Abstract

One of the most important and challenging problems in scientific and engineering applications is to find the solution of the nonlinear equations. Analytical methods for such equations rarely exist and therefore we can only hope to obtain approximate solutions by relying on numerical methods based on iteration procedures. Newton method is probably the best known iterative method for solving nonlinear equations. In recent years, several modifications of Newton method have been proposed and analyzed, which have either equal or better performance than Newton method. The present thesis deals mainly with the development of iterative methods with improved order and efficiency to solve nonlinear equations and systems of nonlinear equations. Work of the thesis is divided into 6 chapters. The main contents of each chapter are furnished in the following text.

In chapter 1, we give brief explanation about the need of iterative methods in scientific and engineering problems. Some classical methods are introduced, their merits and demerits are discussed. The basic definitions and classification of iterative methods are presented. Some basic concepts and definitions regarding multiple roots and systems of nonlinear equations are introduced. The important features of one-point and multipoint methods are stated. The various techniques, which are used by researchers to generate higher order iterative methods such as functional approximation, sampling, composition, geometric and Adomian approaches, are presented. Lastly, the chapter ends with the summery of the main results embodied in the thesis.

In chapter 2, an attempt is made to develop a unified scheme to obtain one-point methods without memory with at least cubic convergence. Our approach is based on a simple modification of Newton method. The scheme is powerful and interesting since it generates almost all available one-point third order methods in literature. Moreover, many new methods can be generated. The convergence analysis is provided to establish third order convergence of the proposed scheme. In order to support the theory, we present some numerical tests and compare the methods of proposed family with Newton method and existing third order methods.

In chapter 3, some multipoint methods of third and fourth order convergence are presented by a variety of techniques. First, we derive a two-parameter family of methods. All the methods of the family have third order convergence, except for one set of values of parameters, for which it has the fourth order convergence. In terms of computational cost,
all these methods require evaluations of one function, one first derivative and one second derivative per iteration. Then, we propose some optimal fourth order methods, which do not require the evaluation of second derivative. Per iteration, the methods require either two functions and one first derivative evaluations or one function and two of first derivative evaluations. Numerical tests are performed and the proposed methods are compared with well-known existing methods.

In chapter 4, the multipoint methods of higher order viz. fifth, sixth, seventh and eighth order are developed. The methods are constructed either by composition of three Newton-like steps or by composition of fourth order methods with Newton-like method. The fifth and sixth order methods require two evaluations of the given function and two evaluations of its first derivative, whereas the seventh and eighth order methods require three evaluations of function and one evaluation of first derivative. In order to check the validity of theoretical results some numerical tests are performed and the performance is compared with existing methods. It is observed that present methods, particularly eighth order methods, show good stability and robustness.

Chapter 5 deals with the iterative methods for computing multiple roots. The proposed methods are generalization of either known methods or newly developed methods for simple roots. Firstly, we present the families of third order methods with and without the evaluation of second derivative. The families include many well-known methods as special cases. Then, we present some optimal fourth order methods, which require one evaluation of function and two evaluations of first derivative. Numerical examples are given to support that the methods thus obtained are competitive with existing methods.

In chapter 6, which is also the concluding chapter of the thesis, some iterative methods to solve systems of nonlinear equations are derived. Recently, many third order iterative methods have been derived and analyzed for systems of nonlinear equations that do not require the computation of second Fréchet derivative. With this aim, first we extend two families of third order methods for simple roots given by Traub [Traub, J.F. 1977. Iterative Methods for the Solution of Equations. Chelsea Publishing Company, New York] to systems of equations. Next, we propose a fourth order method which is based on the composition of two weighted-Newton steps. Then, a sixth order one-parameter family of methods is presented. The proposed methods are compared with the existing methods by performing numerical experiments and by way of graphical presentations.