6.1 Introduction

It is supposed to be a fact that no equipment or a system is perfect in the sense that it will continue to function (without failure) forever, how so ever carefully it might have been designed and manufactured. However it is assumed that the reliability of an equipment or a system may be increased by proper maintenance at regular intervals. Such maintenance is known as Preventive maintenance. It is done periodically, before the failure of the system; hence it is different from the corrective or repair maintenance, which is carried out only after the failure of the item or the system.

In this chapter we examine the effect of preventive maintenance on the reliability of an item that functions until first failure. For this purpose we consider some well-known lifetime distributions of such items. 

In section 6.2 we consider the effect of preventive maintenance on the reliability of an item that follows an exponential failure time distribution. It is shown that the preventive maintenance does not improve the reliability of such an item. We prove that the mean time to failure (MTTF) of such an item is equal to mean lifetime without maintenance and establish it as a characteristic property of exponential failure time distribution. Further definition of MTTF, see section 1.2. In section 6.3 we consider the Power function distribution and obtain the condition under which maintenance reliability exceeds the reliability without maintenance. In section 6.4 we discuss the effect of preventive
maintenance with respect to some other distributions along with a general discussion on preventive maintenance.

6.2 Reliability For Exponential Distribution under Preventive Maintenance

It is well known that exponential distribution has constant failure rate and this is the characteristic property of it. Now we consider the case of reliability under preventive maintenance of equipment following exponential distribution. If the pdf of failure time $T$ is given by

$$f(t) = \lambda e^{-\lambda t}, \quad t > 0$$

(6.2.1)

then reliability of such an equipment is given by

$$R(t) = e^{-\lambda t}, \quad t > 0.$$  \hspace{1cm} (6.2.2)

Using the result on maintenance reliability given in section 1.2, we find the reliability of that equipment with regular preventive maintenance at time $T, 2T, 3T, \ldots$, given by

$$R_M(t) = (e^{-\lambda T})^n e^{-\lambda (t-nT)}$$

for $nT \leq t < (n+1)T$

$$= e^{-\lambda t} = R(t)$$

for $0 \leq t < \infty$

Note that this does not depend on the number of preventive maintenance. We conclude that preventive maintenance does not improve the reliability of equipment having exponential failure distribution. We now prove an important result in the form of a theorem.

**Theorem 6.2.1:** Preventive maintenance does not improve the reliability of an equipment or system iff it has a constant failure rate.

**Proof:** (N) Let us suppose that the reliability does not improve after preventive maintenance or mean time to failure (MTTF) is a constant
with respect to maintenance after time $T$, i.e.

$$MTTF = \int_0^T R(t) dt = \frac{\alpha}{1 - R(T)} = \alpha \quad \text{for all } T > 0.$$ 

or

$$\int_0^T R(t) dt = \alpha \{1 - R(T)\}$$

differentiating both the sides w.r.t. $t$, we get

$$R(T) = -\alpha R'(T)$$

$$\Rightarrow \quad \frac{R'(T)}{R(T)} = -\frac{1}{\alpha}$$

$$\Rightarrow \quad \ln(R(T)) = -\frac{T}{\alpha} + c$$

$$\Rightarrow \quad R(T) = e^{-\left(\frac{T}{\alpha}\right) + c} \quad T \geq 0$$

where $c$ is the constant of integration. However, $R(0) = 1 \Rightarrow c = 0$. Thus

$$R(T) = e^{-T/\alpha}, \quad T > 0$$

which is the reliability at time $T$, of an item following Exponential distribution. Hence it shows that equipment with constant MTTF follows Exponential distribution.

**(S):** Now we consider that equipment follows exponential distribution and has the reliability $R(t) = e^{-t/\alpha}$. Thus the MTTF with respect to maintenance after time $T$ is given by

$$MTTF = \int_0^T e^{-t/\alpha} dt$$

$$= \frac{\alpha(1 - e^{-T/\alpha})}{1 - e^{-T/\alpha}}$$

$$= \alpha$$

This proves the theorem.
6.3 Reliability for the power function distribution under preventive maintenance

The reliability function for power function distribution is given by

\[ R(t) = \frac{k^a - t^a}{k^a}, \quad 0 < t \leq k. \]

Reliability of such an item under preventive maintenance, is given by

\[ R_M(t) = \{R(t)\}^n [R(t-nT)] \]

\[ = \left( \frac{k^a - t^a}{k^a} \right)^n \left[ \frac{k^a - (t-nT)^a}{k^a} \right], \quad nT \leq t < (n+1)T \]

In order that the reliability under preventive maintenance be more than that of without maintenance, we must have \( \{R_M(t) / R(t)\} > 1 \) at the time of preventive maintenance \( t = nT \), where \( n = 1,2,3, \ldots, \) i.e.

\[ \frac{R_M(nt)}{R(t)} = \left[ \frac{1 - \frac{T^a}{k^a}}{1 - \frac{(nT)^a}{k^a}} \right] > 1 \]

\[ \Rightarrow 1 - \frac{nT^a}{k^a} > 1 - \left( \frac{nT}{k} \right)^a \]

\[ \Rightarrow \left( \frac{nT}{k} \right)^a - nT^a > 0 \]

\[ \Rightarrow n^a - n > 0 \]

\[ \Rightarrow n^{a-1} > 1 \quad \Rightarrow a - 1 > 0 \quad \Rightarrow a > 1. \]

This means that preventive maintenance will improve the reliability of the power function system, only when \( a > 1. \)

It simply means that for \( a < 1, \) preventive maintenance may not be useful.

To get a better insight into this result, we have tables 6.3.1 to 6.3.4.
showing $R(t)$, reliability without maintenance and $R_M(t)$, reliability with maintenance for selected values of $a$. Without loss of generality let $k = 1$

**Table 6.3.1:** Showing $R(t)$ and $R_M(t); T = 0.25, a = 0.5$

<table>
<thead>
<tr>
<th>$t$</th>
<th>0.1</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.75</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(t)$</td>
<td>0.6838</td>
<td>0.5000</td>
<td>0.4523</td>
<td>0.3675</td>
<td>0.2929</td>
<td>0.2254</td>
<td>0.1339</td>
<td>0.1056</td>
<td>0.0513</td>
<td>0</td>
</tr>
<tr>
<td>$R_M(t)$</td>
<td>0.6838</td>
<td>0.5000</td>
<td>0.3882</td>
<td>0.3064</td>
<td>0.25</td>
<td>0.1709</td>
<td>0.125</td>
<td>0.0970</td>
<td>0.0766</td>
<td>0.0625</td>
</tr>
</tbody>
</table>

**Table 6.3.2:** Showing $R(t)$ and $R_M(t); T = 0.25, a = 1$

<table>
<thead>
<tr>
<th>$t$</th>
<th>0.1</th>
<th>0.2</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.75</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(t)$</td>
<td>0.9</td>
<td>0.8</td>
<td>0.75</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.25</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>$R_M(t)$</td>
<td>0.9</td>
<td>0.8</td>
<td>0.75</td>
<td>0.7125</td>
<td>0.6375</td>
<td>0.5635</td>
<td>0.5063</td>
<td>0.45</td>
<td>0.4008</td>
<td>0.3586</td>
<td>0.3164</td>
</tr>
</tbody>
</table>

**Table 6.3.3:** Showing $R(t)$ and $R_M(t); T = 0.25, a = 2$

<table>
<thead>
<tr>
<th>$t$</th>
<th>0.1</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.75</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(t)$</td>
<td>0.99</td>
<td>0.91</td>
<td>0.84</td>
<td>0.75</td>
<td>0.64</td>
<td>0.51</td>
<td>0.44</td>
<td>0.36</td>
<td>0.19</td>
<td>0</td>
</tr>
<tr>
<td>$R_M(t)$</td>
<td>0.99</td>
<td>0.9375</td>
<td>0.9352</td>
<td>0.9164</td>
<td>0.8789</td>
<td>0.8701</td>
<td>0.8438</td>
<td>0.8219</td>
<td>0.8054</td>
<td>0.7724</td>
</tr>
</tbody>
</table>

**Table 6.3.4 (a):** Showing $R(t)$ and $R_M(t); T = 0.25, a = 3$

<table>
<thead>
<tr>
<th>$t$</th>
<th>0.1</th>
<th>0.25</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6</th>
<th>0.75</th>
<th>0.8</th>
<th>0.9</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(t)$</td>
<td>0.999</td>
<td>0.984</td>
<td>0.973</td>
<td>0.936</td>
<td>0.875</td>
<td>0.784</td>
<td>0.578</td>
<td>0.488</td>
<td>0.271</td>
<td>0</td>
</tr>
<tr>
<td>$R_M(t)$</td>
<td>0.999</td>
<td>0.9844</td>
<td>0.9842</td>
<td>0.9811</td>
<td>0.9689</td>
<td>0.9680</td>
<td>0.9612</td>
<td>0.9537</td>
<td>0.9506</td>
<td>0.9389</td>
</tr>
</tbody>
</table>

**6.4 Reliability Under Preventive Maintenance for other lifetime Distributions**

In this section we derive the expression for maintenance reliability for some important life-time distributions.
(i) **Weibull Distribution:** The pdf of Weibull distribution is given by

\[ f(t) = \frac{\beta}{\theta} \left( \frac{t}{\theta} \right)^{\beta-1} \exp \left[ -\left( \frac{t}{\theta} \right)^\beta \right], \quad \theta, \beta > 0, \quad t \geq 0. \]

The reliability function for this distribution is given by

\[ R(t) = \exp \left[ -\left( \frac{t}{\theta} \right)^\beta \right]. \]

Reliability under preventive maintenance of such equipment or system is given by

\[ R_M(t) = \left[ \exp \left\{ -\left( \frac{T}{\theta} \right)^\beta \right\} \right]^n \exp \left[ -\left( \frac{t-nT}{\theta} \right)^\beta \right]. \]

In order to see the effect of preventive maintenance we have to find \( R_M(t) / R(t) \) at the time of preventive maintenance \( t = nT \).

\[
\frac{R_M(nt)}{R(nT)} = \frac{\exp \left[ -n \left( \frac{T}{\theta} \right)^\beta \right]}{\exp \left[ -\left( \frac{nT}{\theta} \right)^\beta \right]} > 1 \Rightarrow \beta > 1.
\]

This means that the preventive method is effective for the Weibull system, if the shape parameter, \( \beta > 1 \).

(ii) **Normal Distribution:** The pdf of normal distribution is given by

\[ f(t) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{t-\mu}{\sigma} \right)^2 \right], \quad \mu, \sigma > 0. \]

There is no closed-form solution for the normal reliability function. Solutions can be obtained via the use of standard normal tables. Thus,
\[ R(t) = 1 - \Phi \left( \frac{t - \mu}{\sigma_T} \right) \]

Reliability under preventive maintenance

\[ R_M(t) = \left[ 1 - \Phi \left( \frac{T - \mu}{\sigma_T} \right) \right]^n \left[ 1 - \Phi \left( \frac{t-nT - \mu}{\sigma_T} \right) \right]. \]

(iii) **Gamma Distribution:** The pdf of Gamma distribution is given by

\[ f(t) = \frac{t^{\gamma-1}e^{-t}}{\Gamma(\gamma)}, \quad t \geq 0; \gamma > 0. \]

where \( \gamma \) is the shape parameter, and \( \Gamma \) is the gamma function given by the relation

\[ \Gamma(a) = \int_0^\infty t^{a-1}e^{-t} \, dt. \]

There is no closed-form of reliability function for Gamma distribution also. Thus the reliability function is given by

\[ R(t) = 1 - \frac{\Gamma_t(\gamma)}{\Gamma(\gamma)}, \quad t \geq 0; \gamma > 0 \]

where \( \Gamma \) is the gamma function defined above and \( \Gamma_t(a) \) is the incomplete gamma function given by the relation

\[ \Gamma_t(a) = \int_0^t t^{a-1}e^{-t} \, dt. \]

We can obtain the solutions by using the table of Incomplete Gamma Distribution given by Pearson (1957).
Reliability under preventive maintenance for gamma distribution is given by

\[ R_M(t) = \left[ 1 - \frac{\Gamma_T(\gamma)}{\Gamma(\gamma)} \right]^n \left[ 1 - \frac{\Gamma_{t+nT}(\gamma)}{\Gamma(\gamma)} \right]. \]

No general comment is possible on the behaviour of the reliability under preventive maintenance for Normal and Gamma distributions.