CHAPTER – V

FREE CONVECTIVE FLOW OF IMMISCIBLE PERMEABLE FLUIDS IN A VERTICAL CHANNEL WITH FIRST ORDER CHEMICAL REACTION

5.1 INTRODUCTION

Convection in porous media is applied in utilization of geothermal energy, the control of pollutant spread in ground water, the design of nuclear reactors, compact heat exchangers, solar power collectors, heat transfer associated with the deep storage of nuclear waste and high performance insulators for buildings. Considerable progress in this area was made by Nield and Bejan (2006) and Kaviany (1991). Vafai and Tien (1981) also analyzed the effects of a solid boundary and the inertial forces on flow and heat transfer in porous media. The coupled fluid flow and heat transfer problem in a fully developed composite region of two parallel plates filled with Brinkman-Darcy porous medium was analytically investigated by Kaviany (1985). Rudraiah and Nagraj (1977) studied the fully developed free convection flow of a viscous fluid through a porous medium bounded by two heated vertical plates. Beckerman et al. (1987) studied natural convection in vertical enclosure containing simultaneously fluid and porous layers. Singh et al. (1999) analyzed heat and mass transfer phenomena due to natural convection in a composite cavity containing a fluid layer overlying a porous layer saturated with the same fluid, in which the flow in the porous region was modeled using Brinkman-Forchheimer extended Darcy model that includes both the effect of macroscopic shear (Brinkman effect) and flow inertia (Forchheimer effect).
Forced convection in composite channel is a subject of intensive investigation. This is due to the rapid development of technology and numerous modern thermal applications relevant to this area such as cooling of microelectronic devices. Poulikakos and Kazmierczak (1987) presented analytical solutions for forced convection flow in ducts where the central part is occupied by clear fluid and the peripheral part is occupied by a Brinkman-Darcy fluid saturated porous medium. The results of Poulikakos and Kazmierczak (1987) were extended by Kuznetsov et al. (1998) to account for the Forchheimer (quadratic drag) effects. Prasad (1991) have made an excellent review for composite systems. Alazami and Vafai (2001) reviewed different types of interface conditions between a porous medium and fluid layer.

Some novel designs of heat sinks for cooling microelectronic devices utilize highly porous materials such as aluminum foam (Paek et al., 2000). Nield and Kuznetsov (2000a) considered a forced convection problem in a channel whose center is occupied by a layer of isotropic porous medium (porous layer 1) and whose peripheral part is occupied by another layer of isotropic porous medium (porous layer 2), each of the layers with its own permeability and thermal conductivity. They utilized the Darcy law for the flow in porous layers. Malashetty et al. (2001a, 2004, 2005) studied two fluid flow and heat transfer in an inclined channel containing a porous fluid layer and composite porous medium. Recently, Umavathi et al. (2005b, 2006, 2009b, 2010a, 2010c, 2010d and 2012c), Umavathi and Manjula (2006), Umavathi (2011) and Prathap Kumar et al. (2009, 2010) studied mixed convection in a vertical porous channel.
Combined heat and mass transfer problems with a chemical reaction are of importance in many processes and have received a considerable amount of attention in recent years. In such processes as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occurs simultaneously. Natural convection processes involving the combined mechanisms are also encountered in many natural processes, such as evaporation, condensation and agriculture drying and in many industrial applications, such as the curing of plastics, cleaning and chemical processing of materials relevant to the manufacture of the printed circuitry and the manufacture of pulp insulated cables. In many chemical engineering processes, chemical reactions take place between a foreign mass and a working fluid mass which moves due to the stretch of a surface. The order of the chemical reactions depends on several factors. One of the simplest chemical reactions is the first order reaction in which the rate of the reaction is directly proportional to the species concentration. Chamkha (2003) studied the analytical solutions for heat and mass transfer by the laminar flow of a Newtonian, viscous, electrically conducting and heat generating/absorbing fluid on a continuously moving vertical permeable surface in the presence of a magnetic field and the first order chemical reaction. Muthucumaraswamy and Ganeshan (2002) studied the numerical solution for the transient natural convection flow of an incompressible viscous fluid past an impulsively started semi infinite isothermal vertical plate with the mass diffusion, taking into account a homogeneous chemical reaction of the first order. The analytical solution of the free convection heat and mass transfer from a vertical plate embedded in a fluid saturated porous medium with the constant wall temperature and concentration was obtained by Singh and Queeny
(1997). The heat and mass transfer characteristics of the natural convection about a vertical surface embedded in a saturated porous medium subjected to a chemical reaction taking into account the Soret and Dufour effects was analyzed by Postelnicu (2007). Prathap Kumar et al. (2011b, 2011d, 2012b) studied the effect of homogenous and heterogeneous reaction on the dispersion of a solute for an immiscible fluids.

Keeping in view the wide area of practical applications on multi-fluid flow and effects of chemical reaction as mentioned, the objective of this study is to investigate the heat and mass transfer of two immiscible permeable fluids between vertical parallel plates.

5.2 MATHEMATICAL FORMULATION

Fig. 5.1: Physical configuration.
The geometry under consideration illustrated in figure 5.1, consists of two infinite parallel plates maintained at equal or constant temperature, taking $X$ axis along the midsection of channel and $Y$ axis perpendicular to walls. The region-I $(0 \leq Y \leq h_1)$ is filled with a homogeneous isotropic porous material having permeability $k_1$, density $\rho_1$, viscosity $\mu_1$, thermal conductivity $K_1$, thermal expansion coefficient $\beta_{r_1}$, concentration expansion coefficient $\beta_{c_1}$ and diffusion coefficient $D_1$. The region-II $(-h_2 \leq Y \leq 0)$ is filled with another homogeneous isotropic porous material having permeability $k_2$. This region is saturated with different viscous fluid of density $\rho_2$, viscosity $\mu_2$, thermal conductivity $K_2$, thermal expansion coefficient $\beta_{r_2}$, concentration expansion coefficient $\beta_{c_2}$ and diffusion coefficient $D_2$. The fluids are assumed to have constant property except the density in the buoyancy term in the momentum equation. A fluid rises in the channel driven by buoyancy force. The temperature properties of both the fluids are assumed to be constant. We consider the fluids to be incompressible flow is steady, laminar and fully developed. It is assumed that the fluid viscosity and Brinkman viscosity (i.e., effective viscosity) are same. The flow in both the regions is assumed to be driven by a common constant pressure gradient $dp/dX$ and temperature gradient $\Delta T = T_{w_1} - T_{w_2}$. It is also assumed that at any given instant, the temperature of the fluid and the temperature of solid are same. The temperature and concentration of boundary at $Y = h_1$ is $T_{w_1}$ and $C_{w_1}$, while at $Y = -h_2$ is $T_{w_2}$ and $C_{w_2}$ respectively.

Under these assumptions, the governing equations of motion, energy and concentration for incompressible fluids yields as
Region - I

\[ g \beta_1 (T_1 - T_{w_2}) - \frac{1}{\rho_1} \frac{dp}{dX} + \nu_1 \left( \frac{d^2U_1}{dY^2} \right) + g \beta c_1 (C_1 - \bar{C}_2) - \frac{\nu_1}{\kappa_1} U_1 = 0 \]  
(5.2.1)

\[ K_1 \frac{d^2T_1}{dY^2} + \mu_1 \left( \frac{dU_1}{dY} \right)^2 + \frac{\mu_1}{\kappa_1} U_1^2 = 0 \]  
(5.2.2)

\[ D_1 \frac{dC_1^2}{dY^2} - K_1 C_1 = 0 \]  
(5.2.3)

Region - II

\[ g \beta_2 (T_2 - T_{w_2}) - \frac{1}{\rho_2} \frac{dp}{dX} + \nu_2 \left( \frac{d^2U_2}{dY^2} \right) + g \beta c_2 (C_2 - \bar{C}_1) - \frac{\nu_2}{\kappa_2} U_2 = 0 \]  
(5.2.4)

\[ K_2 \frac{d^2T_2}{dY^2} + \mu_2 \left( \frac{dU_2}{dY} \right)^2 + \frac{\mu_2}{\kappa_2} U_2^2 = 0 \]  
(5.2.5)

\[ D_2 \frac{dC_2^2}{dY^2} - K_2 C_2 = 0 \]  
(5.2.6)

The boundary conditions on velocity are no slip conditions and the two boundaries are held at different temperatures. In addition, continuity of velocity, shear stress, temperature, heat flux, concentration and mass flux at the interface are assumed.

\[ U_1(h_1) = 0, \quad U_2(-h_2) = 0, \quad U_1(0) = U_2(0), \quad \mu_1 \frac{dU_1}{dY}(0) = \mu_2 \frac{dU_2}{dY}(0) \]  
(5.2.7)

\[ T_1(h_1) = T_w_1, \quad T_2(-h_2) = T_w_2, \quad T_1(0) = T_2(0), \quad K_1 \frac{dT_1}{dY}(0) = K_2 \frac{dT_2}{dY}(0) \]  
(5.2.8)

\[ C_1(h_1) = \bar{C}_1, \quad C_2(-h_2) = \bar{C}_2, \quad C_1(0) = C_2(0), \quad D_1 \frac{dC_1}{dY}(0) = D_2 \frac{dC_2}{dY}(0) \]  
(5.2.9)
Equations (5.2.1) to (5.2.6) along with boundary and interface conditions (5.2.7) to (5.2.9) are made dimensionless by using the following transformations:

\[ u_i = \frac{U_i}{U_1}, \quad y_i = \frac{Y_i}{h_i}, \quad \theta_1 = \frac{T_1 - T_{w2}}{T_{w1} - T_{w2}}, \quad \theta_2 = \frac{T_2 - T_{w2}}{T_{w1} - T_{w2}}, \quad \phi_1 = \frac{C_1 - \overline{C}_2}{C_1 - C_2}, \phi_2 = \frac{C_2 - \overline{C}_2}{C_2 - C_2}, \]

\[ Br = \frac{U_1 \mu_l}{K_1 (T_{w1} - T_{w2})}, \quad Re = \frac{U_1 h_1}{\nu_l}, \quad p = \frac{h_1^2}{\mu_l U_1} \frac{dp}{dX}, \quad \sigma_1 = \frac{h_1}{\sqrt{k_1}}, \quad \sigma_2 = \frac{h_2}{\sqrt{k_2}}, \]

\[ \alpha_1^2 = \frac{K_i h_1^2}{D_1}, \quad \alpha_2^2 = \frac{K_i h_2^2}{D_2}, \quad Gr = \frac{g \beta_T h_1^3}{\nu_l^2} \frac{T_{w1} - T_{w2}}{\nu_l^2}, \quad Gc = \frac{g \beta_C h_1^3}{\nu_l^2} \frac{\overline{C}_1 - \overline{C}_2}{\nu_l^2} \]

(5.2.10)

The non-dimensional governing equations (5.2.1) to (5.2.6) can be written in a dimensionless form by employing the dimensionless quantities (5.2.10)

Region - I

\[ \frac{d^2 u_i}{dy^2} + Gr_e \theta_1 + Gr_c \phi_1 - p - \sigma_1^2 u_i = 0 \]  

(5.2.11)

\[ \frac{d^2 \theta_1}{dy^2} + Br \left( \frac{du_i}{dy} \right)^2 + \sigma_1^2 u_i^2 = 0 \]  

(5.2.12)

\[ \frac{d^2 \phi_1}{dy^2} - \alpha_1^2 \phi_1 = 0 \]  

(5.2.13)

Region - II

\[ \frac{d^2 u_2}{dy^2} + Gr_e m r h_j b_i \theta_2 + Gr_c m r h_j b_c \phi_2 - m h_j^2 p - \sigma_2^2 u_2 = 0 \]  

(5.2.14)

\[ \frac{d^2 \theta_2}{dy^2} + Brk \left( \frac{du_2}{dy} \right)^2 + \sigma_2^2 u_2^2 = 0 \]  

(5.2.15)
where,

\[
GR_T = \frac{Gr}{Re}, \quad GR_C = \frac{Gc}{Re}, \quad h = \frac{h_2}{h_1}, \quad m = \frac{\mu_1}{\mu_2}, \quad \beta_T = \frac{\beta_{T_2}}{\beta_{T_1}}, \quad r = \frac{P_2}{P_1}, \quad \beta_c = \frac{\beta_{c_2}}{\beta_{c_1}}, \quad d = \frac{D_2}{D_1},
\]

\[
k = \frac{K_1}{K_2}, \quad \alpha = \frac{\alpha_2}{\alpha_1}
\]

The boundary and interface conditions in non-dimensional form become

\[
\begin{align*}
&u_1(1) = 0, \quad u_2(-1) = 0, \quad u_i(0) = u_2(0), \quad \frac{du_i}{dy}(0) = \frac{1}{mh} \frac{du_2}{dy}(0) \quad (5.2.17) \\
&\theta_1(1) = 1, \quad \theta_2(-1) = 0, \quad \theta_i(0) = \theta_2(0), \quad \frac{d\theta_i}{dy}(0) = \frac{1}{kh} \frac{d\theta_2}{dy}(0) \quad (5.2.18) \\
&\phi_1(1) = 1, \quad \phi_2(-1) = 0, \quad \phi_i(0) = \phi_2(0), \quad \frac{d\phi_i}{dy}(0) = \frac{d}{h} \frac{d\phi_2}{dy}(0) \quad (5.2.19)
\end{align*}
\]

5.3 SOLUTIONS

(i) Perturbation Method (PM)

Equations (5.2.11) (5.2.12) (5.2.14) and (5.2.15) are coupled and highly nonlinear equations because of viscous and Darcy dissipation terms, hence exact solutions cannot be found. The approximate analytical solutions can be found using perturbation method. The Brinkman number can be exploited as the perturbation parameter. Therefore the solutions are assumed in the form

\[
u_i(y) = u_{i0}(y) + Br u_{i1}(y) + Br^2 u_{i2}(y) + \ldots \quad (5.3.1)
\]

\[
\theta_i(y) = \theta_{i0}(y) + Br \theta_{i1}(y) + Br^2 \theta_{i2}(y) + \ldots \quad (5.3.2)
\]

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Using equations (5.3.1) and (5.3.2) in equations (5.2.11), (5.2.12), (5.2.14), (5.2.15) and equating the coefficients of like powers of $Br$ to zero and one we determine zeroth and first order equations as follows

Region - I

Zeroth order equations

\[
\frac{d^2 \theta_{10}}{dy^2} = 0
\]  

(5.3.3)

\[
\frac{d^2 u_{10}}{dy^2} + GR_T \theta_{10} + GR_c \phi_1 - p - u_{10} \sigma_1^2 = 0
\]  

(5.3.4)

First order equations

\[
\frac{d^2 \theta_{11}}{dy^2} + \left( \frac{du_{10}}{dy} \right)^2 + \sigma_1^2 u_{10}^2 = 0
\]  

(5.3.5)

\[
\frac{d^2 u_{11}}{dy^2} + GR_T \theta_{11} - u_{11} \sigma_1^2 = 0
\]  

(5.3.6)

Region - II

Zeroth order equations

\[
\frac{d^2 \theta_{20}}{dy^2} = 0
\]  

(5.3.7)

\[
\frac{d^2 u_{20}}{dy^2} + GR_T \frac{m r h^2 b_1 \theta_{20}}{b_1 \theta_{20}} + GR_c \frac{m r h^2 b_c \phi_2 - m h^2 p - \sigma_2^2 u_{20}}{m h^2 p - \sigma_2^2 u_{20}} = 0
\]  

(5.3.8)

First order equations

\[
\frac{d^2 \theta_{21}}{dy^2} + \frac{k}{m} \left[ \left( \frac{du_{20}}{dy} \right)^2 + \sigma_2^2 u_{20}^2 \right] = 0
\]  

(5.3.9)
\[
\frac{d^2 u_{21}}{dy^2} + GR_m rh^2 b \frac{\partial}{\partial x} - \sigma_z^2 u_{21} = 0
\] (5.3.10)

The corresponding boundary and interface conditions as given in equations (5.2.17), (5.2.18) and (5.2.19) can be written as,

Zeroth order boundary and interface conditions

\[
u_{10} = 0, \quad u_{20} = 0, \quad u_{10} = u_{20} = 0, \quad \frac{du_{10}}{dy}(0) = \frac{1}{mh} \frac{du_{20}}{dy}(0)
\] (5.3.11)

\[
\theta_{10}(1) = 1, \quad \theta_{20}(-1) = 0, \quad \theta_{10}(0) = \theta_{20}(0), \quad \frac{d\theta_{10}}{dy}(0) = \frac{1}{kh} \frac{d\theta_{20}}{dy}(0)
\] (5.3.12)

First order boundary and interface conditions

\[
u_{11}(1) = 0, \quad u_{21}(-1) = 0, \quad u_{11}(0) = u_{21}(0), \quad \frac{du_{11}}{dy}(0) = \frac{1}{mh} \frac{du_{21}}{dy}(0)
\] (5.3.13)

\[
\theta_{11}(1) = 0, \quad \theta_{21}(-1) = 0, \quad \theta_{11}(0) = \theta_{21}(0), \quad \frac{d\theta_{11}}{dy}(0) = \frac{1}{kh} \frac{d\theta_{21}}{dy}(0)
\] (5.3.14)

The solutions for equations (5.2.13) and (5.2.16) using boundary and interface conditions as defined in equation (5.2.19) become,

\[
\phi_1 = B_1 \text{Cosh}(\alpha_1 y) + B_2 \text{Sinh}(\alpha_1 y)
\] (5.3.15)

\[
\phi_2 = B_3 \text{Cosh}(\alpha_2 y) + B_4 \text{Sinh}(\alpha_2 y)
\] (5.3.16)

The solutions of the zeroth and first order equations (5.3.3), (5.3.4), (5.3.7), (5.3.8) and (5.3.5), (5.3.6), (5.3.9), (5.2.10) are obtained by using boundary and interface conditions as defined in equations (5.3.11), (5.3.12), (5.3.13) and (5.3.14) respectively and are given by
\[ \theta_{10} = c_1 y + c_2 \quad (5.3.17) \]

\[ \theta_{20} = c_3 y + c_4 \quad (5.3.18) \]

\[ u_{10}(y) = A_1 \cosh(\sigma_1 y) + A_2 \sinh(\sigma_1 y) + r_1 + r_2 y + r_3 \cosh(\alpha_1 y) + r_4 \sinh(\alpha_1 y) \quad (5.3.19) \]

\[ u_{20}(y) = A_3 \cosh(\sigma_2 y) + A_4 \sinh(\sigma_2 y) + r_5 + r_6 y + r_7 \cosh(\alpha_2 y) + r_8 \sinh(\alpha_2 y) \quad (5.3.20) \]

\[ \theta_{11}(y) = E_2 + E_3 y + q_1 y^2 + q_2 y^3 + q_3 y^4 + q_4 \cosh(\alpha_1 y) + q_5 \sinh(\alpha_1 y) + q_6 \cosh(2\alpha_1 y) + q_7 \sinh(2\alpha_1 y) + q_8 \cosh(2\sigma_1 y) + q_9 \sinh(2\sigma_1 y) + q_{10} \cosh(\sigma_1 y) + q_{11} \sinh(\sigma_1 y) + q_{12} y \cosh(\sigma_1 y) + q_{13} \sinh(\sigma_1 y) + q_{14} y \cosh(\alpha_1 y) + q_{15} \sinh(\alpha_1 y) + q_{16} \cosh(\alpha_1 + \sigma_1 y) + q_{17} \sinh(\alpha_1 + \sigma_1 y) + q_{18} \sinh(\alpha_1 + \sigma_1 y) + q_{19} \sinh(\alpha_1 - \sigma_1 y) \quad (5.3.21) \]

\[ \theta_{21}(y) = E_4 + E_5 y + F_1 y^2 + F_2 y^3 + F_3 y^4 + F_4 \cosh(\alpha_2 y) + F_5 \sinh(\alpha_2 y) + F_6 \cosh(2\alpha_2 y) + F_7 \sinh(2\alpha_2 y) + F_8 \cosh(2\sigma_2 y) + F_9 \sinh(2\sigma_2 y) + F_{10} \cosh(\sigma_2 y) + F_{11} \sinh(\sigma_2 y) + F_{12} y \cosh(\sigma_2 y) + F_{13} \sinh(\sigma_2 y) + F_{14} \cosh(\alpha_2 y) + F_{15} \sinh(\alpha_2 y) + F_{16} \cosh(\alpha_2 + \sigma_2 y) + F_{17} \sinh(\alpha_2 + \sigma_2 y) + F_{18} \sinh(\alpha_2 + \sigma_2 y) + F_{19} \sinh(\alpha_2 - \sigma_2 y) \quad (5.3.22) \]

\[ u_{11}(y) = E_5 \cosh(\sigma_1 y) + E_6 \sinh(\sigma_1 y) + H_1 + H_2 y + H_3 y^2 + H_4 y^3 + H_5 y^4 + H_6 \cosh(\alpha_1 y) + H_7 \sinh(\alpha_1 y) + H_8 \cosh(\alpha_1 y) + H_9 \sinh(\alpha_1 y) + H_{10} \cosh(2\sigma_1 y) + H_{11} \sinh(2\sigma_1 y) + H_{12} y \cosh(\sigma_1 y) + H_{13} \sinh(\sigma_1 y) + H_{14} y \cosh(\sigma_1 y) + H_{15} \sinh(\sigma_1 y) + H_{16} \cosh(\alpha_1 + \sigma_1 y) + H_{17} \cosh(\alpha_1 + \sigma_1 y) + H_{18} \sinh(\alpha_1 + \sigma_1 y) + H_{19} \sinh(\alpha_1 - \sigma_1 y) + H_{20} \cosh(\sigma_1 y) + H_{21} y^2 \sinh(\sigma_1 y) \quad (5.3.23) \]

\[ u_{21}(y) = E_6 \cosh(\sigma_2 y) + E_9 \sinh(\sigma_2 y) + H_{22} + H_{23} y + H_{24} y^2 + H_{25} y^3 + H_{26} y^4 + H_{27} \cosh(\alpha_2 y) + H_{28} \sinh(\alpha_2 y) + H_{29} \cosh(\alpha_2 y) + H_{30} \sinh(\alpha_2 y) + H_{31} \cosh(2\sigma_2 y) + H_{32} \sinh(2\sigma_2 y) + H_{33} y \cosh(\sigma_2 y) + H_{34} \sinh(\sigma_2 y) + H_{35} \cosh(\alpha_2 + \sigma_2 y) + H_{36} \sinh(\alpha_2 + \sigma_2 y) + H_{37} \cosh(\alpha_2 + \sigma_2 y) + H_{38} \sinh(\alpha_2 + \sigma_2 y) + H_{39} \sinh(\alpha_2 + \sigma_2 y) + H_{40} \cosh(\alpha_2 - \sigma_2 y) + H_{41} y^2 \cosh(\sigma_2 y) + H_{42} y^2 \sinh(\sigma_2 y) \quad (5.3.24) \]
The wall heat transfer expression in terms of the Nusselt number becomes

\[ Nu_+ = 1 + h \frac{d\theta_1}{dy} \quad \text{at} \quad y = 1 \]

\[ Nu_- = 1 + \frac{1}{h} \frac{d\theta_2}{dy} \quad \text{at} \quad y = -1 \]  

(5.3.25)

\[ Nu_+ = (1 + h) \left( c_1 + Br \ E_1 + 2q_1 + 3q_2 + 4q_3 + q_4 \alpha_1 \sinh(\alpha_1) + q_5 \alpha_1 \cosh(\alpha_1) + 2 \alpha_1 q_6 \sinh(2\alpha_1) + 2 \alpha_1 q_7 \cosh(2\alpha_1) + 2 \alpha_1 q_8 \sinh(2\sigma_1) + 2 \sigma_1 q_9 \cosh(2\sigma_1) \right) \]

\[ + q_{10} \sigma_1 \sinh(\sigma_1) + q_{11} \sigma_1 \cosh(\sigma_1) + q_{12} \sigma_1 \sinh(\sigma_1) + q_{13} \sigma_1 \cosh(\sigma_1) + q_{14} \alpha_1 \sinh(\alpha_1) + q_{15} \alpha_1 \cosh(\alpha_1) + q_{16} \alpha_1 \sinh(\alpha_1) + q_{17} \alpha_1 \cosh(\alpha_1) + q_{18} (\alpha_1 - \sigma_1) \sinh(\alpha_1 - \sigma_1) + q_{19} (\alpha_1 - \sigma_1) \cosh(\alpha_1 - \sigma_1) \]  

(5.3.26)

\[ Nu_- = 1 + \frac{1}{h} \left( c_3 + Br \ E_3 - 2F_1 + 3F_2 - 4F_3 - F_1 \alpha_2 \sinh(\alpha_2) + F_2 \alpha_2 \cosh(\alpha_2) \right) \]

\[ - 2 \alpha_2 F_6 \sinh(2\alpha_2) + 2 \alpha_2 F_7 \cosh(2\alpha_2) - 2 \sigma_2 F_9 \sinh(2\sigma_2) + 2 \sigma_2 F_8 \cosh(2\sigma_2) \]

\[ - F_{10} \sigma_2 \sinh(\sigma_2) + F_{11} \sigma_2 \cosh(\sigma_2) + F_{12} \sigma_2 \sinh(\sigma_2) + \cosh(\sigma_2) \]

\[ - F_{13} \sigma_2 \cosh(\sigma_2) + \sinh(\sigma_2) + F_{14} \alpha_2 \sinh(\alpha_2) + \cosh(\alpha_2) \]  

(5.3.27)

The dimensionless total volume flow rate is given by

\[ Qv = Qv_1 + Qv_2 \]  

(5.3.28)

where

\[ Qv_1 = \int_0^1 u_1 \, dy \quad Qv_2 = \int_1^0 u_2 \, dy \]
The dimensionless total heat rate added to the fluid is given by

\[ Hr = Hr_1 + Hr_2 \]  \hspace{1cm} (5.3.29)

where

\[ Hr_1 = \int_0^1 u_1 \theta_1 dy, \quad Hr_2 = \int_1^0 u_2 \theta_2 dy \]

The dimensionless total species rate added to the fluid is given by

\[ Cs = Cs_1 + Cs_2 \]  \hspace{1cm} (5.3.30)

where

\[ Cs_1 = \int_0^1 u_1 \phi_1 dy, \quad Cs_2 = \int_1^0 u_2 \phi_2 dy \]

Equations (5.3.17) to (5.3.30) are evaluated for different values of the governing parameters and the results are presented graphically.

(ii) **Finite Difference Method (FDM)**

The approximate analytical solutions obtained in the subsection 5.3 (i) are valid for values of Brinkman number less than one. However in many practical problems especially when viscous dissipation dominates, the Brinkman number takes the values greater than one. In such situations it is required to find the approximate solutions numerically. The governing equations (5.2.11) to (5.2.16) with the boundary and interface conditions (5.2.17), (5.2.18) and (5.2.19) are solved using FDM. In numerical iterations, computation domain is divided into a uniform grid system. The second derivative and the squared first derivatives terms are discritized with central difference of second order accuracy. By replacing the derivatives with the corresponding finite
difference approximation, we obtain a set of $n$ algebraic equations, where $n$ is the number of divisions from -1 to 1. To validate the present numerical method, computed solutions are compared with analytical solutions. The numerical and analytical solutions agree very well in the absence of Brinkman number and as the Brinkman number increases, error between FDM and PM also increases. The solutions obtained by FDM and PM are depicted in table 5.1 and percentage error between FDM and PM is also evaluated.

The constants appeared in all the above equations are presented in the section appendix.

Appendix

$$
a_1 = -GR_r mh^2 b_3, \quad a_2 = -GR_c mh^2 b_4, \quad a_3 = -\frac{k}{m}, \quad c_1 = \frac{1}{1+kh}, \quad c_2 = \frac{kh}{1+kh}, \quad c_4 = \frac{kh}{1+kh}.
$$

$$
c_3 = \frac{kh}{1+kh}, \quad B_1 = \frac{h \sinh \alpha_2}{h \cosh \alpha_2 \sinh \alpha_2 + \alpha d \sinh \alpha_2 \cosh \alpha_2}, \quad B_2 = \frac{\alpha d \beta \cosh \alpha_2}{h}, \quad B_4 = \frac{h B_2}{\alpha d},
$$

$$
B_3 = B_1, \quad r_1 = \frac{GR_r c_2 - p}{M^2}, \quad r_2 = \frac{GR_c c_1}{M^2}, \quad r_3 = -\frac{GR_r B_1}{\alpha_1^2 - M^2}, \quad r_4 = -\frac{GR_c B_2}{\alpha_1^2 - M^2}, \quad r_6 = -\frac{a_c c_3}{B^2}
$$

$$
r_5 = \frac{-mh^2 p - a_c c_4}{B^2}, \quad r_7 = \frac{a_c B_3}{\alpha_2^2 - B^2}, \quad r_8 = \frac{a_c B_4}{\alpha_2^2 - B^2}, \quad T_1 = -r_1 - r_2 - r_3 \cosh \alpha_1 - r_4 \sinh \alpha_1,
$$

$$
T_2 = -r_5 + r_6 - r_7 \cosh \alpha_2 + r_8 \sinh \alpha_2, \quad T_3 = r_1 + r_5 - r_1 - r_5, \quad T_4 = r_6 + \alpha_2 r_8 - mh(\alpha_1 r_4 + r_2),
$$

$$
A_4 = \frac{T_1 - A_2 \sinh M}{\cosh M}, \quad A_1 = A_1 - T_3, \quad A_4 = \frac{mh A_2 - T_4}{B}, \quad p_2 = -2 r_2 M^2, \quad p_3 = -M^2 r_2^2.
$$
\[ p_1 = \frac{-2r_1^2 - 2r_1^2M^2 + (\alpha_1^2 - M^2)(r_3^2 - r_4^2)}{2}, \quad p_4 = -2r_2r_3\alpha_1 - 2r_3r_4M^2, \]

\[ p_3 = -2r_2r_3\alpha_1 - 2r_4r_3M^2, \quad p_6 = \frac{-(r_2^2 + r_4^2)(\alpha_1^2 + M^2)}{2}, \quad p_7 = -r_3r_4(\alpha_1^2 + M^2), \]

\[ A_2 = \frac{T_1B\cosh B - T_1\cosh B \cosh M - BT_1\cosh M + T_1\sinh B \cosh M}{B\sinh M \cosh B + mhM \sinh B \cosh M}, \]

\[ p_8 = -A_1^2M^2 - A_2^2M^2, \quad p_9 = -2A_1A_2M^2, \quad p_{10} = -2A_2r_3M - 2A_1r_4M^2, \]

\[ p_{11} = -2A_2r_2M - 2A_2r_3M^2, \quad p_{12} = -2A_2r_2M^2, \quad p_{13} = -2A_2r_2M^2, \quad p_{14} = -2r_3r_4M^2, \]

\[ p_{15} = -2r_2r_4M^2, \quad q_1 = \frac{p_1}{2}, \quad q_2 = \frac{p_2}{6}, \quad q_3 = \frac{p_3}{12}, \quad q_4 = \frac{p_4\alpha_1 - 2p_{15}}{\alpha_1^3}, \quad q_5 = \frac{p_5\alpha_1 - 2p_{14}}{\alpha_1^3}, \]

\[ p_{16} = -A_2r_4M\alpha_1 - A_1r_5M^2 - A_1r_4M - A_1r_5M^2, \quad q_6 = \frac{p_6}{4\alpha_1^2}, \quad q_7 = \frac{p_7}{4\alpha_1^2}, \quad q_8 = \frac{p_8}{4M^2}. \]

\[ q_{11} = \frac{p_{11}M - 2p_{12}}{M^3}, \quad q_{12} = \frac{p_{12}}{M^2}, \quad q_{13} = \frac{p_{13}}{M^2}, \quad q_{18} = -A_2r_3M\alpha_1 - A_1r_4M^2 - A_1r_5M\alpha_1 - A_2r_5M^2, \]

\[ q_{14} = \frac{p_{14}}{\alpha_1^2}, \quad q_{15} = \frac{p_{15}}{\alpha_1^2}, \quad q_{16} = \frac{p_{16}}{\alpha_1 + M}, \quad q_{19} = -A_2r_5M\alpha_1 - A_1r_4M^2 + A_1r_5M\alpha_1 + A_2r_5M^2, \]

\[ q_{17} = \frac{p_{17}}{\alpha_1^2 - M^2}, \quad q_{18} = \frac{p_{18}}{\alpha_1 + M}, \quad q_{19} = \frac{p_{19}}{\alpha_1 - M^2}, \quad R_2 = 2a_2r_6r_6B^2, \quad R_3 = a_3r_6B^2, \]

\[ R_4 = 2a_3r_6r_6\alpha_2 + 2a_3r_7r_7B^2, \quad R_5 = 2a_3r_6r_7\alpha_2 + 2a_3r_5r_6B^2, \quad R_6 = \frac{a_3(r_6^2 + r_7^2)(\alpha_2^2 + B^2)}{2}, \]

\[ R_7 = \frac{2a_3r_6^2 + 2a_3r_5^2B^2 + (a_3^2 - a_5^2)(\alpha_2^2 - B^2)}{2}, \quad R_8 = a_3A_3B^2 + a_3A_4B^2, \quad R_9 = 2a_3A_3B^2, \quad R_{10} = 2a_3A_3r_6B + 2a_3A_3r_5B^2, \]

\[ R_{11} = 2a_3A_3B_6 + 2a_3A_4B^2, \quad R_{12} = 2a_3A_3r_6B^2, \quad R_{13} = 2a_3A_3r_6B^2, \quad R_{14} = 2a_3r_6r_7B^2, \]

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\[ R_{15} = 2a_3 r_4 r_6 B^2, \quad F_1 = \frac{R_2}{2}, \quad F_2 = \frac{R_3}{6}, \quad F_3 = \frac{R_4}{12}, \quad F_4 = \frac{R_5 \alpha_2 - 2 R_{15}}{\alpha_2^3}, \quad F_5 = \frac{R_5 \alpha_2 - 2 R_{14}}{\alpha_2^3}, \]

\[ F_6 = \frac{R_6}{4 \alpha_2}, \quad F_7 = \frac{R_7}{4 \alpha_2}, \quad F_8 = \frac{R_8}{4 B^2}, \quad F_9 = \frac{R_9}{4 B^2}, \quad F_{10} = \frac{R_{10} B - 2 R_{13}}{B^3}, \quad F_{11} = \frac{R_{11} B - 2 R_{12}}{B^3}, \]

\[ R_{16} = a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B, \quad F_{12} = \frac{R_{12}}{B^2}, \quad F_{13} = \frac{R_{13}}{B^2}, \quad F_{14} = \frac{R_{14}}{B^2}, \]

\[ R_{17} = a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B - a_4 A_8 r_8 \alpha_2 B - a_4 A_8 r_8 \alpha_2 B, \quad F_{15} = \frac{R_{15}}{\alpha_2}, \quad F_{16} = \frac{R_{16}}{(\alpha_2 + B)^2}, \]

\[ R_{18} = a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B, \quad F_{17} = \frac{R_{17}}{(\alpha_2 - B)^2}, \quad F_{18} = \frac{R_{18}}{(\alpha_2 + B)^2}, \]

\[ R_{19} = a_4 A_8 r_8 \alpha_2 B + a_4 A_8 r_8 \alpha_2 B - a_4 A_8 r_8 \alpha_2 B - a_4 A_8 r_8 \alpha_2 B, \quad F_{19} = \frac{R_{19}}{(\alpha_2 - B)^2}, \]

\[ G_1 = \begin{pmatrix}
q_1 + q_2 + q_3 + (q_4 + q_{14}) \cosh \alpha_1 + (q_5 + q_{11}) \sinh \alpha_1 + q_6 \cosh 2 \alpha_1 + q_7 \sinh 2 \alpha_1 \\
+ q_7 \cosh 2 M + q_8 \sinh 2 M + (q_{10} + q_{12}) \cosh M + (q_{11} + q_{13}) \sinh M \\
+ q_{16} \cosh (\alpha_1 + M) + q_{17} \cosh (\alpha_2 - B) + q_{18} \sinh (\alpha_2 + B) + q_{19} \sinh (\alpha_2 - B)
\end{pmatrix}, \]

\[ G_2 = -\begin{pmatrix}
F_1 - F_2 + F_3 + (F_4 - F_{14}) \cosh \alpha_2 + (F_{15} - F_3) \sinh \alpha_2 + F_6 \cosh 2 \alpha_2 \\
- F_7 \sinh 2 \alpha_2 + F_8 \cosh 2 B - F_9 \sinh 2 B + F_{10} \cosh B - F_{11} \sinh B \\
- F_{12} \cosh B + F_{13} \sinh \alpha_2 - F_{14} \cosh \alpha_2 + F_{15} \sinh \alpha_2 + F_{16} \cosh (\alpha_2 + B) \\
+ F_{17} \cosh (\alpha_2 - B) - F_{18} \sinh (\alpha_2 + B) - F_{19} \sinh (\alpha_2 - B)
\end{pmatrix}, \]

\[ G_3 = F_4 + F_5 + F_6 + F_{10} + F_{10} + F_{17} - q_4 - q_5 - q_6 - q_8 - q_{10} - q_{16} - q_{17}, \]

\[ G_4 = \begin{pmatrix}
F_5 \alpha_2 + 2 \alpha_2 F_7 + 2 B F_9 + B F_{11} + F_{12} + F_{14} + (\alpha_2 + B) F_{18} + F_{19} (\alpha_2 - B) - k h \\
(\alpha_1 q_5 + 2 \alpha_1 q_7 + 2 M q_9 + M q_{11} + q_{12} + q_{14} + (\alpha_1 + M) q_{18} + q_{19} (\alpha_i - M))
\end{pmatrix}, \]

\[ E_1 = \frac{G_i - G_2 - G_3 + G_4}{1 + k h}, \quad E_2 = \frac{G_i k h + G_2 + G_3 - G_4}{1 + k h}, \quad E_3 = k h E_1 - G_4, \quad E_4 = E_2 - G_3, \]

\[ H_1 = \frac{G R \epsilon (E_2 M^4 + 2 q_j M^2 + 24 q_1)}{M^6}, \quad H_2 = \frac{G R \epsilon (E_i M^2 + 2 q_j)}{M^4}, \quad H_3 = \frac{G R \epsilon (q_j M^2 + 12 q_3)}{M^4}, \]

\[ H_4 = \frac{G R \epsilon q_2}{M^2}, \quad H_5 = \frac{G R \epsilon q_3}{M^2}, \quad H_6 = \frac{2 \alpha_i q_{15} G R \epsilon - q_{14} G R \epsilon (\alpha_1^2 - M^2)}{(\alpha_1^2 - M^2)^2}, \quad H_8 = \frac{-G R \epsilon q_4}{4 \alpha_1^2 - M^2}, \]
\[ H_7 = \frac{2\alpha_1 q_{14} G_{r_7} - G_{r_7} q_7 (\alpha_1^2 - M^2)}{(\alpha_1^2 - M^2)^2}, \quad H_9 = \frac{-G_{r_7} q_7}{4\alpha_1^2 - M^2}, \quad H_{10} = \frac{-G_{r_7} q_8}{3M^2}, \quad H_{11} = \frac{-G_{r_7} q_9}{3M^2}, \]

\[ H_{12} = \frac{G_{r_7} q_{12} - 2G_{r_7} q_{11} M}{4M^2}, \quad H_{13} = \frac{G_{r_7} q_{13} - 2G_{r_7} q_{10} M}{4M^2}, \quad H_{14} = \frac{-G_{r_7} q_{14}}{\alpha_1^2 - M^2}, \]

\[ H_{15} = \frac{-G_{r_7} q_{15}}{(\alpha_1 - M)^2 - M^2}, \quad H_{16} = \frac{-G_{r_7} q_{16}}{(\alpha_1 + M)^2 - M^2}, \quad H_{17} = \frac{-G_{r_7} q_{17}}{(\alpha_1 - M)^2 - M^2}, \quad H_{18} = \frac{-G_{r_7} q_{18}}{(\alpha_1 + M)^2 - M^2}, \]

\[ H_{19} = \frac{-G_{r_7} q_{19}}{(\alpha_1 - M)^2 - M^2}, \quad H_{20} = \frac{-G_{r_7} q_{13}}{4M}, \quad H_{21} = \frac{-G_{r_7} q_{12}}{4M}, \quad H_{23} = \frac{-a_1 (6F_2 + E_1 B^2)}{B^4}, \]

\[ H_{22} = \frac{-a_1 (E_4 B^2 + 2F_2 B^2 + 24F_3)}{B^4}, \quad H_{24} = \frac{-a_1 (F_1 B^2 + 12F_3)}{B^4}, \quad H_{25} = \frac{-a_2 F_2}{B^2}, \quad H_{26} = \frac{-a_3 F_5}{B^2}, \]

\[ H_{27} = \frac{a_1 (F_5 (\alpha_2^2 - B^2) - 2F_2 F_{15})}{(\alpha_2^2 - B^2)^2}, \quad H_{28} = \frac{a_1 (F_5 (\alpha_2^2 - B^2) - 2F_2 F_{14})}{(\alpha_2^2 - B^2)^2}, \quad H_{29} = \frac{a_6 F_6}{4\alpha_2^2 - B^2}, \]

\[ H_{30} = \frac{a_1 F_7}{4\alpha_2^2 - B^2}, \quad H_{31} = \frac{a_5 F_8}{3B^2}, \quad H_{32} = \frac{a_6 F_9}{3B^2}, \quad H_{33} = \frac{2a_1 F_{10} B - a_5 F_{12}}{4B^2}, \quad H_{34} = \frac{2a_1 F_{10} B - a_3 F_{13}}{4B^2}, \]

\[ H_{35} = \frac{a_1 F_{14}}{4\alpha_2^2 - B^2}, \quad H_{36} = \frac{a_2 F_{15}}{\alpha_2 B^2}, \quad H_{37} = \frac{a_1 F_{16}}{(\alpha_2 + B)^2 - B^2}, \quad H_{38} = \frac{a_1 F_{17}}{\alpha_2 - B^2}, \]

\[ H_{39} = \frac{a_1 F_{18}}{(\alpha_2 + B)^2 - B^2}, \quad H_{40} = \frac{a_1 F_{19}}{(\alpha_2 - B)^2 - B^2}, \quad H_{41} = \frac{a_1 F_{14}}{4B}, \quad H_{42} = \frac{a_1 F_{12}}{4B}, \]

\[ G_5 = - \left( H_1 + H_2 + H_3 + H_4 + H_5 + H_6 \cos \alpha_5 + H_7 \sinh \alpha_5 + H_8 \cosh 2\alpha_5 \right) \]

\[ + H_9 \sinh 2\alpha_4 + H_{10} \cosh 2M + H_{11} \sinh 2M + H_{12} \cosh M + H_{13} \sinh M \]

\[ + H_{14} \cosh \alpha_4 + H_{15} \sinh \alpha_4 + H_{16} \cosh (\alpha_4 + M) + H_{17} \cosh (\alpha_4 - M) \]

\[ + H_{18} \sinh (\alpha_4 + M) + H_{19} \sinh (\alpha_4 - M) + H_{20} \cosh M + H_{21} \sinh M \]

\[ G_6 = - \left( H_{22} - H_{23} + H_{24} - H_{25} + H_{26} + H_{27} \cosh \alpha_2 - H_{28} \sinh \alpha_2 + H_{29} \cosh 2\alpha_2 \right) \]

\[ - H_{30} \sinh 2\alpha_2 + H_{31} \cosh 2B - H_{32} \sinh 2B - H_{33} \cosh B + H_{34} \sinh B \]

\[ - H_{35} \cosh \alpha_2 + H_{36} \sinh \alpha_2 + H_{37} \cosh (\alpha_2 + B) + H_{38} \cosh (\alpha_2 - B) \]

\[ - H_{39} \sinh (\alpha_2 + B) - H_{40} \sinh (\alpha_2 - B) + H_{41} \cosh \alpha_2 - H_{42} \sinh \alpha_2 \]
\[ G_7 = H_{22} + H_{27} + H_{29} + H_{31} + H_{37} + H_{38} - H_4 - H_6 - H_8 - H_{10} - H_{16} - H_{17}, \]
\[
G_8 = \left( H_{23} + \alpha_2 H_{30} + 2\alpha_2 H_{32} + 2BH_{30} + H_{33} + H_{35} + H_{39} (\alpha_2 + B) + H_{40} (\alpha_2 - B) - mh(H_2 + \alpha_1 H_7 + 2\alpha_1 H_9 + 2MH_{11} + H_{12} + H_{14} + H_{18} (\alpha_1 + M) + H_{19} (\alpha_1 - M)) \right), \]
\[
E_5 = E_7 + G_7, \quad E_6 = \frac{G_6 - G_7 \cosh M - E_7 \cosh M}{\sinh M}, \]
\[
E_7 = \frac{G_6 \sinh M + mhMG_5 \sinh B - mhMG_7 \sinh B \cosh M - G_5 \sinh M \sinh B}{mhM \sinh B \cosh M + B \sinh M \cosh B}, \]
\[
E_8 = \frac{-(mhG_6 \cosh M - mhMG_7 \cosh B + mhMG_7 \cosh MCoshB + G_5 \cosh B \sinh M)}{mhM \sinh B \cosh M + B \sinh M \cosh B}. \]

5.4 RESULTS AND DISCUSSION

The problem concerned with the heat and mass transfer in a vertical channel for composite porous medium in the presence of homogeneous first order chemical reaction. The flow is modeled with Darcy-Lapwood-Brinkman equation. The viscous and Darcy dissipation terms are included in the energy equation. The continuity of velocity, temperature, shear stress, heat flux concentration and mass flux at the interface is assumed. The equations governing the flow which are highly nonlinear and coupled are solved analytically using perturbation method (PM) and numerically using finite difference method (FDM). The perturbation solutions are valid for small values of Brinkman number and numerical solutions are valid for all values of Brinkman number.

The effect of thermal Grashof number \( \text{GR}_T \) on the velocity and temperature fields is shown in figures 5.2a and 5.2b respectively, in the presence \( (\alpha = 1) \) and in the absence \( (\alpha = 0) \) of first order chemical reaction. As \( \text{GR}_T \) increases the flow increases in
both the regions. Physically an increase in the value of Grashof number means an increase of the buoyancy force which supports the motion. Further figures 5.2a and 5.2b also reveal that the magnitude of velocity and temperature is large in the absence of chemical reaction when compared with values in the presence of the chemical reaction.

The effect of mass Grashof number $GR_c$ on the velocity and temperature fields shows the similar effect as that of thermal Grashof number, as shown in figures 5.3a and 5.3b respectively. That is to say that as $GR_c$ increases, flow increases in both the regions. The mass Grashof number is the ratio of species buoyancy force to the viscous force. As expected, the fluid velocity and temperature increases due to the increase in the species buoyancy force. The effects of $GR_f$ and $GR_c$ on the flow were the similar results observed by Shivaiah and Anand Rao (2012) for the flow past a vertical porous plate and Malashetty et al. (2001a, 2004, 2005) in the absence of chemical reaction. The variation of velocity and temperature for different values of porous parameter $\sigma (=\sigma_1=\sigma_2)$ is shown in figures 5.4a and 5.4b respectively. As the porous parameter increases the velocity, temperature decreases in both the regions. For large values of $\sigma$ the frictional drag resistance against the flow motion is pronounced and as a result velocity is reduced in both the regions.

The effect of viscosity ratio $m (\mu_1/\mu_2)$ is to increase the velocity and temperature fields in both the regions as shown in figures 5.5a and 5.5b respectively. The viscosity ratio $m$ is defined as the viscosity of the fluid in region-I to the viscosity of the fluid in region-II. It is observed from figure 5.5b that the effect of viscosity ratio on the temperature field is not very significant.
The effect of width ratio $h (h_2/h_1)$ is to enhance velocity and temperature field in both the regions as displayed in figures 5.6a and 5.6b respectively. The width ratio $h$ is defined as the ratio of width of the fluid layer in region-II to the width of the fluid in region-I. It is well known that as $h$ increases, velocity increases which intern enhances the dissipation and hence results in enhancement of temperature field also.

The effect of conductivity ratio $k (K_1/K_2)$ on the flow is similar to the effects on viscosity ratio and width ratio, as seen in figures 5.7a and 5.7b. The conductivity of the permeable fluid layer in region-I is large compared to the conductivity of fluid layer in region-II, larger the amount of heat transfer and hence velocity also increases. The effects of $h$ and $k$ in the presence of first order chemical reaction was the similar results observed by Malashetty (2001a) in the absence of first order chemical reaction.

The effect of first order chemical reaction parameter $\alpha$ on velocity, temperature and concentration fields is depicted in figures 5.8a, 5.8b and 5.8c respectively. It is evident from these figures that as $\alpha$ increases the velocity, temperature and concentration are reduced in both the regions. Physically, an increase in $\alpha$ leads to the increase in the number of solute molecules undergoing chemical reaction resulting in decrease in the fluid flow. This was the similar results observed by Damesh and Shannak (2010) for viscoelastic fluid and Krishnendu Bhattacharyya (2012) for viscous fluid. Further, one can also come to the conclusion from figures 5.9, 5.10 and 5.11 that, as $m, h$ and $k$ increases the total volumetric flow rate, species concentration and heat rate also increases. The values of total volumetric flow rate, species concentration and heat rate
remains the same when $m = h = k = 1$. This is the valid result because considering all the ratios to be equal to one implies the channel is filled with same viscous fluids in both the regions. However, variation of $m$, $h$ and $k$ for values not equal to one show the different profiles for total volumetric flow rate, species concentration and heat rate. In all the three graphs, the magnitude of volumetric flow rate, species concentration and heat rate is large for $k$, when compared with $m$ and $h$. The magnitude of volumetric flow rate, species concentration and heat rate is optimal for $m$ when compared with $h$.

The Nusselt number at the cold ($Nu_-$) and hot walls ($Nu_+$) is shown in figure 5.12 for variations of mass Grashof number $GR_c$. It is seen that as $GR_c$ increases $Nu_-$ and $Nu_+$ increases in magnitude.

The effect of Brinkman number on the velocity and temperature field is shown in table 5.1. It is seen that, as the Brinkman number increases the velocity and temperature increase in both the regions. An increase in Brinkman number results in increase of dissipation effects which result in an increase of temperature and as a consequence velocity increase for the increase in buoyancy force in the momentum equation. This table also shows a comparison of numerical and analytical solutions. It is seen that analytical and numerical solutions are exact to the order of $10^{-4}$ in the absence of Brinkman number and the difference increases as the Brinkman number increases. Further the percentage of error is also calculated and shown in table 5.1.
Table 5.1. Velocity and temperature values for different values of Brinkman number with $p = -1$, $GR_T = 1$, $GR_c = 1$, $\sigma_1 = \sigma_2 = 4$.

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<th>$Br = 0.5$</th>
<th>$Br = 1.5$</th>
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<td>PM</td>
<td>%Error</td>
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<table>
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</tr>
</tbody>
</table>
Fig. 5.2a: Velocity profiles for different values of thermal Grashof number $GR_T$.

$p = 0.2$

Fig. 5.2b: Temperature profiles for different values of thermal Grashof number $GR_T$.

$p = 0.2$
Fig. 5.3a: Velocity profiles for different values of mass Grashof number $GR_c$.

Fig. 5.3b: Temperature profiles for different values of mass Grashof number $GR_c$. 

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Fig. 5.4a: Velocity profiles for different values of porous parameter $\sigma$.

Fig. 5.4b: Temperature profiles for different values of porous parameter $\sigma$. 
Fig. 5.5a: Velocity profiles for different values of viscosity ratio $m$.

Fig. 5.5b: Temperature profiles for different values of viscosity ratio $m$. 
Fig. 5.6a: Velocity profiles for different values of width ratio $h$.

Fig. 5.6b: Temperature profiles for different values of width ratio $h$. 
Fig. 5.7a: Velocity profiles for different values of thermal conductivity ratio $k$.

$p = 0.5$

Region-II

Region-I

$k = 0.5$

Fig. 5.7b: Temperature profiles for different values of thermal conductivity ratio $k$.

$p = 0.5$

Region-II

Region-I

$k = 0.5$
Fig. 5.8a: Velocity profiles for different values of chemical reaction parameter \( \alpha \).

Fig. 5.8b: Temperature profiles for different values of chemical reaction parameter \( \alpha \).
Fig. 5.8c: Concentration profiles for different values of chemical reaction parameter $\alpha$.

Fig. 5.9: Effect of mass Grashof number, viscosity ratio, width ratio and conductivity ratio on the total volumetric flow rate.
Fig. 5.10: Effect of mass Grashof number, viscosity ratio, width ratio and conductivity ratio on total species rate added to the fluid.

Fig. 5.11: Effect of mass Grashof number, viscosity ratio, width ratio and conductivity ratio on total heat rate added to the fluid.
Fig. 5.12: Effect of mass Grashof number on the Nusselt number.