CHAPTER 2
In this Chapter, a review of theoretical studies and the basis of the theories opted for prediction of sound speed and attenuation in sediments in the present study are included. Discussion on the effect of variation of bottom water temperature on sound speed and attenuation in sediments is presented.

2.1. Sound speed in sediments

The sedimentary material found in the seafloor includes high-porosity suspensions of colloidal clay particles at one end to low-porosity porous solids at the other. Many particles, particularly those of clay and silty clay size, are in suspension, while those of sand size are in contact with another. Each particle has an irregular shape. Although certain minerals predominate in sediments, each sediment sample has a unique mineral grain composition. As the acoustic properties of sediments are determined by such properties, any general theory for acoustic wave propagation in sediments must consider these variables.

Earlier studies on the variation of sound speed in marine sediments can be divided into two basic types: theoretical and empirical. Theoretical models have some basis in a physical model and the model parameters are specified priori. Empirical equations assume no a priori model, and the dependent parameters are only determined by reference to some explicit data set. Alternatively, different equations may be proposed for different physical properties connecting sound speed (Hamilton, 1980, Bachman, 1985).

For the derivation of geoacoustic properties from the structural parameters of marine sediments, essentially two different approaches have been applied. The earlier approach (Urick, 1948; McCann and McCann, 1969) is based on the assumption that fine-grained unconsolidated marine sediments can be described as suspensions, and, for the compressional wave speed, uses the formula of Wood (1941) for the effective compressibility of suspensions. Theoretical studies (Wyllie et al., 1956; Wood, 1941) have examined one or more equations, based on some physical model that relate
velocity to the porosity. These theoretical models have often suffered from a lack of
general applicability. For example, the Wood equation is valid for particles in
suspension. The bulk compression is computed as the weighted mean compressibility
of the solid and fluid fractions and thus is only valid for sound speed estimation when
the bulk material has no strength. The time average equation (Wyllie et al., 1956) is
approximately valid for mixtures of fluid media, and thus is only valid for low-porosity
materials. The main draw back of this concept is that the sediment does not have
rigidity. This is an unreasonable assumption, for shear waves are observed even in very
soft high-porosity mud (Hamilton, 1972).

The more comprehensive approach (Stoll and Bryan, 1970; Stoll, 1977; Hovem
and Ingram, 1979; Stoll, 1985; McCann and McCann, 1985) applies theory of acoustics
of porous media. The description of unconsolidated marine sediments with Biot theory
(Biot, 1956a,b) appears to be more realistic than former approach because it accounts
for the occurrence of shear waves. An overview of some of the theoretical
developments related to the sound speed and attenuation in marine sediments is
presented below.

2.1.1. Wood Model

Wood (1941) considered a medium (sediment) composed of grains, some of
which are in contact and some of which are not. The pore space is filled with water. At
higher porosities fewer of the grains are in contact. As the porosity decreases, more and
more grains come in contact forming a rigid matrix. Any given medium may thus be
considered to be composed of pockets of slurries (particles in suspension) and rigid
grain matrices. The fraction of slurry depends directly on the porosity. The travel time
across any given volume of the medium will then be the sum of the travel times across
the individual slurry and rigid particles that make up that volume. Based on this simple
model, Wood (1941) proposed an equation for sound speed \( V_p \) in slurry:

\[
\frac{1}{\rho V_p^2} = \frac{n}{\rho_f V_f^2} + \frac{(1-n)}{\rho_r V_r^2}
\]  

(2.1)
where \( \rho \) is the bulk density, defined by \( \rho = n \rho_w + (1-n) \rho_r \), \( n \) is the porosity, \( V_f \) is the speed of sound in the fluid medium, \( V_r \) is the speed of sound solid medium, \( V_p \) is the speed of sound in the porous medium, \( \rho_r \) is the density of solid medium and \( \rho_f \) is the density of fluid medium. The Wood equation is approximately valid only at high porosities.

2.1.2. Wyllie et al. Time Average Model

Wyllie et al. (1956) studied mixtures of rigid media, and proposed that sound speed was well represented by the average travel time equation

\[
\frac{1}{V_p} = \frac{n}{V_f} + \frac{(1-n)}{V_r}
\]

(2.2)

The parameters are as defined previously. The Wyllie or time average equation is approximately valid at low porosities (less than 30%), where media can be considered as approximately rigid.

2.1.3. Nafe and Drake Model

Nafe and Drake (1957) compared the Wood model to the behavior of compressed springs in series. The Wood model breaks down at low porosities because the increased grain to grain contact stiffens the lattice to the extent that the behavior is more akin to springs in parallel. They derived the relation,

\[
V_p^2 = n V_f^2 + (1-n)^m V_r^2 \rho_r / \rho
\]

(2.3)

The exponent \( m \) is a constant to be determined with experimental data. Nafe and Drake found a good fit to their experimental data using \( m = 4 \) and 5. Boyce (1981) found a value of \( m \) as high as 9 required to fit his data. Nobes et al. used a value of \( 2 \rho \) for \( m \).
Alternately the exponent is allowed to vary with porosity as \( m = a + bn \). The values of 
\( a = 5.3 \pm 0.2 \) and \( b = -3.5 \pm 0.3 \) yielded best fitting model to their data.

### 2.1.4. Nobes et al. Model

Nobes *et al.* (1989) proposed a time averaged model for the variation of sound velocity with porosity and obtained an equation which is a simple combination of Wood and Wyllie equations. Here, instead of assuming the two media as the fluid and solid fractions, the sediments are assumed to be mixtures of slurry and rigid components. The amount of slurry depends directly on the amount of porosity. By taking the weighted mean of the Wood and Wyllie equations, an equation is obtained that is representative of the physical models described above.

\[
\frac{1}{V_p} = \frac{n}{V_w} + \frac{(1-n)}{V_i}
\]  
(2.4)

where \( V_w \) represents the Wood's sound speed and \( V_i \) the fine average sound speed of Wyllie *et al.* Good agreement between results obtained using the equation and experimental data has been reported.

A cursory look at the above equations show that all the models depend on \( V_r \). Nafe and Drake used a constant value of 6000 m/s for \( V_r \). Tosaya and Nur (1982) developed an empirical equation to represent the sound speed in grains, \( V_r \) (in km/s), where \( f_c \) is the clay fraction.

\[
V_r = 5.8 - 2.4 f_c
\]  
(2.5)

If the lithology are accurately known, Nobes *et al.*, (1989) given a better suggestion and the equation can be generalized as

\[
\frac{1}{V_r} = \sum_j \frac{f_j}{V_j},
\]  
(2.6)
where \( f_j \) is the fraction of the medium composed of the \( j \)th component, which has sound speed \( V_j \).

### 2.1.5. Biot theory

Biot (1956a, 1956b) considered a saturated sediment to consist of a porous assemblage of sediment grains (the "skeletal frame"), whose interconnected pores are filled with water or gas (the "pore fluid"). Biot devised a theoretical model of the acoustic behavior of such a material. The Biot model treats both the individual and coupled behavior of the frame and pore fluid. Energy loss is considered to be caused by the inelasticity of the skeletal frame and by the viscosity of the pore fluid as it moves relative to the frame. The model predicts that sound speed and attenuation in sediment will depend on frequency, elastic properties of the sediment grains and pore fluid, porosity, mean grain size, permeability, and effective stress.

The practical application of Biot's theory was rather limited until Stoll and Bryan (1970) and Stoll (1974, 1977) applied the theory to the discussion of sound speed and attenuation as a function of frequency in marine sediments. The modified form of Biot theory by Stoll is presently known as Biot-Stoll theory. In the Biot-Stoll theory the loss terms are included to the elastic parameters of the solid frame and thereby account losses due to grain-to-grain contacts as well as fluid loss. Compressional and shear waves travelling through this structure are attenuated by two physical mechanisms: frictional grain to grain contact sliding and viscous dissipation caused by relative motion between the grains and the interstitial fluid.

The Biot-Stoll theory predicts three kinds of body waves in a fluid-saturated porous medium. One of the compressional waves and a shear wave are the traditional body waves of elastic media. The 'second' compressional wave is highly attenuated and for most of the geophysical applications is not important. However, recent demonstrations of its existence provide a striking success for the Biot-Stoll model.
Ogushwitz (1985a,b) has shown that Biot-Stoll theory can be used to model marine acoustic sound speed and attenuation data for artificial and natural materials with porosities ranging from 2% to 100%.

The mathematical formulation of the Biot-Stoll theory is given below.

Let $u$ be the displacement of the frame, $U$ be displacement of the pore fluid relative to the frame, and $n$ be porosity. Then the dilatation of a volume element attached to the frame is given by

$$ e = \text{div}(u) $$

(2.7)

and the volume of fluid that has flowed into or out of that element is given by

$$ \zeta = n \cdot \text{div}(u - U) $$

(2.8)

Biot nominated the following equations to be constitutive equations for a porous, saturated, isotropic medium

$$ \tau_y = 2\mu e_y + \delta_y [(H - 2\mu)\epsilon - C\zeta], $$

(2.9)

$$ P = M\zeta - Ce, $$

(2.10)

where $\tau_y$ and $e_y$ are the stress and strain components respectively of an element of volume attached to the skeletal frame, $P$ is the pore fluid pressure, $H,C,M,$ and $\mu$ are real constants, and $\delta_y$ is the Kronecker delta.

By assuming that the sediment porosity remains constant under the small strains typical of acoustic waves, Stoll showed that the constants of the Biot theory could be identified with measurable physical properties in the following way:
\[ H = \frac{(K_r - K_b)^2}{(D - K_b)} + K_b + \frac{4}{3} \mu, \quad (2.11) \]

\[ C = \frac{K_r(K_r - K_b)}{(D - K_b)}, \quad (2.12) \]

\[ M = \frac{K_r^2}{(D - K_b)} \quad (2.13) \]

where

\[ D = K_r[1 + n(K_r/K_f - 1)] \quad (2.14) \]

Here, \( K_r \) and \( K_f \) are the bulk moduli of the sediment grains and pore fluid respectively, and \( K_b \) and \( \mu \) are bulk and shear moduli of the skeletal frame respectively. To incorporate the inelasticity of the frame into the model, Stoll permitted \( H, C, M, \) and \( \mu \) to be complex. In particular, he concentrated the inelastic effects in the skeletal frame moduli \( K_b \) and \( \mu \).

Biot (1956a, 1956b) derived wave equations for dilatational and shear waves from the constitutive equations, the equations of motion, and the equations of flow through a porous medium. In particular, the compressional wave equations are

\[ \nabla^2(He - C\zeta) = \frac{d^2}{dt^2}(\rho e - \rho_e \zeta) \quad (2.15) \]

\[ \nabla^2(Ce - M\zeta) = \frac{d^2}{dt^2}(\rho_e e - m\zeta) - \frac{\eta F}{k} \frac{d\zeta}{dt} \quad (2.16) \]
Here $\rho_w$ is the fluid density, $\rho$ is the total density of the volume element, $\eta$ is the fluid viscosity, and $k$ is the dynamic permeability of the skeletal frame. Note that

$$\rho = n\rho_w + (1 - n)\rho, \quad (2.17)$$

where $\rho$ is the density of the solid grains. The parameter $m$ is given by

$$m = a'(\rho_w/n), a' \geq 1 \quad (2.18)$$

The coefficient $a'$, which is called the "structure factor," accounts for the apparent increase in fluid inertia caused by the tortuosity of the pores. The frequency dependence of $\eta/k$ (the viscous resistance to fluid flow) is accounted for by a complex correction factor $F$ which is given by

$$F(\kappa) = \frac{\kappa T(\kappa)}{4[1 + 2iT(\kappa)/\kappa]} \quad (2.19)$$

where $T(\kappa)$ is given by the complex Kelvin function

$$T(\kappa) = \frac{ber'(\kappa) + ibei'(\kappa)}{ber(\kappa) + ibei(\kappa)} \quad (2.20)$$

where

$$\kappa = a(\omega \rho_w / \eta)^{1/2} \quad (2.21)$$

is non dimensional and depends on the pore-size parameter, $a$ the fluid density $\rho_w$, the viscosity $\eta$ and the angular frequency $\omega$. 
If one assumes solutions of the form

\[ e = A_1 \exp[i(\omega t - lx)], \]  
\[ \zeta = A_2 \exp[i(\omega t - lx)], \]

then Eqs.(2.15) yield the compressional wave dispersion relation

\[
\begin{vmatrix}
Hl^2 - \rho \omega^2 & \rho_\omega \omega^2 - C_l^2 \\
C_l^2 - \rho_\omega \omega^2 & m \omega^2 - M_l^2 - i\omega \eta F/K
\end{vmatrix} = 0
\]

For shear waves, one obtains the analogous dispersion relation,

\[
\begin{vmatrix}
\rho \omega^2 - \mu l^2 & \rho_\gamma \omega^2 \\
\rho_\omega \omega^2 & m \omega^2 - i\omega \eta F/K
\end{vmatrix} = 0
\]

The dispersion relation of Eq. (2.24) has two distinct complex roots of the form, \( l = l_r + il_i \). The wave speed for each root is given by \( \omega l_r \) and the attenuation by \( l_i \) (Np/m). One solution corresponds to a relatively slow wave with high attenuation, the so-called Biot "slow wave" while the other root corresponds to the usual, higher speed, acoustic wave. The attenuation of this wave is very high and the second wave has therefore limited practical importance except that is directly connected to the viscous loss of the first wave (Hovem, 1980). Johnson and Plona (1982) and Chotiros (1995)
verified the existence of the compressional waves of the “second kind” predicted by the Biot theory.

Eq. (2.25) indicates that only one kind of shear wave can propagate. It also shows that this wave may suffer some dispersion because the solid and fluid components can move out of phase with one another causing viscous dispersion. However, this is not considered in this study.

The theory predicts that attenuation, when dominated by viscous flow losses in the pore space of the sediment, will vary as the second power of frequency for low frequencies and will vary as the square root of frequency for high frequencies. Frictional loss mechanisms, in contrast, exhibit a linear dependence of attenuation. (The ratio of the cross-sectional length scale of the pore space in the sediments to the wavelength of the acoustic wave determines the boundary between high and low frequencies).

Biot-Stoll theory of wave propagation in porous, saturated material is complicated, with more than ten parameters affecting dispersion relation. Some of the parameters are directly measurable or known *a priori* (e.g., grain density, saturated bulk density, porosity, and fluid density). However, the parameters that are directly related to the grain geometry and nature of the grain contacts are not well known (e.g., pore size parameter, mass factor, and elastic moduli of the frame) and their values are usually inferred from empirical data. Ogushwitz (1985) and Holland and Brunson (1988) in their studies discussed different methods (empirical and theoretical) to predict these parameters for natural sediment materials.

Murty and Pradeep Kumar (1989,1990) studied the applicability of sound speed models of Wood, Wyllie *et al.*, Nafe and Drake, Nobes *et al.*, and Biot-Stoll for predicting sound speeds in sediments of two environments - backwaters and continental shelf off Cochin. They compared the laboratory measured sound speeds with predicted values of each model. Biot-Stoll model showed good agreement for the entire range of sediments. Nobes *et al.* model fitted well with the laboratory data for carbonate rich sediments. Wood’s equation also agreed well with the sediments from backwaters
indicating that these sediments behave like suspensions and lack rigidity. In this study, Biot-Stoll model is considered for the prediction of sound speed in sediments. The details are discussed in Chapter 5.

2.2 Attenuation in sediments

When sound energy passes through saturated sediment, energy is lost through a number of mechanisms. Some of these such as friction between mineral grains and the relative movement of the mineral frame and pore fluid are fundamental to the material and are usually referred to as intrinsic attenuation. In the seabed itself other factors play a part in the absorption process. Gas bubbles, shells, boulders, and other inhomogeneities can produce losses through scattering. Energy conversion between compressional, shear and interface waves and multiple inter-bed reflections also introduce significant attenuation. The total of all losses is called “effective” attenuation and it is that to be considered in any practical situation.

An approach developed by Stoll (1956a, 1956b) assumes that two key mechanisms control the dynamic response of fluid-saturated sediments under insonification. One mechanism produces energy loss through intergranular friction at the contact area between particles of the frame and the other through the viscosity of the pore fluid. The effects of viscosity manifest themselves in two different ways depending on the permeability of the sedimentary material. They differ from those associated with intergranular friction in that the overall motion of the fluid relative to the skeletal frame of the sediment is predicted to result in a frequency-dependent damping.

Considering all these effects are real, the overall intrinsic damping in sediments will involve all the three mechanisms just described namely, friction, overall fluid motion, and local fluid motion. The first mechanism would lead to attenuation proportional to the first power of frequency. The two losses associated with the fluid are believed to produce attenuation that varies at $f^1$, $f^2$, or $f^{1/2}$, depending on the frequency range involved. Attenuation should thus vary in a complex manner when all
the mechanisms are combined. This proposed nonlinear relation between frequency and attenuation is a subject of the debate among researchers.

Major obstacles to the wider acceptance by the acoustical community of Biot-Stoll approach to sound propagation in sediments have been the lack of data to adequately verify the model's predictions and relatively small magnitude of the effects involved compared with the scatter in attenuation measurements. Moreover, individual measurement programs are usually confined to a narrow frequency range, a linear dependence of attenuation and frequency has often appeared justified for the limited data obtained.

Literature survey shows that the theories fall into one of the two broad groups. One considers the medium as a continuum with viscoelastic properties representative of the bulk material as a whole. In the second category wave propagation is assumed to depend upon the properties of individual constituents of the material and on structural characteristics of the skeleton. In this approach acoustical properties of the sediments are related to its observable physical properties.

2.2.1. Viscoelastic models

a) The Hamilton viscoelastic model

In his model Hamilton (1974a) assumes that sediments can be represented by an isotropic two-phase system composed of sediment grains and water. The mechanics of attenuation are not specified but provision is made for velocity dispersion and a nonlinear dependence of attenuation and frequency.

The basic derivation of this model leads to an equation for both the shear and compressional attenuation which, with appropriate changes in notation, is

\[
\frac{1}{Q} = \frac{aV}{\left(\pi f - \frac{a^2 V}{4\pi f}\right)}
\] (2.26)
where $1/Q$ is the specific attenuation factor, $\alpha$ is the attenuation coefficient, $V$ is the wave velocity ($V_p$ or $V_s$), and $f$ is the frequency.

When energy losses are small, the term $\alpha^2V^2/4\pi f$ is negligible and we obtain

$$\frac{1}{Q} = \frac{\alpha V}{\pi f} = \frac{2\alpha V}{\omega} = \frac{\Delta}{\pi} = \tan \theta$$  \hspace{1cm} (2.27)

Additionally,

$$\frac{1}{Q_p} = \tan \theta_p = \frac{\Delta_p}{\pi},$$  \hspace{1cm} (2.28)

$$\frac{1}{Q_s} = \tan \theta_s = \frac{\Delta_s}{\pi},$$  \hspace{1cm} (2.29)

$$\alpha_p = \frac{8.686\pi f}{Q_p V_p} dB/m$$  \hspace{1cm} (2.30)

$$\alpha_s = \frac{8.686\pi f}{Q_s V_s} dB/m$$  \hspace{1cm} (2.31)

where $\Delta$ is the logarithmic decrement (natural log of the ratio of the amplitudes of two successive cycles an exponentially decaying sine wave), $\theta$ is the loss angle, and $\alpha=8.686\alpha$ is the attenuation coefficient in decibels per unit length (dB/m). $V_p$ is the compressional wave speed, $V_s$ is the shear wave speed, and $Q_p$ and $Q_s$ are the specific attenuation factors for compressional and shear waves.

The wave speed, specific attenuation factor, and logarithmic decrement are independent of frequency if the attenuation coefficient is proportional to the first power of frequency. Otherwise, if $1/Q$ is independent of frequency, $\alpha$ will be linearly related
to that parameter. In recent literature (Mitchell and Focke, 1980; Kibblewhite, 1989) it is reported that Q is independent of frequency only in very dry rocks or over a restricted bandwidth in wet rocks. It has been demonstrated that pore fluids control attenuation in porous sands and sandstones and that the sound speed and attenuation of elastic waves in these media are dependent on pressure and the degree of saturation involved.

b. Kelvin-Voigt model

The homogeneous viscoelastic model generally recognized in connection with rocks and sediments is the solid model of Kelvin-Voigt. This is often represented by springs and viscous dashpot in parallel, an arrangement that leads to a viscoelastic relationship in which stress is not directly proportional to strain, as it is in Hookean elasticity, but is proportional to time rate of change of strain as well. Such models are rarely used because of the mathematical complexity involved (Kibblewhite, 1989).

2.2.2. Physical sediment models

In porous, permeable sediments, three mechanisms are generally considered to account for most of the observed response to acoustic waves: scattering, frictional losses at grain-grain contacts, and viscous losses due to relative motion between pore fluid and sediment frame.

a. Suspension models

In this approach, sediment is considered as a composite medium consisting of an emulsion of solid particles in a continuous liquid phase. The theory assumes that the medium has no frame rigidity. While successful in some applications, the model is not applicable in fluid-saturated media in which the skeletal frame must be attributed with appreciable rigidity (Kibblewhite, 1989). Later, skeletal effects are included in describing the interactions between the two components of a saturated porous medium. He formulated a “closed system” in which no pore fluid motion is allowed to take place. This model predicts sound speed in sediments if the moduli of the sediment
components are known. Attenuation due to frame friction has to be assessed by experimental observations.

b. Biot-Stoll sediment model

This approach to sediment modeling was discussed earlier (section 2.1.5). The initial Biot model assumed a perfectly elastic sediment frame. Stoll allowed for losses due to frictional effects at grain-grain contacts. The Biot-Stoll theory, despite its limitations, remains one of the most promising approaches to this complex problem.

Discrepancies between the literature data and predictions using Biot-Stoll Model (BSM) lead to the assumption that in unconsolidated fine-grained marine sediments, an additional absorption mechanism occurs that is activated during the excitation by compressional waves that is not covered by the BSM. Leurer (1997) introduced such a mechanism into the BSM. The formulation of the Effective Grain Model (EGM) is based on the assumption that the description of the grain material by a normally assumed constant real bulk modulus is not adequate in the case, in which the sediment has a significant clay fraction and swelling clay minerals are present. The grain material is therefore treated as an effective medium with an anelastic response to stress waves, described by a complex frequency-dependent bulk modulus. The associated relaxation mechanism consists of the stress-induced motion of the interlayer water into the pore space and its reentry into the interlayer space of the crystallite. This process includes the fluid motion caused by the squeezing of the thin interlayer-water films and can therefore be regarded as a squirt-flow process (Mavko and Nur, 1975), a phenomenon that is generally characterized by locally restricted fluid flow.

Swelling clay minerals to various amounts, encountered in nearly all clay-bearing marine sediments. In the case that the clay fraction of the sediment consists totally of nonswelling clay minerals the EGM is insignificant and the original BSM applies. Including the EGM leads to a comparatively good fit to existing literature data and is in agreement with the observed linear frequency dependence of the attenuation coefficient in the range of a few kHz to about 1MHz. The limitation of the applied distribution
causes the EGM curves to gradually converge with the curves of the BSM at lower frequencies and with those of Biot model at higher frequencies. Although the EGM showed comparatively good fit to existing data, a reliable test of its capability can only be carried out by further experiments on physically well-described sediment samples.

c. Buckingham’s Theory

Toksoz et al. (1979), Johnston et al. (1979), and Winkler and Nur (1982) experimentally identified some form of internal friction as being responsible for the characteristic attenuation exhibited by granular materials. Buckingham (1997) pointed out that the Biot-Stoll theory does not account for the characteristic attenuation in granular materials over an extended frequency range. The term characteristic attenuation was introduced by Buckingham (1997) to identify the component of attenuation that scales accurately as $f^4$ corresponding to a constant quality factor $Q$. In a series of publications he introduced a unified theory of sound propagation in saturated marine sediments on the basis of a linear wave equation. In his first publication, Buckingham (1997) included a new dissipation term representing internal losses arising from inter-particle contacts. In his paper Buckingham (1997) considered pore-fluid viscosity to be insignificant compared to inter-granular friction in saturated marine sediments. A new model of the mechanical properties of sediments is developed based on the idea of randomly packed rough mineral grains. When the mechanical model is coupled to the wave theory through a simple relationship between a frictional coefficient and the grain size, expressions are obtained that relate the acoustic properties (wave speed and attenuation) to the mechanical properties (grain size, density, and porosity). The resultant relationships between the acoustical and mechanical properties (e.g., sound speed and porosity) of marine sediments are shown to follow the trends of published experimental data sets very closely.

In his second paper of the series, Buckingham (1998) considered the theory of compressional and shear waves in fluid-like marine sediments in which medium is treated as a fluid that supports a dissipative rigidity, which is capable of supporting shear. This behavior is distinct from that of a viscous fluid, for which the shear
equation is diffusion like in character, giving rise to critically damped disturbances rather than propagating waves. Buckingham (1998) compared the predicted values of attenuation coefficient for both compressional and shear waves. He claims that the attenuation coefficient for both the compressional and shear wave is proportional to the first power of frequency, in accord with published data. However, the data used by Buckingham (1998) for the comparison mostly represented measurements above 1 kHz and he extended the fitted power line to the lower frequency end. Stoll (1985) reported that in the frequency range from 10-500 Hz, many new in situ measurements from different geologic settings fall well below the data used to justify the assumption of first power dependency. It is true in the case of Buckingham's comparison study also. Below 1 kHz, few data points shown in his comparison study indicate more deviation from the fitted line. So, Buckingham's conclusions on attenuation coefficient for both compressional and shear waves that scales with the first power of frequency is unacceptable in the case of entire frequency range. As the new theory needs more evaluation for predicting sound speed and attenuation in marine sediments this is not considered further in this study.

d. Relaxation time–attenuation model

Haumeder (1986) derived the relaxation time for a porous, fluid-filled material on the basis of the Biot theory. The porous material acts as a band-pass filter with respect to broad-band acoustic energy and the fraction of energy that is transferred into the interior of the porous material is used to drive the internal flow. For many frequencies this internal flow will follow the excitation by the pressure wave with a delay. Such a delay results in energy dissipation due to relaxation. Relaxation can be described best in terms of a characteristic response time, the so-called relaxation time, \( \tau \). The porous material acts as a band-pass filter with respect to broad-band acoustic energy, and the fraction of energy that is transferred into the interior of the porous material is used to derive the internal flow. For many frequencies this internal flow will follow the excitation by the pressure wave with a delay. Such a delay results in energy dissipation due to relaxation. Relaxation can be described best in terms of a
characteristic response time, the so-called relaxation time, $\tau$. Relaxation times are expressed entirely in terms of flow parameters, which give evidence that the underlying microscopic process is a flow process. With a knowledge of the relaxation time, the response of the attenuating medium can be completely characterized and hence the equation of Haumeder (1986) is opted for computing compressional wave attenuation in this study.

Haumeder (1986) derived equation for compressional wave attenuation coefficient,

$$\alpha = \frac{\omega}{V\sqrt{2}} \sqrt{-1 + \frac{1}{1 + \left[\omega \tau + \frac{F_i}{F_r}\right]^2} \left\{ \left[ K \omega^2 \tau^2 + \frac{F_i}{F_r} \right] \pm \sqrt{\left[ 1 + \left( \omega \tau + \frac{F_i}{F_r} \right)^2 \right] \left[ 1 + \left( \omega \tau + \frac{F_i}{F_r} \right) - K \omega \tau \right]^2} \right\} \right]}$$

(2.32)

where

$$K = \frac{n \rho_r}{\rho}$$

(2.33)

and

$$F(\kappa) = F_r(\kappa) + F_i(\kappa)$$

(2.34)

$$\tau = \frac{Dk}{n \eta} F_r$$

(2.35)

where $\rho$ is the bulk density, $n$ is the porosity in decimal fraction and $k$ is the coefficient of permeability of the porous frame with dimension (L$^2$). $\tau$ has the dimension of a time.
and can be identified as a relaxation time. This typical response time of the porous material is entirely expressed in terms of parameters that are descriptive of the internal flow of the pore fluid through the frame, i.e., permeability $k$, fluid density, viscosity, and the real part of the frequency correction factor $F_r$. $K$ is a dimensionless factor that is determined by the density ratio of the fluid and porous material. This factor introduces an altered frequency into the attenuation formula.

The above equation valid for all frequencies takes the form

$$
\alpha = \frac{\omega}{V \sqrt{2}} \sqrt{-1 + \frac{1}{1 + \omega^2\tau^2} \left\{ K\omega^2\tau^2 \pm \sqrt{[1 + \omega^2\tau^2][1 + \omega^2\tau^2(1 - K)^2]} \right\}}
$$

(2.36)

when $f \to 0$, $F_r/F_r \to 0$

For the case of a pure liquid ($n = 1$), $K$ equals one and the above equation takes the form of the well-known formula for relaxation attenuation in a viscous fluid:

$$
\alpha = \frac{\omega}{V \sqrt{2}} \sqrt{-1 + \frac{1}{1 + \omega^2\tau^2} \left\{ \omega^2\tau^2 \pm \sqrt{[1 + \omega^2\tau^2]} \right\}}
$$

(2.37)

This shows that the formula is capable of producing proper results in the limit $n = 1$. The derivation of complex correction factor is valid only for frequencies where the wave length is large compared with the pore size. For sands, this puts the upper limit on frequencies at about $10^5$ to $10^6$ Hz (Haumeder, 1985).

2.2.3. Effect of temperature on sound speed and attenuation in marine sediments

Rajan and Frisk (1992) conducted a study on the variation of compressional wave speed in sediments due to the temperature variability in the water column of the shallow waters of Gulf of Mexico. They used temperature data collected at different
seasons and at the same location for the study. It was hypothesized that heat flow from the bottom of the water column into the sediment affects the sediment pore water temperature, thereby influencing the temperature structure and thus the compressional wave speed in the sediments. Since this heat flux varies with season, the effect on sediment wave speed should also change seasonally.

In the shallower depths (< 30m) of the Gulf of Mexico, seasonal fluctuations in ocean bottom temperature as great as 15°C have been observed (Rajan and Frisk, 1992). They investigated the heat flux across the water/sediment interface and using Biot-Stoll model for the sediments, assessed its effects on the compressional wave speed in the sediment layers. They reported that the compressional wave speed varies approximately linearly with pore water temperature, independent of both the porosity and sediment type. The study showed that temperature induced variations in the bottom compressional speeds can have important effects on the prediction of the pressure field in the water column (especially at higher frequencies), and also on source localization schemes like matched-field processing.

Ali (1993) made a study on the oceanographic variability in shallow water acoustics and the dual role of the sea bottom. He reported that, while sea bottom plays a significant part in degrading a waterborne signal, it could also provide an additional seismic path for the propagation of sound—particularly at very low frequencies.

Due to the variability in oceanographic conditions, density of pore water (seawater at the ocean bottom) varies with the change in temperature and salinity of the bottom water. As the saturated bulk density of the sea floor sediment depends on the pore water density for a given porosity, density of the sediment also indicates corresponding variation. Shumway (1958) reported that in seawater of 35 PSU salinity, density changes by about 6% when temperature varied between 0°C and 100°C. In quartz, the variation in density is 0.36% when temperature varies between 20°C and 100°C, while in calcite density changes by 0.08% in the same temperature range. Accordingly, sound speed in sediment also varies with the change in oceanographic conditions of the bottom water.
A number of laboratory and *in situ* studies of sound speed in unconsolidated water saturated sediments have been reported in literature (Hamilton, 1963, 1970, 1972; Fu et al., 1996, Best et al., 1998). But very few sound speed measurements at different temperatures have been reported (Sutton et al., 1957, Shumway, 1958, Leroy et al., 1986).

Sutton *et al.* (1957) made sound speed measurements at different temperatures on two samples and the effect of temperature on compressibility was reported. Shumway (1958) measured sound speed as a function of temperature in different types of sediments using the resonant chamber technique. He concluded that temperature effect on these water-saturated sediments was approximately the same as that for water alone and is due to the large water compressibility dominating over the water-sediment mixture. The temperature effect on water compressibility is considerable whereas it is small for calcite and quartz.

Leroy *et al.* (1986) made laboratory measurements of sound speed against temperature variation. They reported a variation of about 150 m/s (1725 to 1875 m/s) in sound speed in sand for a temperature variation of 60°C (5 to 65°C).

Pradeep Kumar and Murty (1993) in their study, considered the effect of variation of temperature on pore water density and sediment density for computing sound speed in sediment using Biot-Stoll model. The data on the variability of the temperature at the sea bottom is obtained from the monthly hydrocast and mechanical bathythermograph (BT) data of 12 months collected from the same location, off Cochin. The temperature of the bottom water 1 meter above the sea floor is taken as the pore water temperature at the time of measurement.

Pradeep Kumar (1997) studied the seasonal variation of relaxation time and attenuation in sediment at the sea bottom interface based on Biot-Stoll and Haumender models. Biot-Stoll model is used for computing sound speed in marine sediments and
relaxation time-attenuation model (Haumender, 1986) is used for computing attenuation.

Pradeep Kumar (1997) in his study included the effect of variation of temperature on the viscosity of pore water to account for the variability in viscous loss. Viscosity of seawater decreases rapidly and non-linearly with temperature rise (Sverdrup et al., 1942; Dera, 1992). Increasing salinity raises the viscosity of seawater slightly. The relationship between viscosity and pressure is a more complex one, highly dependent on the temperature and salinity (Dera, 1992). The pressure rise at low temperatures slightly reduces the viscosity; but this reduction in viscosity is only slight. For estimating viscosity of pore water at the bottom temperature, the data given for different values of the temperature at salinity of 35 PSU by Sverdrup et al. (1942) are used. Effect of variation of salinity and pressure on viscosity is very small and hence neglected in this study. The viscosity of the sea bottom water is computed by fitting the following equation to the Sverdrup et al. data. The fitted equation (Pradeep Kumar, 1997) is

\[ n_{t,35} = 0.0183534e^{-0.02574t} \]

(2.38) here \( n_{t,35} \) is the viscosity of sea water at a temperature \( t \), and salinity 35 PSU.

In this study, Haumender’s (1986) equation is used to compute the variation in compressional wave attenuation due to the change in bottom water temperature and the above equation is used for estimating pore water viscosity. It is also hypothesized that heat flow from the bottom of the water column into the sediment affects the sediment pore water temperature, thereby influencing the temperature structure, pore water viscosity and thus the compressional wave speed and attenuation in the sediments. Details of the study are included in Chapter 6.