pulses [13]. XPM also opens the ways to study the dynamics of ultra-fast nonlinear processes, including multi-photon ionization, and to characterize ultra-short light pulses through phase measurements on a short probe pulse [14].

iii) **Four wave mixing:** FWM occurs when two or more waves propagate in the same direction in the same (single-mode) fiber. The signals mix to produce new signals at wavelengths which are spaced at the same intervals as the mixing signals. This is easier to understand if we use frequency instead of wavelength for the description. In general FWM (figure 1.6), three laser fields with frequencies \( \omega_1, \omega_2, \) and \( \omega_3 \) generate the fourth field with a frequency \( \omega_4 = \omega_1 \pm \omega_2 \pm \omega_3 \). In the case when all the three laser fields have the same frequency (when all the three pump photons are taken from the same laser field), \( \omega_1 = 3\omega \), and deal with third-harmonic generation, which is considered in greater detail for short-pulse interactions. If the frequency difference \( \omega_1 - \omega_2 \) of two of the laser fields is tuned
Figure 1.6: The effects of four wave mixing on two or three channel transmission through a single mode fiber.

to a resonance with a Raman-active mode of the nonlinear medium, the FWM process $\omega_4 = \omega_1 - \omega_2 + \omega_3 = \omega_R$ is referred to as coherent anti-Stokes Raman scattering. An FWM process involving four fields of the same frequency with $\omega_4 = \omega = \omega - \omega + \omega$ corresponds to degenerate FWM. The FWM field in this nonlinear-optical process is the phase conjugate of one of the laser fields, giving rise to another name of this type of FWM is called optical phase conjugation. For the general-type FWM $\omega_4 = \omega_1 + \omega_2 + \omega_3$, we represent the pump fields as

$$E(r, t) = Q_j(z, t)exp(ik_jz - i\omega_jt) + c.c., \quad (1.15)$$

where $j = 1, 2, 3$, $c.c.$ represent the complex conjugate, $k_j$ are the wave numbers of the pump fields and $Q_j$ represent the slowly varying envelopes. By substituting equation (1.15) into equation (1.8), the pump fields give the following expression (centro-symmetric material) for the envelope of the FWM field as [10]

$$Q_4 = \frac{\omega n_{2f}}{c}(Q_1Q_2Q_3e^{i\theta_+} + Q_1Q_2Q_3^*e^{i\theta_-} + ...), \quad (1.16)$$

where

$$n_{2f} = (3/4n_0)\chi^{(3)}(\omega_4; \omega_1, \omega_2, \omega_3)$$

$$\theta_+ = (k_1 + k_2 + k_3 - k_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t,$$

$$\theta_- = (k_1 + k_2 - k_3 - k_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t.$$
Phase matching, as can be seen from (1.16), is the key requirement for high efficiency of frequency conversion in FWM. The fields involved in FWM can be phase matched by choosing appropriate angles between the wave vectors of the laser fields and the wave number of the nonlinear signal or using the waveguide regime with the phase mismatch related to the material dispersion compensated by the waveguide dispersion component.

Inelastic scattering effect is where there is an energy transfer between the matter involved and the optical wave. Two types of inelastic scattering processes in fibers are distinguished: Raman scattering and Brillouin scattering. These are scattering processes either at the optical (Raman) or acoustic (Brillouin) phonon branch. In either case, in principle there can be downshift or upshift of frequency. The medium usually consists of atoms or molecules which are in or near their energetic ground states, as given by the thermal energy and the Boltzmann distribution of occupation numbers. Then the medium can only absorb, but not release energy. Photons can be scattered spontaneously in both cases, but the rate is low. On the other hand, beginning at a certain threshold intensity a stimulated scattering process sets on. This is then called stimulated Raman scattering (SRS), or stimulated Brillouin scattering (SBS). The process can become stimulated when a sufficient number of spontaneously generated photons is already present and interacts with the pump wave. A fundamental difference is that SBS in optical fiber occurs only in the backward direction whereas SRS can occur in both forward and backward directions.

i) **Stimulated Raman scattering:** The energy level diagram of SRS is illustrated in figure 1.7. When a light wave is incident (pump wave) in a fiber at the carrier frequency $\omega_0$ in the presence of some resonance level $\omega_R$ of the fiber material, the incident light results in the downshift (the energy transfer from the higher energy to the lower energy of the frequency components of the light
pulse) of the carrier frequency resulting in an entirely new frequency $\omega_0 - \omega_R$, known as the mode frequency (Stokes). Such a process is known as Raman effect [10]. The principle of SRS is that a lower wavelength pump-laser light traveling down an optical fiber along with the signal, scatters off atoms in the fiber, loses some energy to the atoms, and then continues its journey with the same wavelength as the signal. Therefore the signal has additional photons representing it and, hence, is amplified. This new photon can now be joined by many more from the pump, which continue to be scattered as they travel down the fiber in a cascading process. When the frequency beating between the incident and scattered waves collectively enhance the optical photon, the scattering process is stimulated and the amplitude of the scattered wave grows exponentially in the direction of propagation, a phenomenon known as SRS [15].

Raman gain, resulting from SRS, depends on the frequency separation between the pump and the Stokes modes, becomes zero when the frequency separation is zero and increases with frequency separation. As a result, the corresponding Raman gain spectrum spans over an extremely broad frequency range wider than about 40 THz with broad peak located near 13 THz [10]. Raman amplification [10,15], one of the effects due to SRS, aims to boost the power of signals.

Figure 1.7: The energy level diagram of stimulated Raman scattering.
In particular, the Raman effect in fibers gives rise to a continuous frequency downshift of solitons propagating in optical fibers.

ii) **Stimulated Brillouin scattering:** By considering the case of SBS in which an acoustic wave is generated [16]. Pump and Stokes waves oscillate at optical frequencies while the acoustic wave has a considerably lower frequency. Therefore the wave vectors for pump and Stokes wave will be similar and much larger than that of the acoustic wave. This extremely low value renders SBS the nonlinear process with the lowest threshold [16]. One important consequence is a severe limitation of the fibers ability to transmit power: As soon as the pump wave exceeds threshold, the excess is transferred into a Stokes wave which travels back to the light source. Continuous wave light with adjustable power from a dye laser is launched into a single mode fiber. Power scattered back inside the fiber is diverted with a beam splitter at the near fiber end. The threshold can be lowered even further when power traveling in the fiber is recycled by reflection at the fiber ends. The main effect is that by back reflection, a coherent wave can seed the Stokes wave; this is more efficient than a spontaneous photon. For short pulses of light, the threshold is generally higher because the pulse is spectrally wider than the SBS line width. The consequence is that for short, i.e., broadband pulses, the threshold becomes much higher than the SRS threshold for the case of pulse have picosecond width. In that case SBS loses its importance.

### 1.6 Optical soliton

Optical solitons are an interesting subject in optical fiber communication because of their capability of propagation over long distance without attenuation and changes in shapes. The intensive optical pulse propagating along the fiber experiences group
velocity dispersion (GVD) and the SPM [17–19]. GVD is a linear effect and the refractive index varies with frequency components. Different frequency components travel with different velocity. This gives rise to differential transit time and thus a pulse gets distortion. The SPM is an intensity-induced refractive index change. The SPM is said to be positive if the refractive index increases with increasing intensity. This index change leads to a time-dependent phase shift. Since the time derivative of phase is related to the frequency, the SPM effect usually produces a change of the pulse spectrum. The combination of positive (normal) GVD and a positive SPM leads to temporal and spectral broadening of the pulse. A fiber with negative (anomalous) GVD and a positive SPM can propagate a pulse with no distortion (figure 1.8) [17–19]. This was surprising, at first, since GVD affects the pulse in the time domain, the SPM effect in the frequency domain. However, a small time-dependent phase shift added to a Fourier transform-limited pulse does not change the spectrum to first-order. If this phase shift is canceled by GVD in the same fiber, the pulse does not change its shape or its spectrum as it propagates. Solitons are therefore a remarkable breakthrough in the field of optical fiber communications. In 1973, Hasegawa and Tappert predicted the possibility of stationary transmission of bright or dark soliton in the anomalous or normal dispersion regimes, respectively, in the single mode fiber [17, 18]. This prediction was then successfully verified by the experiments of Mollenauer et al in 1980 [19]. The force of attraction between the dark solitons is always repulsive whereas it is either attractive or repulsive in the case of bright solitons depending upon their relative phase [20, 21]. The dark pulses undergo soliton like pulse propagation and emerge from the fiber almost unchanged even in the presence of significant broadening and chirping [21–23]. Solitons are localised solitary waves which have very special properties [24]: (i) They propagate at constant speed without changing their shape. (ii) They are extremely stable to perturbations, and in particular to collisions with small amplitude linear waves. (iii) They are even stable with respect to collisions with
other solitons. When two solitons collide, they pass through each other and emerge out by retaining their original speed and shape after the interaction. It is noteworthy that the interaction is not a simple superposition of the two waves. Moreover, after the collision, the trajectories of the two waves will be shifted with respect to trajectories without the collision. In other words, the outcome of the collision of two solitons is only a simple phase shift of each wave. Such a collision between two solitary waves resembles elastic collision between two particles. Hence, they are termed as solitons, a name based on their particle nature.

![Diagram of soliton pulse](image)

**Figure 1.8:** Soliton pulse neither broadens in time nor in its spectrum.

Solitons were first observed by J. Scott Russell in 1834 [25], whilst riding on horseback beside the narrow Union canal near Edinburg, Scotland. There are a number of discussion in the literature describing Russell’s observations. Subsequently, Russel did extensive experiments in a laboratory scale wave tank in order to study this phenomenon more carefully. Included amongst Russel’s results are the following: (i) he observed solitary waves, which are long, shallow, water waves of permanent form, hence he deduced that they exist; this is his most significant result; (ii) the speed of propagation, $v$, if a solitary wave in a channel of uniform depth $h_d$ is given by $v^2 = g(h_d + a_m)$, where $a_m$ is the amplitude of the wave and $g$ is the force
due to gravity. Further investigations were undertaken by Airy [26], Stokes [27], Boussinesq [28] and Rayleigh [29] in an attempt to understand this phenomenon. Boussinesq [28] and Rayleigh [29] independently obtained approximate descriptions of the solitary wave; Boussinesq [28] derived a one dimensional nonlinear evolution equation, which now bears his name, in order to obtain his result. These investigations provoked much lively discussion and controversy as to whether the inviscid equations of water waves would possess such solitary wave solutions. The issue was finally resolved by Korteweg and de Vries in the year 1895 [30]. They derived a nonlinear evolution equation governing one dimensional with lowest-order of dispersion and nonlinearity [30]. When the equation was numerically solved in a periodic boundary condition by Zabusky and Kruskal in 1965, a set of solitary waves was found to emerge and stably pass each other. They named these solitary waves solitons because of their stability [31]. The term “soliton” was justified when the KdV equation was solved analytically by means of inverse scattering transform (IST) and the solution was described by a set of solitons [32]. Solitons are regarded as a fundamental unit of mode in nonlinear dispersive medium and play a role similar to the Fourier mode in a linear medium. In particular, a soliton being identified as an eigenvalue in IST supports its particle concept.

The soliton is an innovative area of research that deserves intense investigation in many branch of physics. Different class of soliton-bearing models have been proposed in many physical problem in various context of physical system and still continues to flourish as an area of intense research. The development has brought in many novel modifications to the classical methods. Here we will point out briefly some important soliton possessing ubiquitous nonlinear evolution equations as follow:

i) Korteweg and de Vries equation: In 1985, Korteweg and de Vries (KdV) analyzed theoretically for first time about the wave phenomenon observed by Scott Russel, latter called as KdV equation, from first principles to dynamics
The KdV equation is considered to be the simple form of nonlinear wave equation which is of the form

\[
\frac{\partial q}{\partial t} + \frac{\partial^3 q}{\partial x^3} + 6q \frac{\partial q}{\partial x} = 0, 
\]

(1.17)

where \(q(x,t)\) is the slowly varying envelope, \(t\) is the time and \(x\) is the space coordinate in the relevant direction. It describes how waves evolve under the competing; but comparable effects of weak nonlinearity and weak dispersion. The KdV equation (1.17) admits an exact solitary wave solution in the context of fluid dynamics which seeds for the latter evidence of the existence of soliton, a particle like aspect of the wave that undergoes like elastic collision by Zabusky and Kruskal [31]. Also, the KdV equation is an ubiquitous system in the sense, that it occurs not only in the description of shallow water waves, but also in many other physical systems involving unidirectional dispersive wave propagation such as anharmonic lattices, ion-acoustic plasma waves, stratified internal waves, collision-free hydrodynamic waves and so on.

ii) Sine-Gordon equation: In 1939, Frenkel and Kontorova introduced a seemingly an eccentric type of problem in the context of solid state physics to model the relationship between dislocation dynamics and plastic deformation of a crystal [33]. The dislocation motion in solids can be modeled as:

\[
\frac{\partial^2 q}{\partial t^2} - \frac{\partial^2 q}{\partial x^2} + \sin q = 0, 
\]

(1.18)

where \(q(x,t)\) is the atomic displacement in the \(x\)-direction and the \(\sin\) function represents periodicity of the crystal lattice. In 1962, Perrin and Skyrme reported numerical results showing perfect recovery of shapes and speeds after a collision and went on to discover an exact analytical description of equation
The sine-Gordon equation has found a variety of applications, including Bloch wall dynamics in ferromagnetics and ferroelectrics, the chain-like magnetic compounds and transmission lines made out of arrays of Josephson junctions of superconductors, self-induced transparency (SIT) in nonlinear optics and a simple one-dimensional model for elementary particles.

iii) **Toda lattice equation**: The Toda lattice is named after Morikazu Toda, who discovered in the year 1967 [35]. It consists of an one-dimensional array of equal mass points connected by means of coupling with neighbouring masses that exercise a nonlinear forces of exponential type which is described as

\[
\frac{d^2 q_n}{dt^2} = \exp[-(q_n - q_{n-1})] - \exp[-(q_{n+1} - q_n)],
\]

(1.19)

where \(q_n(t)\) is the longitudinal displacement of the \(n^{th}\) atom from its equilibrium position. Equation (1.19) admits solitary wave solutions which can be written in terms of Jacobi elliptic functions [36]. The Toda lattice behaves in a very similar way to that of Fermi-Pasta-Ulam lattice. The Toda lattice is much easier to analyze than the Fermi-Pasta-Ulam lattice, because it is a completely integrable Hamiltonian system [36]. In principle, it is a conserved system. These evidences are the salient features of a growing panorama of nonlinear wave activities that became gradually less parochial during the 1960s. This remarkable discovery in solid state physics leads laid down the stepping stone in the other class of physical problems like classical hydrodynamics, oceanography etc.,

iv) **Landau-Lifshitz equation**: The Landau-Lifshitz equation was first used by Soviet scientists Lev Landau and Evgeni Lifshitz for mapping the dynamics of a small velocity domain wall, the magnetic susceptibility of ferromagnets with a domain structure, and ferromagnetic resonance in 1935 [37]. The principal assumption of macroscopic ferromagnetism theory is that a magnetic crystal
state is typically described by the magnetization vector \( \mathbf{M} \), so the dynamics and kinetics of a ferromagnet are determined by variations in its magnetization. The magnetization of a ferromagnet as a function of space coordinates and time \( \mathbf{M}(x, t) \) is a solution of the Landau-Lifshitz equation as

\[
\frac{\partial \mathbf{M}}{\partial t} = - \left( \frac{2\mu_0}{\hbar} \right) \mathbf{M} \times \mathbf{H}_{\text{eff}} - \gamma \mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}}),
\]

where \( \mu_0 \) is the Bohr magneton and \( \gamma \) is the relaxation constant determining the motional damping of the vector \( \mathbf{M} \). The effective magnetic field \( \mathbf{H}_{\text{eff}} = \frac{\delta E}{\delta \mathbf{M}} \) is equal to the variational derivative of the magnetic crystal energy \( E \) with respect to the vector \( \mathbf{M} \). Equation (1.20) is a highly non-trivial nonlinear partial differential equation of the vector form. The 1-dimensional version of equation (1.20) is a completely integrable in both isotropic [38, 39] and easy-axial [40] cases and possesses a set of multiple-soliton solutions. At present, the Landau-Lifshitz equation is the theoretical foundation of phenomenological magnetization dynamics in magnetically ordered solids, including ferromagnets, antiferromagnets, and ferrites.

v) **Nonlinear Schrödinger equation:** The intense light wave propagation in the optical fiber is described by the nonlinear Schrödinger (NLS) equation of the form:

\[
i \frac{\partial q}{\partial t} \pm \frac{\partial^2 q}{\partial x^2} + 2|q|^2 q = 0,
\]

where \( q(x, t) \) is the slowly varying complex envelope, \( z \) and \( t \) are the space and time coordinates respectively, + and − signs represent anomalous and normal dispersion respectively, the second term and third signifies, the GVD and SPM of the system. The NLS equation (1.21) was proposed by Zakharov in 1968 as a model for the envelope of a weakly nonlinear deep-water wave train [41]. Equation (1.21) found to be integrable by Zakharov and Shabat [42], by means
of IST, and the solution is given by a set of solitons and dispersive waves. The existence of an optical soliton in fibers is made possible by the theoretical realization of solving the evolution equation for the complex light wave envelope by retaining the GVD and the nonlinearity, which, for a glass fiber, is cubic and originates from the Kerr effect. In 1973, Hasegawa and Tappert were the first to model the propagation of a guided mode in a perfect nonlinear single mode fiber by the NLS equation [17, 18]. The NLS equation used to model wave packets in diverse fields of science such as hydrodynamics, nonlinear optics, nonlinear acoustics, plasma waves and bio-molecular dynamics [17, 18, 41, 43–45].

vi) **Maxwell-Bloch equations:** In 1967, McCall and Hahn [46] explained a special type of lossless pulse propagation in two-level resonant media. They showed that if the energy difference between the two levels of the medium coincides with the optical wavelength, then coherent absorption takes place. The medium becomes optically transparent to that particular wavelength, called the self-induced transparency (SIT). The process of SIT was discovered by Maxwell-Bloch (MB) equations which can be expressed as [46]:

\[
\frac{\partial q}{\partial z} = i < p >, \quad (1.22a)
\]

\[
\frac{\partial p}{\partial t} = i \Delta p - i R w q, \quad (1.22b)
\]

\[
\frac{\partial w}{\partial t} = i R (p^* q - q^* p), \quad (1.22c)
\]

where \(q\) is a complex envelope, \(p\) and \(w\) represent the resonant polarization and the population inversion and \(R\) describes the character of interaction between the propagating field and the two-level resonant atoms. The bracketed term \(< \cdots >\) means, averaging over the entire frequency range. If the fibers are doped with erbium atoms, then SIT can also be induced in optical fibers. This type of soliton pulse propagation was shown for the first time by Maimistov.
and Manykin in 1983 [47]. Nakazawa et al [48] experimentally observed the coexistence of NLS solitons and MB solitons in erbium doped resonant fibers. In Refs. [49, 50], the possibility of coexistence of the NLS soliton and the MB soliton with some higher-order terms have also studied.

The above mentioned nonlinear evolution equations are completely integrable through the IST procedure and bilinearizable, possess Lax pairs, infinite conservation laws, Bäcklund transformation and so on [51]. Among these branches, we have aspired in nonlinear fiber optics communication. In this context, in addition to the well known NLS model there are also few successful models describing the nonlinear pulse propagation in the fiber such as: coupled NLS equations, NLS-MB equations, nonlinear coupled mode (NLCM) equations and complex Ginzburg-Landau equation for a class of fibers like birefringent fiber, resonant fiber, photonic band gap (PBG) materials and dissipative fiber respectively. In this thesis, from the above mentioned physical models, we have studied the NLS soliton, SIT soliton and SIT gap soliton by using coupled NLS equations, NLS-MB Equations and nonlinear coupled mode-MB (NLCM-MB) equations.

1.6.1 Optical soliton transmission

Since solitons do not suffer distortion from nonlinearity and dispersion, which are inherent in fibers, the natural next step is to construct an all-optical transmission system in which fiber loss is compensated by amplification [52]. In the absence of realistic optical amplifiers, Hasegawa [53] in 1983 proposed to use the Raman gain of the fiber itself. The idea was used in the first long distance all-optical transmission experiment by Mollenauer and Smith [54] in 1988. This experimental result has attracted serious interest in the optical communication community. In fact, the optical transmission systems prior to 1991 required repeaters periodically installed in transmission lines to regenerate optical pulses which have been distorted by fiber
dispersion and loss. A repeater consists of a light detector and light pulse generators. Consequently, it is the most expensive unit in a transmission system and also the bottleneck to increase in the transmission speed. Repeater less transmission is a very innovative concept that people believed to be possible. In particular, the invention of EDFA have enhanced the concept of all-optical transmission systems to more realistic levels as initiated by Nakazawa et al. [55] in the first reshaping experiment of solitons. In experiments of long distance soliton transmission with loss compensated periodically by EDFAs, it was recognized that the initial soliton amplitude had to provide enough nonlinearity to overcome for fiber loss [56–58]. In this case, however, the pulse shape at an arbitrary position along the fiber deviates from a fundamental soliton. Fiber amplifiers are commonly used to overcome transmission losses. Optical amplifiers solve the loss problem but, at the same time, worsen the dispersion problem since, an optical amplifier does not restore the amplified signal to its original state. Dispersion-induced degradation of the optical signal is caused by the phase factor, acquired by spectral components of the pulse during its propagation in the fiber. Dispersion-management (DM) schemes attempt to compensate this dispersion problem so that the input signal can be restored back to its original shape. Actual implementation can be carried out at the transmitter, at the receiver, or along the fiber link. DM of transmission fiber is a key issue to realize high-speed long-distance optical fiber communication. The optical solitons can propagate without attenuation and distortion which is facilitated by passing the pulse through an active region doped with two-level resonant atoms, and they are called SIT solitons. SIT solitons are coherent optical pulses propagating through a resonant fiber without loss and distortion [46,48,59–62]. It has been demonstrated that solitons exist in doped fiber with two-level resonant atoms through the combined effects of the GVD, SPM of the host medium and the resonant interaction with two-level atoms [46,48,59–62].
The practical needs of current development in optical fiber communication requires an increase in the information rates and consequently a denser packing of information. For this purpose, optical communication systems require extremely short pulses called ultra-short pulses. Normally, modulational instability (MI) is used to generate ultra-short pulses. MI refers to a parametric process in which a continuous wave (CW) or quasi-CW undergoes a modulation of its amplitude or phase in the presence of weak perturbations [63, 64]. This instability is also called as Benjamin-Feir instability [65]. The perturbation can either originate from quantum noise or from a frequency shifted signal wave. The former and later are referred to as spontaneous MI and induced MI respectively. MI of a light wave in an optical fiber was suggested by Hasegawa and Brinkman [66] as a means to generate a far infrared light source and since then has attracted extensive attention because of its fundamental and applied interests [66, 67]. When MI is used in practical applications, such as for the generation of ultra-short light pulses [10, 68], the control of the sidebands frequencies and optimization of the MI power gain become rather crucial concern. In this context, one of the simplest MI processes in fibers is the one that results due to the interplay between negative GVD (anomalous dispersion regime) and SPM [69]. The phase matching of such a process, results from the compensation of the second-order chromatic dispersion by the self-focusing Kerr nonlinearity. To get more insight, MI can be analyzed in both temporal and frequency domains. In the temporal domain, break-up of the perturbed CW takes place owing to MI (CW undergoes exponential growth) and as a result there is formation of train of ultra-short pulses. In the frequency domain, the generation of high-repetition-rate pulse trains resulting from the MI is determined by the growth of a cascade of sidebands. The repetition rate of the pulses is determined by the frequency detuning between the signal and the perturbation frequency. It has been shown that the temporal shape of these ultra-short pulses depends not only on the powers of the different waves but also on the
perturbation frequency [70]. Thus, the MI effect find wide source of application in the fiber-optics telecommunication. Based on the field component involved in the MI process, MI is further classified into two types. The first one is the induced MI involving a single pump wave propagating in a standard non-birefringent fiber. The other one is known as the vector MI, which involves more than one field component. Vector MI occurs in a birefringent fiber due to XPM between two modes that extends the instability domain to the normal dispersion regime, which is in contrast to the result of the NLS equation. This vector MI is also called XPM induced MI. As regards applications, MI provides a natural means of generating ultra-short pulses at ultrahigh repetition rates and is thus potentially useful for the development of high-speed optical communication systems [71]. MI manifest itself in the exponential growth of small perturbations in the amplitude or the phase of initial optical waves leads to the breakup of a beam or a quasi-coherent pulse into filaments that can evolve into trains of solitons [72]. The MI find its applications in fluids [65], plasma [43], hydrodynamics [73] and atomic Bose-Einstein condensates [74,75]. The realm of applications also includes optical communications systems, the generation of ultra-high repetition-rate trains of soliton-like pulses [69,70] and all-optical logical devices [76].

1.7 Nonlinear Schrödinger equation

The propagation of light through a dielectric waveguide can be described by using Maxwell’s equations for electromagnetic waves. The propagation of electromagnetic fields in the optical fiber whose electric and magnetic field vectors are given by \( \mathbf{E}(\mathbf{r}, t) \) and \( \mathbf{H}(\mathbf{r}, t) \), their corresponding flux densities are given by \( \mathbf{D}(\mathbf{r}, t) \) and \( \mathbf{B}(\mathbf{r}, t) \), respectively, is governed by the following four Maxwell’s equations [10]:

\[
\nabla \cdot \mathbf{D}(\mathbf{r}, t) = 0, \quad (1.23)
\]
\[
\nabla \cdot \mathbf{B}(\mathbf{r}, t) = 0, \quad (1.24)
\]
\[ \nabla \times \mathbf{E}(\mathbf{r}, t) + \frac{\partial \mathbf{B}(\mathbf{r}, t)}{\partial t} = 0, \quad (1.25) \]
\[ \nabla \times \mathbf{H}(\mathbf{r}, t) - \frac{\partial \mathbf{D}(\mathbf{r}, t)}{\partial t} = 0. \quad (1.26) \]

where \( \mathbf{r} \) and \( t \) correspond to space and time coordinate. In the optical fiber, the electric and the magnetic flux density vectors are defined as

\[ \mathbf{D}(\mathbf{r}, t) = \varepsilon_0 \mathbf{E}(\mathbf{r}, t) + \mathbf{P}(\mathbf{r}, t), \quad (1.27) \]
\[ \mathbf{B}(\mathbf{r}, t) = \mu_0 \mathbf{H}(\mathbf{r}, t), \quad (1.28) \]

where \( \mu_0 \) is the permeability of the free space, \( \varepsilon_0 \) is the permittivity of the free space and \( \mathbf{P}(\mathbf{r}, t) \) is the induced electric polarization vector. By taking the curl of equation (1.25) and using equations (1.26), (1.27) and (1.28), one can eliminate \( \mathbf{D}(\mathbf{r}, t) \) and \( \mathbf{B}(\mathbf{r}, t) \) in favour of \( \mathbf{E}(\mathbf{r}, t) \) and \( \mathbf{P}(\mathbf{r}, t) \) and obtain

\[ \nabla \times \nabla \times \mathbf{E}(\mathbf{r}, t) + \frac{1}{c^2} \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} + \mu_0 \frac{\partial^2 \mathbf{P}(\mathbf{r}, t)}{\partial t^2} = 0, \quad (1.29) \]

where \( c (= 1/\sqrt{\mu_0 \varepsilon_0}) \) is the light velocity in the vacuum. The induced polarization, \( \mathbf{P} \) has both a linear and a nonlinear response which can be written as

\[ \mathbf{P}(\mathbf{r}, t) = \mathbf{P}_L(\mathbf{r}, t) + \mathbf{P}_{NL}(\mathbf{r}, t), \quad (1.30) \]

where

\[ \mathbf{P}_L(\mathbf{r}, t) = \varepsilon_0 \int_{-\infty}^{t} \chi^{(1)}(t-t').\mathbf{E}(\mathbf{r}, t')dt', \quad (1.31) \]

\[ \mathbf{P}_{NL}(\mathbf{r}, t) = \varepsilon_0 \int \int \int_{-\infty}^{t} \chi^{(3)}(t-t_1, t-t_2, t-t_3) : \mathbf{E}(\mathbf{r}, t_1)\mathbf{E}(\mathbf{r}, t_2)\mathbf{E}(\mathbf{r}, t_3)dt_1dt_2dt_3, \]

where the \( : \) represent the multiplication between the \( \mathbf{E}(\mathbf{r}, t) \) and \( \chi^{(3)} \) is for the respective time coordinates and \( \chi^{(1)} \) and \( \chi^{(3)} \) are the material dielectric response tensors for linear and nonlinear respectively. The second-order susceptibility, \( \chi^{(2)} \) is not considered because it vanishes due to the molecular (centro) symmetry of silica glass.
Inserting equation (1.30) into equation (1.29) and using $\nabla \times \nabla \times E = \nabla(\nabla \cdot E) - \nabla^2 E$ and $\nabla \cdot E = 0$, we obtain the Maxwell’s wave equation as:

$$\nabla^2 E(r, t) - \frac{1}{c^2} \frac{\partial^2 E(r, t)}{\partial t^2} - \mu_0 \frac{\partial^2 P_L(r, t)}{\partial t^2} - \mu_0 \frac{\partial^2 P_{NL}(r, t)}{\partial t^2} = 0. \quad (1.32)$$

From the above equation (1.32), one can derive the NLS equation that describes propagation of optical pulses in nonlinear dispersive fiber. Assuming that the nonlinear response of the third-order susceptibility, $\chi^{(3)}$, in the fiber is instantaneous as follow

$$\chi^{(3)}(t - t_1, t - t_2, t - t_3) = \chi^{(3)}(t - t_1)\delta(t - t_2)\delta(t - t_3), \quad (1.33)$$

where $\delta$ represents the delta function. Substituting equation (1.33) into equation (1.32), we get

$$P_{NL} = \epsilon_0 \chi^{(3)}(t) E(r, t) E(r, t). \quad (1.34)$$

By taking the Fourier transform of equation (1.32) and using equation (1.34), we get

$$\nabla^2 \tilde{E}(r, \omega) + k_0^2 \tilde{n}(\omega)^2 \tilde{E}(r, \omega) = 0, \quad (1.35)$$

where $\tilde{E}$ is the Fourier transform of $E$, $k_0 = \omega/c$ and $\tilde{n}(\omega)$ is defined by

$$\tilde{n}(\omega) = n_0(\omega) + \Delta n, \quad (1.36)$$

where

$$\Delta n = n_2|\tilde{E}(r, \omega)|^2 + i\alpha + i\alpha_2|\tilde{E}(r, \omega)|^2,$$

$$n_0 = 1 + \frac{1}{2} \Re[\chi^{(1)}], \quad n_2 = \frac{3}{8n} \Re[\chi^{(3)}],$$

$$\alpha = \frac{\omega}{nc} \Im[\chi^{(1)}], \quad \alpha_2 = \frac{3\omega}{4nc} \Im[\chi^{(3)}],$$

$n_0(\omega)$ is the frequency-dependent refractive index of the optical fiber, $n_2$ is the intensity-dependant refractive index of the optical fiber, $\alpha$ is the linear absorption and $\alpha_2$ is the two-photon absorption. As $\alpha_2$ is relatively small for silica fibers, it is
often neglected [10]. To obtain the pulse propagation equation for the optical fiber, we use the perturbation theory of distributed feedback [10] to reduce equation (1.35). Using equation (1.36), the refractive index, $\tilde{n}(\omega)^2$, in equation (1.35) is approximated by

$$\tilde{n}(\omega)^2 \simeq n_0(\omega)^2 + 2n_0(\omega)\Delta n,$$  \hspace{1cm} (1.37)

On substituting equation (1.37) into equation (1.35), we obtain

$$\nabla^2 \tilde{E} + k_0^2n_0(\omega)\left(n_0(\omega) + 2\Delta n\right) \tilde{E} = 0.$$  \hspace{1cm} (1.38)

We now specialize to the optical fiber geometry by allowing $z$ to correspond to distance along the fiber. The electric field $E$ can be expressed as:

$$E(r,t) = \frac{1}{2} \hat{x} F(x, y) \left(Q(z, t)e^{i(\beta_0 z - \omega t)} + c.c.\right),$$  \hspace{1cm} (1.39)

where $\hat{x}$ is the polarization unit vector, $F(x, y)$ is the transverse modal distribution, $Q$ is the slowly varying envelopes of electric field. By inserting equation (1.39) into equation (1.38), we obtain

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + (\ddot{\beta}(\omega)^2 - \beta_0^2) F = 0,$$  \hspace{1cm} (1.40)

$$\frac{\partial^2 \tilde{Q}}{\partial z^2} + 2i\beta_0 \frac{\partial \tilde{Q}}{\partial z} + (\ddot{\beta}(\omega)^2 - \beta_0^2) \tilde{Q} = 0.$$  \hspace{1cm} (1.41)

It is usually permissible to neglect the first term on the left-hand side of equation (1.41) on the grounds that it is very much smaller than the second and the fourth terms. This approximation is known as the slowly-varying envelope approximation (SVEA) and is valid whenever

$$\left|\frac{\partial^2 \tilde{Q}}{\partial z^2}\right| \ll \left|\beta_0 \frac{\partial \tilde{Q}}{\partial z}\right| \ll \left|\beta_0^2 \tilde{Q}\right|.$$  \hspace{1cm} (1.42)
This condition requires that the fractional change in $\tilde{Q}$ in a distance of the order of an optical wavelength must be much smaller than unity. When this approximation is made, equation (1.41) becomes

$$2i\beta_0 \frac{\partial \tilde{Q}}{\partial z} + (\tilde{\beta}(\omega)^2 - \beta_0^2) \tilde{Q} = 0,$$

(1.43)

where $\tilde{\beta}(\omega)(= \beta(\omega) + \Delta \beta)$ and $k_0$ are the wave numbers that are determined according to the eigenvalues of equation (1.40). The transverse mode function $F(x, y)$ can be averaged out by introducing the effective core area $A_{eff}$ [10]. Likewise the effective core area, $A_{eff}$ and $\Delta \beta$ can be described by

$$A_{eff} = \frac{\left(\int \int_{-\infty}^{\infty} |F(x, y)|^2 dxdy\right)^2}{\int \int_{-\infty}^{\infty} |F(x, y)|^4 dxdy},$$

(1.44)

$$\Delta \beta = \frac{\omega_0 \int \int_{-\infty}^{\infty} \Delta n |F(x, y)|^2 dxdy}{c \int \int_{-\infty}^{\infty} |F(x, y)|^2 dxdy}.$$

(1.45)

In equation (1.43), we have approximated $(\tilde{\beta}(\omega)^2 - \beta_0^2)$ by $2\beta_0(\tilde{\beta}(\omega) - \beta_0)$ and $\tilde{\beta}(\omega)/\beta_0 \approx 1$, which indicate that the wave number is close to the mode-propagation constant. We expand both $\beta(\omega)$ and $\omega^2$ in a Taylor series as

$$\beta(\omega) = \beta_0 + (\omega - \omega_0)\beta_1 + \frac{1}{2!}(\omega - \omega_0)^2 \beta_2 + \ldots,$$

(1.46)

$$\omega = \omega_0 + \omega_0(\omega - \omega_0) + \ldots,$$

(1.47)

where $\beta_l = d^l\beta/d\omega^l |_{\omega=\omega_0}$ and $l = 1, 2, \ldots$. The cubic and higher-order terms in equation (1.46) are generally negligible if the spectral width $\Delta \omega \ll \omega_0$ and the second and other terms in equation (1.47) are negligible for the picosecond regime. Their neglect is consistent with the quasi-monochromatic assumption used in the derivation of equation (1.43). $\beta_1(= d\beta/d\omega = 1/v_g)$ is related inversely to the group velocity ($v_g$) and $\beta_2$ is GVD. We substitute equations (1.46) and (1.47) in equation (1.43) and take the inverse Fourier transform. During the Fourier transform operation, $(\omega - \omega_0)^n$ is
replaced by the differential operator $i^n \partial^n / \partial t^n$. The resulting equation can be written in the time domain as

$$i \frac{\partial Q}{\partial z} + i \beta_1 \frac{\partial Q}{\partial t} - \frac{\beta_2}{2} \frac{\partial^2 Q}{\partial t^2} + \gamma |Q|^2 Q + i \frac{\alpha}{2} Q = 0,$$

where $\gamma (= n_2 \omega_0 / (c A_{\text{eff}}))$ is SPM. Using the following transformation $Q(z, t) = q(z, \tau)$ and $\tau = t - \beta_1 z$ into equation (1.48), we obtain [10, 17, 18]

$$i \frac{\partial q}{\partial z} - \frac{\beta_2}{2} \frac{\partial^2 q}{\partial \tau^2} + \gamma |q|^2 q + i \frac{\alpha}{2} q = 0.$$

Equation (1.49) is the NLS equation which describes the nonlinear optical pulse propagates inside the fiber at picosecond regime. The GVD parameter $\beta_2$ can be a positive (normal dispersion) or negative (anomalous dispersion). We have considered the following assumptions which led us to equation (1.49) as:

- The optical field is assumed to be quasi-monochromatic, i.e., the pulse spectrum, centered at $\omega_0$, is assumed to have a spectral width $\Delta \omega$ such that $\Delta \omega / \omega \ll 1$.

- The slowly varying envelope approximation. This assumption underpins the entire theoretical development. Most of the subsequent assumptions are only reasonable in the context of this assumption.

- The optical field is assumed to maintain its polarization along the fiber length so that a scalar approach is valid. This is not really the case, unless polarization-maintaining fibers are used, but the approximation works quite well in practice.

- $P_{NL}$ is treated as small perturbation to $P_L$. This is justified because nonlinear changes in the refractive index are less than $10^{-6}$ in practice.

- Truncation of the Taylor expansions of $\beta(\omega)$ and $\chi^{(3)}$. We stopped at second-order in the former case and zeroth-order in the latter case. From the standpoint of linear propagation, the first-order term governs the overall velocity
of the optical pulse, while the second-order term governs its spreading. Both
theses effects are readily visible. By contrast, the effect of the third and higher-
order terms are not readily visible unless the second-order term is zero. Since
a soliton forms by balancing dispersion ($\beta_2$) with the zeroth nonlinearity ($\gamma$),
higher-order nonlinear effects will not be visible unless the pulse duration is
quite small.

- Since $\omega_0 \sim 10^{15}$ Hz which is valid for optical pulses as short as 0.1 ps.

1.8 Outline of the Thesis

This thesis is devoted to the study of the optical soliton propagation through different
nonlinear optical fiber media. Analytical and numerical methods are used to show
the propagation of solitons in the respective media. The coupled amplitude-phase
method, the projection operator method and the linear stability analysis technique are
employed for the analytical investigation throughout the thesis. Numerical method
based on the split-step Fourier method of the original partial differential equations
are utilized for the numerical analysis.

Chapter two gives an overview of the birefringent optical fiber and the mathemat-
ical frame work required for a theoretical understanding of the femtosecond soliton
pulse propagation in birefringent optical fibers. Starting from Maxwell’s equations,
the wave equation in a nonlinear medium is used to obtain the coupled higher-order
NLS (HNLS) equations satisfied by the amplitude of the pulse envelope. The later
part of the chapter provides the solitary wave solutions of the coupled HNLS equations
for anomalous as well as normal dispersion regimes.

Chapter three gives an introduction to DM system and the mathematical descrip-
tion of ultra-short optical pulse propagation in birefringent DM optical fiber. For
Gaussian pulse, we have derived the collective variational equations that describe the
evolution of the pulse parameters along the birefringent DM optical fiber through the projection operator method. We have shown that the temporal and frequency shift due to the presence of SRS effect. Finally, we have also verified the accuracy of our analytical results using numerical simulations.

Chapter four deals with the study of distortionless SIT soliton pulse propagation in the nonlinear optical fiber doped with two-level resonant atoms which is described by NLS-MB equations. In the first part, the detailed derivation of MB equations is presented for the two-level system. The second part of the chapter discusses the MI process, required for the generation of high-repetition arrays of ultra-short optical pulses in both anomalous and normal dispersion regimes. We have also examined the significance of higher-order dispersion effect in this chapter. Finally, this chapter gives the periodic as well as the exact SIT solitary wave solutions for both anomalous and normal dispersion regimes.

Chapter five describes the nonlinear optical pulse propagation in uniformly doped fiber Bragg grating (FBG) with two-level resonant atoms which is governed by NLCM-MB. In this chapter, we have derived NLCM-MB equations for the propagation of forward and backward directions respectively. This chapter provides the MI process for the generation of ultra-short optical pulses without the significant requirement of threshold conditions and generate SIT gap soliton solutions at the PBG edges. We have also shown the MI gain spectra, in both the anomalous and the normal dispersion regimes, as predicted by the computation of the growth rates for eigen modes of small perturbations, the results agree well with the direction simulations.

Chapter six summarizes the crucial results of the thesis and endowed with a quick review of the results. The future studies are also presented at the end of this chapter.