The decision regarding Investment plays a vital role in any type of organization. Normally these decisions are based on risk-return patterns. Involvement of the risk factor leads to a decision to be made under uncertainty. The main objective of this chapter is how one can eliminate the risk factors in decision making with the help of Mathematical Models available in the field of Decision Sciences. I have considered a hypothetical situation of making financial investment and for the same a mathematical model based on Integer Programming has been constructed. An example is presented with the real life data to demonstrate the application.

6.1 Introduction

In today’s competitive environment for business, companies strive for quality and effective utilization of the fund available. Efforts have been directed towards the utilization of the available fund resources to optimize the earnings. A key function which influences utilization of the financial resources is the proper planning and scheduling the investment decisions. The decision maker cannot take the decisions immediately because the situation is uncertain, that it contains with it the risk element. To reduce the risk factor, careful analysis and past experience is needed. In order to eliminate the risk factor completely one may make use of Mathematical Programming approach. Jarvis [1980] has developed a computerized maintenance system for planning and scheduling. Roberts and Escudero [1983] proposed an
integer linear programming model for scheduling the maintenance work. Panwalker [1977] proposed specific rules for planning and scheduling. Hsuan-shih Lee [2002] proposed a new method for aggregating individual fuzzy opinions into an optimal group consensus. Rotab Khan [1999] has developed a methodology of structuring a garment production simulation model to minimize the average daily production cost. The objective of this chapter is to address the problem of eliminating the risk in investment decisions. The Mathematical Programming approach for modeling and making decisions is hereby presented.

6.2 Notations and Assumptions

Notations

\( j \) : various projects available for consideration \( [ j = 1,2,\ldots,n ] \)

\( i \) : number of years for completion of the selected project \( [ i = 1,2,\ldots,m ] \)

\( m, n \) : finite positive integers

\( E_{rj} \) : present value of the expected return based on the \( j^{\text{th}} \) project selected

\( I_{ij} \) : present value of the investment needed for the \( i^{\text{th}} \) project in the \( i^{\text{th}} \) year
A_i : total availability of the present value of the cash for the i^{th} year

K : number of projects to be selected out of the n projects

\[ k \leq n \]

X_j : refers the j^{th} project

Assumptions

1. The values for all the constants i, j, E_{ri}, I_{ij}, A_i and k must be known in advance.
2. If the value of X_j is 1 the j^{th} project is selected for investment if X_j is 0 it is rejected.

6.3 The Structure of the Investment Analysis Problem

Consider the decision making situation where in which the decision maker has to decide with the available fund, the various projects to be selected for investment. In such a way that the over all return should be maximum. This type of decision making clearly comes under the case of decision making under uncertainty. This is because the amount to be invested and the return for the investment can not be evaluated exactly. All the necessary data can be estimated only approximately. Also the investment runs through \( 'm' \) number of years and the return can be expected only at the \([m+1]^{th}\) year. It needs the utilization of the present value factor. We have to evaluate the
present value of the total cash outflow and the present value of the cash inflow for various projects. Also the decision maker should decide the possible number of different projects can be opted for investment with the available fund to earn a maximum return altogether. The above stated information can be put into a comprehensive Table- 11:

6.4 Mathematical Model

Mathematical Model is a structure, which contains within it a unique objective function, one or more constrains and with the non-negative restrictions over the variables used in the model. If a mathematical model satisfies the condition of linearity then it is referred to as a Linear Programming Model. In a Linear Programming Model, if all the variables are restricted to take only the integer values then the model is referred as Integer Programming Model. For a detailed study of these mathematical models one may refer to Winston [2004], Wagner [2003], Taha [2003] and Mariappan [2002].
### Table: 11 The structure of the investment problem

<table>
<thead>
<tr>
<th>Project [j]</th>
<th>Assumed variables</th>
<th>Year1 [Rs.]</th>
<th>Year2 [Rs.]</th>
<th>......</th>
<th>Yearm [Rs.]</th>
<th>Expected Return [Rs.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$X_1$</td>
<td>$I_{11}$</td>
<td>$I_{12}$</td>
<td>....</td>
<td>$I_{1m}$</td>
<td>$E_{r1}$</td>
</tr>
<tr>
<td>2</td>
<td>$X_2$</td>
<td>$I_{21}$</td>
<td>$I_{22}$</td>
<td></td>
<td>$I_{2m}$</td>
<td>$E_{r2}$</td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td>....</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td>....</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>$X_n$</td>
<td>$I_{n1}$</td>
<td>$I_{n2}$</td>
<td></td>
<td>$I_{nm}$</td>
<td>$E_m$</td>
</tr>
<tr>
<td>Amount Available [RS.]</td>
<td></td>
<td>$A_1$</td>
<td>$A_2$</td>
<td></td>
<td>$A_m$</td>
<td></td>
</tr>
</tbody>
</table>

Out of the n projects one can select k projects as a maximum, k <= n.
The current situation requires a decision to be made in order to select the projects for the investment in such a way that the overall returns are maximized. This model contains a flexibility of choosing the specified number of projects by the end user for the related application. The decision making situation can be transferred into a Mathematical Model called Integer Programming Model.

\[
\text{Maximize} \quad Z = \sum_{j=1}^{n} E_j X_j \\
\text{Subject to} \\
\sum_{j=1}^{n} I_j X_j \leq A_i \quad \text{for all} \quad i=1,2,\ldots,m \\
\sum_{j=1}^{n} X_j \leq k \\
X_j \leq 1; \quad \text{for all} \quad j=1,2,\ldots,n \\
X_j \geq 0 \quad \text{and Integers} \quad \text{for all} \quad j=1,2,\ldots,n
\]

6.5 Solution Procedure

Depending on the formulated model, packages are available for solving Integer Programming Models. Examples of these are Linear Interactive and Discrete Optimizer [LINDO], General Integer and
non-Linear Optimizer [ GINO], Optimizer Software Library [ OSL] and TORA. Results of the models will yield implement able solution.

6.6 Outcome of the Mathematical Model

Based on the final optimum solution, one can decide easily, what are all the projects to be considered, if considered what will be the maximum profit the organization is going to obtain? The decision is risk free.

6.7 Numerical Example

A company considers four projects on which investments can be made for three years with expected revenues, required cash outlays, and maximum allowable capital investment for each of the three years

<table>
<thead>
<tr>
<th>Projects</th>
<th>Net Present Values of Revenues [‘000Rs.]</th>
<th>Present value of Cash Outlay [‘000 Rs.]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Net Present Values of Revenues [‘000Rs.]</td>
<td>Present value of Cash Outlay [‘000 Rs.]</td>
</tr>
<tr>
<td></td>
<td>Year 1</td>
<td>Year 2</td>
</tr>
<tr>
<td>1</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>170</td>
<td>21</td>
</tr>
<tr>
<td>4</td>
<td>140</td>
<td>13</td>
</tr>
<tr>
<td>Maximum Allowable Budget [‘000 Rs.]</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

With the notations already defined,
Maximize \[ Z = 100 X_1 + 120 X_2 + 170 X_3 + 140 X_4 \]

\[ 11 X_1 + 16 X_2 + 21 X_3 + 13 X_4 \leq 40 \]

\[ 13 X_1 + 19 X_2 + 23 X_3 + 15 X_4 \leq 40 \]

\[ 25 X_1 + 21 X_2 + 25 X_3 + 17 X_4 \leq 40 \]

\[ X_1 + X_2 + X_3 + X_4 \leq k \]

\[ X_j = 0 \text{ or } 1 ; \quad \text{for all } j, k = 1, 2, 3, 4 \]

Here \( k \) refers number of projects that can be selected at a time. It is optional to the decision maker.

Whenever the option to the decision maker is to select at the maximum only one project, the optimum solution is select the project -3 which yields a maximum expected return of Rs. 1, 70,000.

If the option to the decision maker is to select any number of projects out of the four possible projects, the solution is select exactly two projects namely project-2 and project-4, which yields a total return of Rs. 2,60,000.
Table: 12  Optimum Solution for various values of  $k$

<table>
<thead>
<tr>
<th>$k$</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$X_4$</th>
<th>Optimum Z [Rs.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2,60,000</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2,60,000</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2,60,000</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1,70,000</td>
</tr>
</tbody>
</table>
6.8 Conclusion

The Mathematical model suggested in this chapter helps to take an effective decision in the phase of uncertainty. One relevant Mathematical Programming approach namely Integer Programming has been presented. An Example illustrating the utility of the proposed model has been given. The example shows that the model can offer a valuable approach for formulating and obtaining an implementable solution for the investment analysis.