Chapter – II

MULTICRITERIA DECISION MAKING IN FUZZY ENVIRONMENT USING TOPSIS METHOD
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In this chapter a methodology for solving multi – person – multi criteria decision making in fuzzy environment is discussed using Technique for Order Performance by Similarity to Ideal Solutions (TOPSIS) method with the help of a new distance function defined.

2.1. Preliminaries

Decision making is the process of finding the best option from all feasible alternatives. Much of the decision making in the real world take place in an environment in which goals, constraints and the consequences of possible actions are not known precisely and fuzzy set theory can be used to deal with imprecision in decision making.

In classical multi criteria decision making (MCDM) methods the ratings and the weights of the criteria are known precisely [15, 22]. TOPSIS was one of the classical methods, first developed by Hwang and Yoon [22] for solving a MCDM problem wherein the process of TOPSIS the performance ratings and the weights of the criteria were given as crisp values.

C.T. Chen [7] extended the concept of TOPSIS to develop a methodology for solving multi person multi criteria decision making problems in fuzzy environment. Considering the fuzziness in the decision data and group decision – making process linguistic variables were used to assess the weights of all criteria and the ratings of each alternative with respect to each criterion.

According to the concept of TOPSIS, the fuzzy positive ideal solution (FPIS) and the fuzzy negative ideal solution (FNIS) were defined and the distance of each alternative from the FPIS and FNIS were calculated. Finally a closeness coefficient was defined to determine the ranking order of the alternatives.

In this chapter a new distance is proposed to calculate the distance between two triangular fuzzy ratings. Using this proposed distance, the distance of each alternative from the FPIS and FNIS are calculated. The ranking order of the alternatives is determined using the closeness coefficient. Numerical example is given to clarify the theory.

2.2. Proposed Distance

2.2.1. Definition

Let \( \tilde{\alpha} = (a_1, a_m, a_2) \) and \( \tilde{\beta} = (b_1, b_m, b_2) \) be two triangular fuzzy numbers. Then the distance between \( \tilde{\alpha} \) and \( \tilde{\beta} \) is defined as

\[
d(\tilde{\alpha}, \tilde{\beta}) = \frac{1}{2} \left\{ \max(|a_1 - b_1|, |a_2 - b_2|) + |a_m - b_m| \right\}
\]
2.2.2. Definition

Let $\tilde{A}$ and $\tilde{B}$ be two triangular fuzzy numbers. Then the fuzzy number $\tilde{A}$ is closer to fuzzy number $\tilde{B}$ as $d(\tilde{A}, \tilde{B})$ approaches 0.

2.3. Some properties of the distance

2.3.1. Property 1

If both $\tilde{A}$ and $\tilde{B}$ are real numbers, then the distance measurement $d(\tilde{A}, \tilde{B})$ is identical to the Euclidean distance.

Proof

Suppose $\tilde{A}$ and $\tilde{B}$ are two real numbers. Then let $a_1 = a_m = a_2 = a$ and $b_1 = b_m = b_2 = b$

\[
d(\tilde{A}, \tilde{B}) = \frac{1}{2} \left\{ \max (|a_1 - b_1|, |a_2 - b_2|) + |a_m - b_m| \right\}
\]

\[
= \frac{1}{2} \left\{ \max(|a - b|, |a - b|) + |a - b| \right\}
\]

\[
= \frac{1}{2} \left\{ |a - b| + |a - b| \right\} = |a - b|
\]

2.3.2. Property 2

Two triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical iff $d(\tilde{A}, \tilde{B}) = 0$

Proof

Let $\tilde{A} = (a_1, a_m, a_2)$ and $\tilde{B} = (b_1, b_m, b_2)$ be two triangular fuzzy numbers. If $\tilde{A}$ and $\tilde{B}$ are identical then $a_1 = b_1, a_2 = b_2$ and $a_m = b_m$.

\[
d(\tilde{A}, \tilde{B}) = \frac{1}{2} \left\{ \max (|a_1 - b_1|, |a_2 - b_2|) + |a_m - b_m| \right\}
\]

\[
= \frac{1}{2} \left\{ \max (0, 0) + 0 \right\} = 0
\]
If \( d(\tilde{A}, \tilde{B}) = 0 \), then \( \frac{1}{2} \left\{ \max (|a_1 - b_1|, |a_2 - b_2|) + |a_m - b_m| \right\} = 0 \)

That is, \( \{ \max (|a_1 - b_1|, |a_2 - b_2|) + |a_m - b_m| \} = 0 \)
\[
\max (|a_1 - b_1|, |a_2 - b_2|) = 0 \text{ and } |a_m - b_m| = 0
\]

\[ |a_1 - b_1| = 0, |a_2 - b_2| = 0 \text{ and } |a_m - b_m| = 0 \]

Therefore \( \tilde{A} \) and \( \tilde{B} \) are identical.

**2.3.3. Property 3**

Let \( \tilde{A} \) and \( \tilde{B} \) and \( \tilde{C} \) be three triangular fuzzy numbers. The fuzzy number \( \tilde{B} \) is closer to fuzzy number \( \tilde{A} \) than the other fuzzy number \( \tilde{C} \) if and only if \( d(\tilde{A}, \tilde{B}) < d(\tilde{A}, \tilde{C}) \).

**Fig. 2.1. Three Triangular Fuzzy Numbers \( \tilde{A}, \tilde{B} \) and \( \tilde{C} \)**

If \( \tilde{A} = (1, 3, 5), \tilde{B} = (2, 5, 8), \tilde{C} = (4, 6, 9) \) are three triangular fuzzy numbers then we can see that fuzzy number \( \tilde{B} \) is closer to fuzzy number \( \tilde{A} \) than the other fuzzy number \( \tilde{C} \).

Distance measurement is calculated as
\[ d(\tilde{A}, \tilde{B}) = \frac{1}{2} \{ \max (|1 - 2|, |5 - 8|) + |3 - 5| \} = \frac{1}{2} \{ (1, 3) + 2 \} = \frac{1}{2} \{ 5 \} = \frac{5}{2} \]
\[ d(\tilde{A}, \tilde{C}) = \frac{1}{2} \{ \max (|1 - 4|, |5 - 9|) + |3 - 6| \} = \frac{1}{2} \{ (3, 4) + 3 \} = \frac{7}{2} \]

According to the distance measurement and property 3, we conclude that the fuzzy number \( \tilde{B} \) is closer to fuzzy number \( \tilde{A} \) than the other fuzzy number \( \tilde{C} \).

### 2.3.4. Property 4

Let \( \tilde{0} = (0,0,0) \) be origin. If \( d(\tilde{A}, \tilde{0}) < d(\tilde{B}, \tilde{0}) \) then fuzzy number \( \tilde{A} \) is closer to origin than the other fuzzy number \( \tilde{B} \).

**Proof**

According to property 3, for any three fuzzy numbers \( \tilde{A}, \tilde{B} \) and \( \tilde{C} \), if \( d(\tilde{A}, \tilde{B}) < d(\tilde{A}, \tilde{C}) \) then \( \tilde{B} \) is closer to \( \tilde{A} \) than \( \tilde{C} \).

Therefore if \( d(\tilde{A}, \tilde{0}) < d(\tilde{B}, \tilde{0}) \) then \( \tilde{A} \) is closer to the origin than \( \tilde{B} \).

### 2.4. Ranking order using TOPSIS method

This method is very suitable for solving group decision making under fuzzy environment. Here the importance weights of various criteria are given as linguistic variables whose values are given as fuzzy numbers.

The assessments of criteria are given as linguistic variables whose values given as fuzzy numbers.

The decision makers use the linguistic variables and their given values to evaluate the importance of the criteria.
Let the decision group has k persons, then the importance of the criteria and the ratings of alternatives with respect to each criterion can be calculated as

\[
\tilde{w}_j = \frac{1}{k}[\tilde{W}_1^j + \tilde{W}_2^j + \cdots + \tilde{W}_k^j]
\]

\[
\tilde{x}_{ij} = \frac{1}{k}[\tilde{x}_{1j}^i + \tilde{x}_{2j}^i + \cdots + \tilde{x}_{kj}^i]
\]

where \( \tilde{x}_{ij}^k \) and \( \tilde{w}_j^k \) are rating and the importance weight of the \( k^{th} \) decision maker.

A fuzzy multi criteria group decision making problem can be concisely expressed in matrix format as

\[
\tilde{D} = \begin{bmatrix}
A_1 & C_1 & C_2 & \cdots & C_n \\
\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\
\tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \\
\end{bmatrix}
\]

\[
\tilde{w} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n] \text{ where } \tilde{x}_{ij}, \forall \ i, j \text{ and } \tilde{w}_j, j = 1, 2, \ldots, n \text{ are linguistic variables.}
\]

These linguistic variables can be described by the triangular fuzzy numbers \( \tilde{x}_{ij} = (a_{ij}, b_{ij}, c_{ij}) \) and \( \tilde{w} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n] \). We obtain the normalized fuzzy decision matrix denoted by \( \tilde{R} \) as \( \tilde{R} = [\tilde{r}_{ij}]_{mn} \) as follows:

Let B and C be the set of benefit criteria and cost criteria respectively.

\[
\tilde{r}_{ij} = \begin{bmatrix}
\frac{a_{ij}}{c_j}, & \frac{b_{ij}}{c_j}, & \frac{c_{ij}}{c_j}
\end{bmatrix} \quad j \in B
\]

\[
\tilde{r}_{ij} = \begin{bmatrix}
\frac{a_j}{c_{ij}}, & \frac{a_j}{b_j}, & \frac{a_j}{a_{ij}}
\end{bmatrix} \quad j \in C
\]
The normalization method mentioned above is to preserve the property that the ranges of normalized triangular fuzzy numbers belong to [0,1]. Given different importance to each criterion, we construct the weighted normalized fuzzy decision matrix as

\[
\tilde{V}^* = \{\tilde{v}_{ij}^*\}_{ij}, \ i = 1, 2, ..., m, j = 1, 2, ..., n, \text{ where } \tilde{v}_{ij}^* = \tilde{v}_{ij}(\alpha)\tilde{w}_j
\]

By the definition of the weighted normalized fuzzy decision matrix, the elements \(\tilde{v}_{ij}, \forall \ i, j\) are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0,1]. Then we define the fuzzy positive ideal solution (FPIS, \(P^*\)) and the fuzzy negative – ideal solution (FNIS, \(N\)) as

\[
P^* = (\tilde{v}_1^*, \tilde{v}_2^*, ..., \tilde{v}_n^*)
\]

\[
N = (\overline{v}_1, \overline{v}_2, ..., \overline{v}_n)
\]

where \(\tilde{v}_j^* = (1, 1, 1)\) and \(\overline{v}_j = (0, 0, 0)\), \(j = 1, 2, ..., n\).

The distance of each alternative from \(P^*\) and \(N\) can be calculated as

\[
d_i^* = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_{ij}^*) \quad i = 1, 2, ..., m
\]

\[
d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \overline{v}_j) \quad i = 1, 2, ..., m
\]

where \(d(, , )\) is the distance measurement between two fuzzy numbers.
Once the $d_i^+$ and $d_i^-$ of each alternative $A_i$ ($i = 1, 2, \ldots, m$) has been calculated, a closeness coefficient is defined to determine the ranking order of all alternatives.

The closeness coefficient of each alternative is calculated as

$$CC_i = \frac{d_i^+}{d_i^+ + d_i^-}, \ i = 1, 2, \ldots, m$$

An alternative $A_i$ is closer to FPIS ($P^*$) and farther from FNIS ($\overline{N}$) as $CC_i$ approaches to 1. Therefore according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

### 2.5. ALGORITHMIC APPROACH OF THE ABOVE METHOD

**Step 1:** Form a committee of decision makers and then identify the evaluation criteria.

**Step 2:** Choose the appropriate linguistic variables for the importance weight of the criteria and the linguistic ratings for alternatives with respect to criteria.

**Step 3:** For the criterion $c_j$, aggregate the weight of criteria to get the aggregated fuzzy weight $w_j$ and pool the decision makers opinions to get the aggregated fuzzy rating $\tilde{x}_{ij}$ of the alternative $A_i$ under criterion $c_j$. 
Step 4: Construct the fuzzy decision matrix and the normalized fuzzy decision matrix.

Step 5: Construct the weighted normalized fuzzy decision matrix.

Step 6: Construct the FPIS and FNIS.

Step 7: Calculate the distance of each alternative from FPIS and FNIS respectively.

Step 8: According to the closeness coefficient, the ranking order of all alternatives can be determined.

2.6. NUMERICAL EXAMPLE

Selecting the best Marine Paint

One of the factors in preserving a ship in a serviceable condition is painting, since sea water is highly corrosive. The criteria used by ship owners in selecting marine paints are (i) cost (c), (ii) corrosive resistance (COR), (iii) durability (Dur), (iv) availability (Av) and (v) toxicity (Tox). A shipping company is presented with three choices of paint $B_i = (i = 1, 2, 3)$. The company needs to select one of the brands so that the purchasing department can negotiate a price with the paint manufacturers. The engineering department of three experts $D_1 D_2 D_3$ evaluated the three brands of paint with respect to the objectives. The hierarchical structure of this decision problem is shown in the Figure. The proposed method is currently applied to solve this problem.
Fig. 2.2. Selecting the best Marine Paint

Three Engineering experts A₁, A₂ and A₃ evaluate three types of brands of paint B₁, B₂ and B₃ and give their evaluations in linguistic variables with respect to the objectives (i.e.) criteria cost (C₁), Corrosive Resistance (C₂), Durability (C₃), Availability (C₄) and Toxicity (C₅).

The three decision makers use the seven points scale linguistic variables whose values are given as triangular fuzzy numbers to express the importance weight / priority to five criteria given by

- Very Low (VL) \((0,0,0.1)\)
- Low (L) \((0,0.1,0.3)\)
- Medium Low (ML) \((0.1,0.3,0.05)\)
- Medium (M) \((0.3,0.5,0.7)\)
- Medium High (MH) \((0.5,0.7,0.9)\)
- High (H) \((0.7,0.9,1.0)\)
- Very High (VH) \((0.9,1.0,1.0)\)

The assessment of the criteria importance by the decision makers are given by
### Table – 2.1

<table>
<thead>
<tr>
<th></th>
<th>D₁</th>
<th>D₂</th>
<th>D₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>H</td>
<td>VH</td>
<td>VH</td>
</tr>
<tr>
<td>C₂</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>C₃</td>
<td>MH</td>
<td>H</td>
<td>MH</td>
</tr>
<tr>
<td>C₄</td>
<td>MH</td>
<td>MH</td>
<td>MH</td>
</tr>
<tr>
<td>C₅</td>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

Based on the above assessment and using the given values of the linguistic variables, the fuzzy weight of each criterion \( j \) is found as

\[
\tilde{w}_j = \frac{1}{3} [w_j^{(1)} + w_j^{(2)} + w_j^{(3)}]
\]

\[\therefore \tilde{w}_i = \frac{1}{3} [H + VH + VH]
\]

\[= \frac{1}{3} [(0.7, 0.9, 1.0) + (0.9, 1.0, 1.0) + (0.9, 1.0, 1.0)]
\]

\[= \frac{1}{3} [(2.5, 2.9, 2.9)]
\]

\[= (0.83, 0.97, 1)
\]

Similarly we can calculate

\[
\tilde{w}_2 = \frac{1}{3} [H + H + H] = (0.7, 0.9, 1)
\]

\[
\tilde{w}_3 = \frac{1}{3} [MH + H + MH]
\]

\[= \frac{1}{3} [(0.5, 0.7, 0.9) + (0.7, 0.9, 1.0) + (0.5, 0.7, 0.9)]
\]

\[= (0.57, 0.77, 0.93)
\]
\[ \tilde{w}_4 = \frac{1}{3} [MH + MH + MH] \]

\[ = \frac{1}{3} [(0.5,0.7,0.9) + (0.5,0.7,0.9) + (0.5,0.7,0.9)] \]

\[ = (0.5, 0.7, 0.9) \]

\[ \tilde{w}_5 = \frac{1}{3} [H + H + H] \]

\[ = \frac{1}{3} [(0.7,0.9,1.0),(0.7,0.9,1.0),(0.7,0.9,1.0)] \]

\[ = (0.7, 0.9, 1.0) \]

Hence the fuzzy weight vector

\[ \tilde{W} = (\tilde{w}_1, \tilde{w}_2, \tilde{w}_3, \tilde{w}_4, \tilde{w}_5) \] whose values are given as above.

The three brands of paint are assessed by the three decision makers on a seven point linguistic scale whose values are given as:

- **Very Poor (VP)**: (0,0,1)
- **Poor (P)**: (0,1,3)
- **Medium Poor (MP)**: (1,3,5)
- **Fair (F)**: (3,5,7)
- **Medium Good (MG)**: (5,7,9)
- **Good (G)**: (7,9,10)
- **Very Good (VG)**: (9,10,10)

The ratings or evaluations of the 4 brands by the 3 decision makers under the five criteria are given below.
Table – 2.2

The rating of the three brands by the decision makers under all criteria

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Brands</th>
<th>Decision makers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>D_1</td>
</tr>
<tr>
<td>C_1 (Rupees)</td>
<td>B_1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>B_2</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>B_3</td>
<td>4</td>
</tr>
<tr>
<td>C_2</td>
<td>B_1</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>B_2</td>
<td>VG</td>
</tr>
<tr>
<td></td>
<td>B_3</td>
<td>MG</td>
</tr>
<tr>
<td>C_3</td>
<td>B_1</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>B_2</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>B_3</td>
<td>G</td>
</tr>
<tr>
<td>C_4</td>
<td>B_1</td>
<td>VG</td>
</tr>
<tr>
<td></td>
<td>B_2</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>B_3</td>
<td>G</td>
</tr>
<tr>
<td>C_5</td>
<td>B_1</td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>B_2</td>
<td>G</td>
</tr>
<tr>
<td></td>
<td>B_3</td>
<td>G</td>
</tr>
</tbody>
</table>

Combining the opinion of all the decision makers for each criterion, the fuzzy decision matrix of the three alternatives (i.e.) three brands is given by:

For the brand B_1, under the criterion C_1, the evaluation is

\[ \tilde{x}_{11} = \frac{6 + 8 + 7}{3} = 7 \]

Under criterion C_2,

\[ \tilde{x}_{12} = \frac{1}{3} [G + VG + F] \]

\[ = \frac{1}{3} [(7,9,10) + (9,10,10) + (3,5,7)] = (6.3, 8, 9) \]
Under criterion $C_3$,

$$\tilde{x}_{13} = \frac{1}{3}[F + G + G]$$

$$= \frac{1}{3}[3(3,5,7) + (7,9,10) + (7,9,10)]$$

$$= (5.7, 7.7, 9)$$

For criterion $C_4$,

$$\tilde{x}_{14} = \frac{1}{3}[VG + G + G]$$

$$= \frac{1}{3}[(9,10,10) + (7,9,10) + (7,9,10)]$$

$$= (7.7, 9.3, 10)$$

For criterion $C_5$,

$$\tilde{x}_{15} = \frac{1}{3}[F + F + F]$$

$$= \frac{1}{3}[(3,5,7) + (3,5,7) + (3,5,7)] = (3, 5, 7)$$

Similarly for the brands $B_2$ and $B_3$ under the five criteria we can calculate the evaluations

$$\tilde{x}_{ij}$$ where $i = 1, 2, 3$ and $j = 1, 2, 3, 4, 5$

The fuzzy decision matrix $\tilde{F} = (\tilde{x}_{ij})$ is given by

$$\tilde{D} = \begin{pmatrix}
B_1 & (7.0, 7.0, 7.0) & (6.3, 8, 9) & (5.7, 7.7, 9) & (7.7, 9.3, 10) & (3,5,7) \\
B_2 & (4.0, 4.0, 4.0) & (9,10,10) & (7,9,10) & (7,9,10) & (5.7,7.7,9) \\
B_3 & (50, 5.0, 5.0) & (7,9,10) & (7,9,10) & (8.3,9.7,10) & (7,9,10)
\end{pmatrix}$$
To find the normalized decision matrix \( \tilde{R} = (\tilde{r}_i^j) \) for the cost criteria \( C_1 \),

For the brand \( B_1 \)

\[
\tilde{x}_{11} = (7, 7, 7)
\]

\( B_2 \)

\[
\tilde{x}_{21} = (4, 4, 4)
\]

\( B_3 \)

\[
\tilde{x}_{31} = (5, 5, 5)
\]

\[
\min \overline{a}_1 = \min \{a_{ij}\} = \min(7, 4, 5) = 4
\]

\[
\tilde{r}_{11} = \left( \frac{4}{7}, \frac{4}{7}, \frac{4}{7} \right) = (0.57, 0.57, 0.57)
\]

\[
\tilde{r}_{21} = \left( \frac{4}{4}, \frac{4}{4}, \frac{4}{4} \right) = (1, 1, 1)
\]

\[
\tilde{r}_{31} = \left( \frac{4}{5}, \frac{4}{5}, \frac{4}{5} \right) = (0.8, 0.8, 0.8)
\]

For the benefit criterion \( C_2 \),

\[
C'_2 = \max_i \{C_{ij}\} = \max \{9, 10, 10\} = 10
\]

\[
\therefore \quad \tilde{r}_{22} = \left( \frac{6.3}{10}, \frac{8}{10}, \frac{9}{10} \right) = (0.63, 0.8, 0.9)
\]

\[
\tilde{r}_{22} = \left( \frac{9}{10}, \frac{10}{10}, \frac{10}{10} \right) = (0.9, 1, 1)
\]

\[
\tilde{r}_{22} = \left( \frac{7}{10}, \frac{9}{10}, \frac{10}{10} \right) = (0.7, 0.9, 1)
\]

Similarly for the other benefit criteria \( C_3, C_4 \) and \( C_5 \), we can calculate the values of \( \tilde{r}_i^j \) for the three alternatives

Hence the normalized fuzzy decision matrix

\[
\tilde{R} = \tilde{r}_i^j \text{ is given by}
\]
The fuzzy normalized decision matrix $\bar{v} = (\bar{v}_{ij})$

where $\bar{v}_{ij} = (\bar{r}_{ij})(.)\bar{w}_j$

For the criterion $C_1$,

$$\bar{v}_{11} = \bar{r}_{11}(.)\bar{w}_1 = (0.57, 0.57, 0.57)(.) (0.83, 0.97, 1) = (0.4731, 0.5529, 0.57)$$

$$\bar{v}_{21} = \bar{r}_{21}(.)\bar{w}_1 = (1, 1, 1)(.) (0.83, 0.97, 1) = (0.83, 0.97, 1)$$

$$\bar{v}_{31} = \bar{r}_{31}(.)\bar{w}_1 = (0.8, 0.8, 0.8)(.) (0.83, 0.97, 1)$$

$$= (0.664, 0.776, 0.8)$$

For the criterion $C_2$,

$$\bar{v}_{12} = \bar{r}_{12}(.)\bar{w}_2 = (0.63, 0.8, 0.9)(.) (0.7, 0.9, 1) = (0.441, 0.72, 0.9)$$

$$\bar{v}_{22} = \bar{r}_{22}(.)\bar{w}_2 = (0.9, 1, 1)(.) (0.7, 0.9, 1) = (0.63, 0.9, 1)$$

$$\bar{v}_{32} = \bar{r}_{32}(.)\bar{w}_2 = (0.7, 0.9, 1)(.) (0.7, 0.9, 1) = (0.49, 0.81, 1)$$

Similarly for the other criteria $C_3$, $C_4$ and $C_5$ we can calculate the values $\bar{v}_{ij}$.

$\therefore$ The weighted normalized fuzzy decision matrix is determined to be

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>(0.57,0.57,0.57)</td>
<td>(0.63,0.8,0.9)</td>
<td>(0.57,0.77,0.9)</td>
<td>(0.77,0.93,1)</td>
<td>(0.3,0.5,0.7)</td>
</tr>
<tr>
<td>$B_2$</td>
<td>(1,1,1)</td>
<td>(0.9,1,1)</td>
<td>(0.7,0.9,1)</td>
<td>(0.7,0.9,1)</td>
<td>(0.57,0.77,0.9)</td>
</tr>
<tr>
<td>$B_3$</td>
<td>(0.8,0.8,0.8)</td>
<td>(0.7,0.9,1)</td>
<td>(0.7,0.9,1)</td>
<td>(0.83,0.97,1)</td>
<td>(0.7,0.9,1)</td>
</tr>
</tbody>
</table>
\[
\bar{v} = (\bar{v}_j) = \\
\begin{array}{cc}
B_1 & \begin{pmatrix} 0.4731, \\ 0.5529, \\ 0.57 \end{pmatrix} \\
B_2 & \begin{pmatrix} 0.83, 0.9, 1 \end{pmatrix} \\
B_3 & \begin{pmatrix} 0.664, \\ 0.776, 0.8 \end{pmatrix}
\end{array}
\]

\[
\begin{array}{cc}
& C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \\
B_1 & (0.4731, \\ 0.441, 0.72, \\ 0.3249, \\ 0.385, \\ 0.21) \\
B_2 & (0.5529, \\ 0.5929, \\ 0.9, 0.651, 0.693) \\
B_3 & (0.57, \\ 0.837) \\
& (0.9) \\
& (0.9) \\
& (0.9)
\end{array}
\]

Take the fuzzy positive and fuzzy negative ideal solutions to be

\[
P^* = (\bar{v}_1^*, \bar{v}_2^*, \bar{v}_3^*, \bar{v}_4^*, \bar{v}_5^*) \quad \text{and}
\]

\[
\bar{N} = (\bar{v}_1, \bar{v}_2, ..., \bar{v}_5) \quad \text{respectively such that}
\]

\[
\bar{V}_j^* = (1, 1, 1) \quad \text{and} \quad \bar{V}_j = (0, 0, 0)
\]

The distance of each alternative \(B_i\) from the positive solution is

\[
d_i^* = \sum_{j=1}^{n} d(V_j, V_j^*)
\]

The distance of the 1\textsuperscript{st} alternative from (1, 1, 1) is

\[
d_1^* = d((0.4731, 0.5529, 0.57), (1, 1, 1)) + \\
d((0.441, 0.72, 0.9) (1, 1, 1)) + \\
d((0.3249, 0.5929, 0.837) (1, 1, 1)) + \\
d((0.385, 0.651, 0.9) (1, 1, 1)) + \\
d((0.21, 0.45, 0.7) (1, 1, 1))
\]
\[
\begin{align*}
&= \frac{1}{2} \{ \max(|0.4731 - 1|, |0.57 - 1|) + |0.5529 - 1| \} \\
&+ \frac{1}{2} \{ \max(|0.441 - 1|, |0.9 - 1|) + |0.72 - 1| \} \\
&+ \frac{1}{2} \{ \max(|0.3249 - 1|, |0.837 - 1|) + |0.5929 - 1| \} \\
&+ \frac{1}{2} \{ \max(|0.385 - 1|, |0.9 - 1|) + |0.651 - 1| \} \\
&+ \frac{1}{2} \{ \max(|0.21 - 1|, |0.7 - 1|, |0.45 - 1|) \}
\end{align*}
\]

\[= 0.487 + 0.4195 + 0.5411 + 0.482 + 0.67 = 2.5996\]

The distance of the 1\textsuperscript{st} alternative from (0, 0, 0) is

\[
d_1^+ = d[(0.4731, 0.5529, 0.57) (0, 0, 0)] \\
+ d[(0.441, 0.72, 0.9), (0, 0, 0)] \\
+ d[(0.3249, 0.5929, 0.837), (0, 0, 0)] \\
+ d[(0.385, 0.651, 0.9) (0, 0, 0)] \\
+ d[(0.21, 0.45, 0.7) (0, 0, 0)] 
\]

\[= 0.5615 + 0.81 + 0.71495 + 0.7755 + 0.575 = 3.43695\]

Similarly the distances of the 2\textsuperscript{nd} alternative from (1, 1, 1) and from (0, 0, 0) are

\[
d_2^+ = 1.753 \\
d_2^- = 4.308 \text{ respectively.}
\]

The distance of the 3\textsuperscript{rd} alternative from (1, 1, 1) and from (0, 0, 0) are

\[
d_3^+ = 1.887 \\
d_3^- = 4.199 \text{ respectively.}
\]

\[\therefore \text{ The separation measures from the positive and negative solution are given by}\]
Table – 2.3
Separation Measures

<table>
<thead>
<tr>
<th></th>
<th>$d_i^+$</th>
<th>$d_i^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>3.08</td>
<td>3.437</td>
</tr>
<tr>
<td>$B_2$</td>
<td>1.753</td>
<td>4.308</td>
</tr>
<tr>
<td>$B_3$</td>
<td>1.887</td>
<td>4.199</td>
</tr>
</tbody>
</table>

The closeness coefficient $CC_i = \frac{d_i^-}{d_i^+ + d_i^-}$

$CC_1 = \frac{3.437}{3.437 + 2.5996} = \frac{3.437}{6.0366} = 0.5694$

$CC_2 = \frac{4.308}{4.308 + 1.753} = \frac{4308}{6.061} = 0.71077$

$CC_3 = \frac{4.199}{4.199 + 1.887} = \frac{4.199}{6.086} = 0.68994$

According to the closeness coefficient, the ranking order of the three alternatives is $B_2 > B_3 > B_1$. Therefore, the best alternative is the brand $B_2$. 