CHAPTER 3

CAD NEURAL ANALYSIS OF MECHANICAL MODELING OF FIXED - FIXED BEAM OF RF MEMS SWITCH

In this chapter, the proposed neural network based CAD model has been discussed for the following analysis related to RF MEMS devices.

- Static analysis of mechanical modeling for fixed - fixed beam RF MEMS switches – spring constant.
- Prediction of electrical and mechanical behaviour of fixed - fixed beam RF MEMS switches – Critical stress and pull down voltage.
- Stabilization analysis of RF MEMS devices using neural model.

3.1 CAD NEURAL ANALYSIS OF SPRING CONSTANT

With the recent rapid growth of RF MEMS switches, an emergent requirement for more theoretical models to predict their electrical and mechanical behaviours has been developed. In this work an efficient approach based on neural network for static analysis of mechanical modelling for RF MEMS switches has been proposed. Spring constant for fixed - fixed beam with force distributed in three different ways has been analyzed.
Neural model is proposed for calculating the spring constant of MEMS switch for two different materials with three different force distributions pattern. The neural model is trained with Quasi - Newton (QN), Scaled Conjugate Gradient (SCG), Conjugate Gradient with Flechther (CGF), Levenberg - Marquart (LM) and Bayesian Regulation (BR) learning algorithms to obtain better performance with simpler structure. Although the extensive time and efforts for preparing the training data base are needed, once it is trained the ANN accurately predicts the device parameters for arbitrary input datasets within the range of ANNs data sets.

3.1.1 Fixed - Fixed Beam over Cantilever

Usually the spring constant of fixed - fixed bridge is higher than cantilever, because the bridges are rigidly anchored at both sides. In circuit point of view, bridges are more useful in shunt configuration and cantilevers are more useful in series configuration (Rebeiz 2003). The cantilever can be used both as a DC and capacitive contact switch. For DC contact switch, a separate actuation electrode is required. For capacitive contact switch the same electrode may be used both for actuation and capacitive contact.

3.1.2 Analysis of Spring Constant of Fixed – Fixed Beams

The first step in understanding the mechanical operation of RF MEMS switches is to derive the spring constants of the fixed - fixed beam. If the operation of the structure is limit to small deflections, as it is the case for most RF MEMS devices, the mechanical behaviour, can be modeled using a linear spring constant. The spring constant for the fixed - fixed beam can be modeled in two parts. One part $k'$ is due to the stiffness of the bridge which accounts for the material characteristics such as Young’s modulus $E$(Pa) and the area moment of inertia $I$(m$^4$). The area moment of inertia $I$ for a
rectangular cross section is given by \( wt^3/12 \) where \( w \) and \( t \) are the width and thickness of the beam.

The other part of the spring constant is due to the biaxial residual stress \( \sigma \text{(Pa)} \) within the beam and is a result of the fabrication processes (Liang et al 2005). In MEMS applications, the load is typically distributed across the beam and the deflection of the beam at the center is used to determine the spring constant. To find the deflection for a distributed load, the principle of superposition is used.

The configuration of fixed-fixed beam with the force distributed over the entire beam is shown in Figure 3.1 (a). For instance, in the case where the load is distributed across the entire beam, the deflection for a concentrated load at a point \( a \) is found by evaluating the integral.

\[
y = \frac{2}{EI} \int_0^{\lambda} \frac{\bar{\xi}}{48} \left( l^3 - 6l^2a + 9la^2 - 4a^3 \right) da
\]

where \( \bar{\xi} \) is the load per unit length so that the total load is \( P = \bar{\xi} \lambda \), \( l \) is the length of the beam. For a gold beam (\( E = 80 \text{ GPa}, \nu = 0.42 \)), where \( \nu \) is Poisson’s ratio. The spring constant (\( K_a' \)) is found to be

\[
K_a' = \frac{P}{y} = \frac{\bar{\xi}l}{y} = 32Ew\left(\frac{l^3}{L}\right)
\]

Another configuration of fixed-fixed beam with force distributed evenly over the center portion (Rebeiz 2003) is shown in Figure 3.1 (b). For this case, the above integral should be evaluated with limits from \( l/2 \) to \( x \).
The general expression for this spring constant \( k'_c \) is

\[
k'_c = 32Ew \left( \frac{t}{l} \right)^3 \frac{1}{8(x/l)^2 - 20(x/l)^2 + 14(x/l) - 1}
\]  \hspace{1cm} (3.3)

Figure 3.1 (c) shows another configuration in which the load is distributed at the ends of the beam, rather than in the centre. Here the spring constant \( k'_c \) is given by

\[
k'_c = 4Ew \left( \frac{t}{l} \right)^3 \frac{1}{(x/l)(1-(x/l))^2}
\]  \hspace{1cm} (3.4)

Figure 3.1 Configuration of fixed-fixed beam with force distributed
(a) Over the entire beam (b) Evenly about the centre of the beam (c) Along the ends (Rebeiz 2003)
3.1.3 Development of ANN Model for Fixed - Fixed Beam

Neural networks with their remarkable ability to derive meaning from complicated or impressive data can be used to extract patterns and solve problems that are too complex to be by either humans or other computer techniques. The MLP neural models have been successfully used to compute the spring constant \( k_{a,ANN} \) \& \( k_{e,ANN} \) of fixed - fixed beam with three different force distribution methods. The ranges of inputs are \( 0 \leq t/l \leq 0.01 \), \( 0.5 \leq x/l \leq 1 \), \( w=100 \mu m \) and \( E =80 \) GPa and 69 GPa. Differences between the expected and the actual outputs of the neural model \( k_{a,ANN} \) \& \( k_{e,ANN} \) are calculated through the network to adopt its weights. The adaptation is carried out after presenting each dataset \((E, w, x/l \& t/l)\) until the calculation accuracy of the network is deemed satisfactory to the intension. The criterion includes RMS (Root Mean Square) errors for all training set or the maximum allowable number of epochs to be reached.

After many trial, two hidden layered neural model as shown in Figure 3.2 (a) which provides high accuracy have been selected. The suitable network configuration is 4 x 12 x 10 x 2. This means that the number of neurons were 4 for the input layer, 12 and 10 for the first and second hidden layers and 2 for the output layer.

The hyperbolic tangent sigmoid was used for input and hidden layers and linear activation functions was used in output layer. Figure 3.2 (b) shows the RMS error comparison of five training algorithms for the calculation of spring constant, critical stress and pull down voltage. When the performance of neural models is compared with each other, the best results were obtained from the models trained with LM algorithm.
3.1.4 Results and Discussion

To obtain better performance, faster convergence and a simpler structure, the proposed ANN was trained with five different training algorithms. The spring constant for a concentrated load at the centre of the beam, over the entire beam and at the ends are calculated and the comparison between the neural and MATLAB calculated results is shown in Figure 3.3. Typical dimensions are beam lengths ranging from 200 to 500 µm and thickness ranging from 0.5 to 0.2 µm. A 300 µm long gold (E = 80 GPa, ν = 0.42) beam with \(t = 1\) µm and \(w = 100\) µm has a spring constant of \(k = 9.5\) N/m. If the same beam is made of aluminum (E = 69 GPa, ν = 0.33), the spring constant is 8.2 N/m. If the beam thickness is increased to 2 µm, the spring constant increases to 76 N/m and 65 N/m for gold and aluminum respectively.

As can be seen in Figure 3.3 (a), concentrating the load more toward the centre of the beam results in a lower spring constant than the case where the load is evenly distributed over the entire beam. By using Equation (3.3), the spring constant of 300 x 1 x100 µm beam is calculated
with the load concentrated over the portion of center from one third point of beam length. It shows that the spring constant drops to 5.2 N/m for gold and 4.5 N/m for aluminum and concludes that the spring constant of gold is higher than aluminium. Figure 3.3 (b) shows the comparison of neural result (symbol) with calculated spring constant (solid line) of a gold beam where the force is distributed at the ends of the beam and toward the centre.

It can be noted that the conditions $x/l = 1$ for $k'_c$ (spring constant of beam with force distributed at the centre) and $x/l = 0.5$ for $k'_e$ (spring constant of beam with force distributed at the ends) result in the same spring constant because they both represent the case where the force is distributed over the entire beam.

![Comparison of neural and MATLAB result of spring constant of a gold beam](image)

(a) Force distributed over the entire beam and at the centre of beam (b) Force applied over the centre and at the ends of the beam

We have calculated the spring constant versus the thickness of the bridge for two different materials (gold and aluminum) as shown in Figure 3.4 (a) & (b) respectively with two different lengths of 200 and 400 μm (solid and dashed line respectively), assuming the width of the bridge as 35 μm.
Figure 3.4 (c) shows the total spring constant versus the ratio of bridge thickness to length for a residual stress of 0 MPa, 30 MPa and 60 MPa with gold and aluminum beam. In graph the solid line (MATLAB results) and dashed (ANN results) overlie each other. Typical dimensions are beam lengths ranging from 200 to 500 µm and thicknesses ranging from 0.5 to 2 µm.

Figure 3.4  Spring constant for 200 µm (solid) and 400 µm (dashed) bridges length. (a) Gold and (b) Aluminum bridge versus bridge thickness (c) Comparison of same for gold and aluminium with residual stress
The CPW center conductor width is two third of the length of the beam and the force is distributed above the center conductor. The spring constant for aluminum beams are similar to gold due to nearly equal Young’s modulus and Poisson’s ratio.

3.2 RF MEMS SWITCH ELECTRO STATIC ACTUATED PULL DOWN VOLTAGE ANALYSIS

The operating principle of an electrostatic actuated RF MEMS switch is very simple. A beam (bridge or cantilever) is suspended from the anchor with an actuation electrode placed underneath. When a DC voltage is applied between the beam and actuation electrode, the beam moves down due to an electrostatic force. The DC actuation voltage at which the beam fully moves down is called the pull down voltage.

To calculate the required pull down voltage for a beam, it is necessary to have an accurate mechanical model for the spring constant and pull down voltage of the beam. The pull down voltage and spring constant also depends on the position and orientation of the actuation electrode.

In this work, neural model is proposed for calculating the critical stress and pull down voltage of MEMS switch for Gold and Aluminum materials with the force distributed evenly above the CPW centre conductor. The neural model is trained with different training algorithms and based on low error, LM algorithm has been chosen to obtain better performance and fast convergence with simpler structure.

3.2.1 Pull - Down Voltage of RF MEMS Switch

For the case where the residual stress within the beam is compressive, the model for spring constant \(k\) is no longer valid. The primary
concern with compressive stress is the tendency for the beam to buckle. Due to the stiffness of the beam, a certain amount of compressive stress can be withstood before buckling occurs. This stress, known as the critical stress, is given for a fixed-fixed beam by Afrang (2004)

$$\sigma_{cr} = \frac{\pi^2 E t^2}{3t^2(1-\nu)}$$

(3.5)

When a voltage is applied between a fixed-fixed beam and the pull-down electrode, an electrostatic force is induced on the beam. It is well known electrostatic force which exists on the plates of a capacitor under an applied voltage. In order to approximate this force, the beam over the pull-down electrode is modeled as a parallel-plate capacitor. Given that the width of the beam w and the width of the pull-down electrode W, the parallel-plate capacitance is given by

$$C = \frac{\varepsilon_0 A}{g} = \frac{\varepsilon_0 Ww}{g}$$

(3.6)

where g is the height of the beam above the electrode. The electrostatic force applied to the beam is found by considering the power delivered to a time dependent capacitance and is given by (Zhan 1979)

$$F_e = \frac{1}{2} V^2 \frac{d}{dg} C(g) = \frac{1}{2} \varepsilon_0 WwV^2$$

(3.7)

where V is the voltage applied between the beam and the electrode.

Equating the applied electrostatic force \((F_e)\) with the mechanical restoring force due to the stiffness of the beam \((F = kx)\), we find
\[
\frac{1}{2} \frac{\varepsilon_0 W w}{g} V^2 = k (g_0 - g)
\]  
\[(3.8)\]

where \( g_0 \) is the zero-bias bridge height and \( \varepsilon_0 \) is the permittivity of air. Solving this equation for the voltage results in

\[
V = \sqrt{\frac{2k}{\varepsilon_0 W w}} g^2 (g_0 - g)
\]
\[(3.9)\]

By taking the derivative of the Equation (3.9) with respect to the beam height and setting that to zero, the height at which the instability occurs is found to be exactly two-thirds of the zero-bias beam height. Substituting this value back into Equation (3.9), the pull down voltage is found to be

\[
V_p = V \left(\frac{2g_0}{3}\right) = \sqrt{\frac{8k}{27\varepsilon_0 W w g_0^3}}
\]
\[(3.10)\]

3.2.2 Results and Discussion

Figure 3.5 (a) shows the comparison of the critical stress with bridge thickness between the proposed ANN model and MATLAB calculated results. It is seen that for a bridge thickness of 1 to 2 µm and a length of 400 µm, a compressive stress of 4 to 12 MPa for gold and 2 to 8 MPa for aluminum can be tolerated. For a shorter length of 200 µm with thickness of 2 µm, a gold bridge can withstand 45 MPa while aluminum can withstand 34 MPa. It is clear that shorter beam lengths can withstand a large compressive stress, but the penalty paid is higher pull down voltage as shown in Figure 3.6. The Figure 3.5 (b) shows two possible bridge heights for every applied voltage with the result of the bridge becoming unstable at \( 2g_0 / 3 \).
Figure 3.5  (a) Critical stress of gold and aluminum bridge versus bridge thickness for the bridge length of 200 µm and 400 µm. (b) Bridge height versus applied voltage with zero bias for bridge height of 1.2 µm (dashed) and 3 µm (solid)

This instability is due to positive feedback in the electrostatic actuation. At $2g_0/3$ the increase in electrostatic force is greater than the increase in the restoring force resulting in the beam position become unstable and collapse of the beam to the down state position.

Figure 3.6 (a) presents the pull-down voltage for a gold bridge with residual stress of 0 MPa, 30 MPa and 60 MPa calculated using Equation (3.10). Pull-down voltage is also obtained for aluminum beams since they have nearly the same spring constant values (Figure 3.6 (b)). In order to support the resultant graph the pull down voltage has also been calculated using MATLAB for bridge length of 500 µm with residual stress of 75 MPa and the values are shown in Table 3.1. The spring constant scales linearly with the width which is held constant at 35 µm. It is seen that beams with $g > 3$ µm and $\sigma = 30$ MPa result in a very large pull-down voltage.
Figure 3.6  Pull down voltage of a (a) Gold and (b) Aluminum bridge versus bridge thickness for the bridge length of 200 µm (solid) and 400 µm (dashed)

Table 3.1 Comparisons of pull down voltage values from MATLAB and ANN training for aluminum and gold bridge having length of 500 µm and biaxial stress of 75 MPa

<table>
<thead>
<tr>
<th>Bridge Thickness (µm)</th>
<th>Pull Down Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aluminum</td>
</tr>
<tr>
<td>0.25</td>
<td>4.226</td>
</tr>
<tr>
<td>0.75</td>
<td>12.807</td>
</tr>
<tr>
<td>1.75</td>
<td>31.373</td>
</tr>
<tr>
<td>2.25</td>
<td>41.870</td>
</tr>
<tr>
<td>2.75</td>
<td>53.516</td>
</tr>
</tbody>
</table>
From these figures and tables, we observed that there are very good agreement between the results of the neural model and the results computed by MATLAB. This mechanical modeling of gold and aluminum fixed-fixed beam over a CPW line is derived with the force distributed above the CPW centre conductor. This good agreement supports the validation of the neural model for the determination of the various parameters for mechanical modeling of fixed-fixed beam.

3.3 RF MEMS SWITCH ELECTRO STATIC ACTUATED STABILIZATION ANALYSIS

In this work, an efficient neural network to analyze the position stabilization of electrostatically actuated beam with force distributed evenly across the section of beam used in MEMS switches has been proposed.

3.3.1 Stabilization of Bridge

It has been demonstrated that by placing a series capacitance in the DC path of the MEMS bridge capacitor, as shown in Figure 3.7(a), the instability in the electrostatic actuation can be reduced (Liang et al 2005). As the applied voltage increased, the voltage applied to the MEMS capacitor \( C_b \) increases, causing the bridge to pull down and \( C_b \) to increase.

This results in the positive feedback leading to instability at \( 2g_0/3 \) (Afrang 2004). However the increase in \( C_b \) results in less voltage across \( C_b \) and more across the switch capacitance \( C_s \) thereby achieving negative feedback. The position at which the instability occurs is determined by finding the force on the MEMS capacitor in terms of \( V_s \).
On solving for $V_s$ and taking the derivative of $V_s$ with respect to the bridge height, the force on the bridge (Rebeiz 2003) is found by considering the change in energy of both capacitors with respect to the height of bridge, $g$.

\[
F_e = \frac{dU}{dg} = \frac{V_s^2}{2} \frac{d}{dg} \left( \frac{C_b C_s}{C_b + C_s} \right) - \frac{\varepsilon_o A V_s^2}{2 \left( g + k g_o \right)^2}
\]

(3.11)

where $C_b$ is the bridge capacitance and

\[
k = \frac{C_{bo}}{C_s} \quad \text{and} \quad C_{bo} = \varepsilon_o A / g_o
\]

(3.12)
called the zero bias bridge capacitance. By equating this force with the force due to the stiffness of the bridge

\[
F = k(g_0 - g)
\]

(3.13)
the applied voltage can be obtained in terms of the bridge height $g$ as

\[
V_s = \sqrt{\frac{2k g_o^3}{\varepsilon_o A} \left( \frac{g}{g_0} + k \right)^2 \left( 1 - \frac{g}{g_0} \right)}
\]

(3.14)
where $\varepsilon_o$ is the permittivity of air. In some cases it may be possible to isolate a single capacitor without a large fringing field component. If a capacitor is placed in parallel with variable capacitor as shown in Figure 3.7 (b), it is found that the effect of series feedback capacitor is reduced, and the instability point must be solved again. The force on the capacitor can be determined from the change in energy with respect to $g$ (Rebeiz 2003), and is given by

\[
F_s = \frac{dU}{dg} = \frac{V_s^2}{2} \frac{d}{dg} \left( \frac{C_b C_s + C_s C_f}{C_b + C_s + C_f} \right) - \frac{\varepsilon_o A V_s^2}{2 \left( g + k g_o \right)^2}
\]

(3.15)
Figure 3.7  Stabilization circuit for an electro statically actuated MEMS Bridge (a) without fringing capacitance (b) with fringing capacitance

where $C_f$ is the fixed capacitance to ground in parallel with the MEMS capacitor. Equating this force to the restoring force of the bridge, the applied voltage after the addition of fringing capacitance is found to be

$$V_f = \sqrt{\frac{2kg_o^3}{\varepsilon_o A}} \left( \frac{g}{g_o} \left(1 + \frac{C_f}{C_s}\right) + k \right) \left(1 - \frac{g}{g_o}\right)$$  \hspace{1cm} (3.16)

3.3.2  Results and Discussion

To obtain better performance, faster convergence and a simpler structure, the proposed ANN as shown in Figure (3.2) was trained with five different training algorithms. Figure 3.8 (a) shows the change in bridge height versus applied voltage with and without the fringing parallel capacitance $C_f = C_b/3$ with the series capacitance varied from $0.5 \ C_{b_0}$ to $3 \ C_{b_0}$. The zero bias bridge height is taken as $1.2 \ \mu m$.

As can be seen, if twice the initial pull down voltage is acceptable, then a series capacitance of $C_s = 1.7 \ C_{b_0}$ (k = 0.6) can be used, resulting in stable operation up to $0.5 \ C_b$. For the case with fringing capacitance
(Figure 3.8 (b)), it is seen that even at 0.5 $C_{b0}$, the instability is still present at 0.32 $\mu$m while the pull down voltage has been increased to 120 V (from an initial value of 22 V). Since the parallel capacitance is less than 0.5$C_{b0}$, it is possible to completely eliminate the instability by setting the series capacitance to 0.5 $C_{b0}/6$.

Figure 3.8  Plot of change in bridge height versus applied voltage (a) without fringing capacitance (b) with fringing capacitance