CHAPTER I

INTRODUCTION

This chapter is an exhaustive review of the literature on fuzzy sets, intuitionistic fuzzy sets, ranking of fuzzy numbers, fuzzy graphs, hypergraphs, directed hypergraphs and fuzzy hypergraphs. For intuitionistic fuzzy hypergraphs (both directed and undirected) we use hypergraph techniques intertwined with intuitionistic fuzzy graphs.

1.1 FUZZY SETS

In classical set theory, membership of an object belonging to a set can only be one of the two values 0 or 1. An object either completely belongs to a set or does not at all. No partial membership is allowed.

Crisp sets handle black and white concepts such as Yes or No, True or False where little ambiguity exists. Nevertheless, in our daily lives, there exist countless vague concepts that human can easily describe, understand and communicate with each other; but conventional mathematics, including the set theory, fails to handle in a rational way. For example, the concept young, for any specific person, age is precise. However, relating a particular age to young involves fuzziness and is sometimes difficult.
A fundamental problem is the abrupt change of the membership value from 1 to 0 at a certain age, (35 in this case), which means that a 34.9 years old person is completely young whereas a 35.1 years old person is not young at all. Hence, imprecision plays an important role in information representation in real process where increase in precision would otherwise become unmanageable. To address issues like this, fuzzy set theory, invented by Lotfi A Zadeh [99, 100] in the year 1965, generalizes 0 and 1 membership values of a crisp set to a membership function of a fuzzy set. Using the theory, one relates an age to young with a membership value ranging from 0 to 1; 0 means no association at all, 1 indicates complete association and any number in between means partial association.

Fuzzy sets and fuzzy logic form the foundations for fuzzy mathematics, which may be viewed as an extension of the traditional mathematics subjects like logic, topology, algebra, analysis etc. to pattern recognition, information theory, artificial intelligence, operations research, neural networks, planning etc. Consequently, fuzzy set theory has emerged as a potential area of interdisciplinary research.

The theory of fuzzy set has achieved successful applications [11, 45] in various fields. This is because this theory is an extraordinary tool for representing human knowledge, perception, and so forth. Two years after the concept of fuzzy set was proposed, it was generalized and developed as L - fuzzy set by Gogeun. In 1973, Zadeh established knowledge representation by means of some generalizations of
fuzzy sets. The so-called extensions of fuzzy set theory arise in this way. Cornelis et al. [27] highlighted the applications of fuzzy techniques in image processing and also reviewed the developments of fuzzy set theory and intuitionistic fuzzy set theory over the past 35 years.

Ranking of fuzzy numbers is an important issue in the study of fuzzy set theory. In order to rank fuzzy numbers, one fuzzy number needs to be compared with the others. Because of the importance and applicability of the ranking process, several methods [10, 20, 24, 26, 28, 31, 53] have been developed. Fuzzy numbers are approximated by real numbers, real intervals, triangular or trapezoidal fuzzy numbers.

The first definition of fuzzy graph was given by Kauffman [47] in 1973, based on Zadeh’s fuzzy relations [100]. But Rosenfeld [83] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975. During the same time, Yeh and Bang have also introduced various connectedness concepts in fuzzy graphs.

Fuzzy graph theory is now finding numerous applications in modern science and technology especially in the fields of neural networks [14], expert systems [40], cluster analysis [59], medical diagnosis [90], etc. Fuzzy modeling is an essential tool in all branches of science, engineering and medicine. Fuzzy models give more
precision, flexibility and compatibility to the system when compared to the classic models [99, 100].

Rosenfeld obtained the fuzzy analogues of several basic graph-theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties [83]. Bhattacharya [15] established connectivity concepts regarding fuzzy cutnodes and fuzzy bridges. He also introduced fuzzy groups and metric notion in fuzzy graphs. Bhutani [17] studied automorphisms on fuzzy graphs and certain properties of complete fuzzy graphs. Bhattacharya and Suraweera [16] introduced an algorithm to find the connectivity of a pair of nodes in a fuzzy graph. Saibal Banerjee [84] developed an optimal algorithm to find the degrees of connectedness in a fuzzy graph. Fuzzy intersection graphs were introduced by McAllister [63] and fuzzy line graphs by Mordeson [65].

Fuzzy trees were characterized by Sunitha and Vijayakumar [91]. A sufficient condition for a node to be a fuzzy cutnode is also established in [91]. Center problems in fuzzy graphs [94], blocks in fuzzy graphs [92] and properties of self complementary fuzzy graphs [93] were also studied by the same authors. They derived a characterization for blocks in fuzzy graphs using the concept of strongest paths. Bhutani and Rosenfeld [19] introduced the concepts of strong arcs and fuzzy end nodes in fuzzy graphs [18]. Mordeson and Peng [67] discussed operations

Shortest path problem in a network is a great deal of applications such as routing, communication and transportation. The fuzzy shortest path problem was introduced by Dubois and Prade [33]. Klein [50] proposed a dynamical programming recursion-based fuzzy algorithm. Lin and Chen [57] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Okada [77,78] defined a new comparison index between the sums of fuzzy numbers by considering interactivity among fuzzy numbers and presented an algorithm to determine the degree of possiblity for each arc on a network. Yao and Lin [98] developed two types of fuzzy shortest path network problems.

1.2 INTUITIONISTIC FUZZY SETS

A membership function in fuzzy sets assign a number from the unit interval to each element in a universe of discourse to indicate the degree of belongingness to the set under consideration. The degree of non belongingness in fuzzy sets is simply the complement to 1 of the membership degree. However, human who express the degree of membership of a given element in a fuzzy set very often do not express a corresponding degree of nonmembership as the complement to 1. Thus, Atanassov [4] introduced the concept of an intuitionistic fuzzy set (IFS), which is a generalization of a fuzzy set. Since an IFS can present the degrees of both membership and nonmembership with a degree of hesitancy, knowledge and semantic representation becomes more meaningful and applicable [4, 5, 6].

The intuitionistic fuzzy set theory has received more and more attention since its appearance. In 1993, Gau and Buehrer [39] introduced the concept of vague sets which is another generalization of fuzzy sets. On its foundation, Hong and Choi [41] researched the fuzzy multi-criteria decision making problem based on vague sets. Bustince and Burillo [23] pointed out that the notion of vague set is the same as that of IFS.

Among various extensions of fuzzy sets, IFSs have captured the attention of many researchers in the last few decades. This is mainly due to the fact that IFSs have been widely studied and applied in various areas such as logic programming
and reasoning [46], decision making problems [7, 41, 58], medical diagnosis [30], pattern recognition [42, 55, 56] and clustering [80, 97, 101].

In fact, interval-valued fuzzy graphs and intuitionistic fuzzy graphs (IFGs) are two different models that extend theory of fuzzy graphs. The motivation for introducing IFGs is due to [4, 5, 87]. The first definition and concept of intuitionistic fuzzy graph was introduced by Krassimir T. Atanassov [86]. Nagoor Gani and Shajitha Begum discussed the various types of degrees and some properties of IFGs [70]. R.Parvathi and M.G.Karunambigai analyzed the properties of minmax IFGs in [48, 79]. Shannon and Atanassov [4, 5, 87] defined intuitionistic fuzzy graphs using five types of Cartesian products. Atanassov introduced the index matrix representation of IFG [3]. Nagoor Gani, et al., discussed the properties of isomorphism on fuzzy graphs and properties of isomorphism on strong fuzzy graphs in [68, 69]. M. Akram and Bijan Davvaz discussed the properties of strong intuitionistic fuzzy graphs [1] and also they introduced the concept of intuitionistic fuzzy line graphs.

1.3 RANKING FUNCTIONS

Nan [71], Burillo et al [22] defined intuitionistic fuzzy number (IFN). Shu et al. [88] gave the definition and operational laws of triangular intuitionistic fuzzy number (TriIFN) and proposed an algorithm of the intuitionistic fuzzy fault tree analysis. Wang and Zhang [96] proposed the trapezoidal intuitionistic fuzzy num-
ber (TraIFN) which is an extension of TriIFN and also defined their operational laws. B.S.Mahapatra and G.S.Mahapatra [61] presented intuitionistic fuzzy fault tree analysis using TraIFN.

On the front of ranking intuitionistic fuzzy numbers, some work has been reported in the literature. Mitchell [64] considered the problem of ranking a set of intuitionistic fuzzy numbers to define a characteristic vagueness factor for each intuitionistic fuzzy number. Also, he extended the natural ordering of real numbers to triangular intuitionistic fuzzy numbers (TriIFN) by adopting a statistical viewpoint and interpreting each intuitionistic fuzzy number as an ensemble of ordinary fuzzy numbers.

Very recently, Nehi [73] put forward a new ordering method for IFNs in which two characteristic values for IFNs are defined by the integral of the inverse fuzzy membership and nonmembership functions multiplied by the grade with powered parameter. Almost parallel, Li [54] introduced a new definition of the TriIFN which has an appealing and logically reasonable interpretation. He defined two concepts, value and the ambiguity of a TriIFN, similar to those for a fuzzy number introduced by Delgado et al. [32].

The ranking of intuitionistic fuzzy number plays a main role in modeling many real life problems involving intuitionistic fuzzy decision making, intuitionistic fuzzy clustering. V. Lakhsmanagomathinayagam et al., [72] introduced the new
intuitionistic fuzzy scoring method which has been applied to clustering problem where the data collected is in terms of intuitionistic fuzzy linguistic term and was converted into intuitionistic fuzzy number.

1.4 HYPERGRAPHS AND DIRECTED HYPERGRAPHS

The concept of hypergraphs was introduced by Berge [12] and has been considered as a useful tool to analyze the structure of a system and to represent a partition, covering and clustering.

Hypergraphs are not as common as graphs, but they do arise in many application areas. In relational databases, there is a natural correspondence between database schemata and hypergraph, with attributes corresponding to vertices and relations to hyperedges [35]. Hypergraphs are used in VLSI design for circuit visualization [34, 85] and also appear in computational biology [60, 81] and social networks [21].

Directed hypergraphs (Ausiello et al.,[8]; Gallo et al.,[37]) are a generalization of directed graphs (digraphs) and they can model binary relations among subsets of a given set. Such relationships appear in different areas of computer science such as database systems (Ausiello et al.), expert systems (Ramaswamy et al.)[82], parallel programming (Nguyen et al.)[74], Scheduling (Lin and Sarrafzadeh, Gallo and Scutella) [38], routing in dynamic networks (Pretolani)[29] and data mining
Subfamilies of directed hypergraphs, as defined in Gallo et al., [37], can be associated with same earlier definitions, as the one presented in Ausilo et al.,[8]. Such subfamilies can be defined as B-graph, F-graph and BF-graph. A digraph is a particular case of BF-graphs.

The visual representation of a hypergraph is as important as the same problem for graphs and digraphs. Makinen [62] introduced two notions of hypergraph drawing based on methods for describing a hypergraph, the subset standard and the edge standard. The first one uses the fact that a hypergraph is a collection of subsets which can be viewed as a Venn diagram. Bertault and Eades [13] presented a drawing system that focuses on the representation of hypergraphs with this standard. In the edge standard, a hyperedge $e$ is represented by connecting the points that represent the vertices that define $e$ by curve lines which must intersect in a unique point, emphasizing the image of a unique edge.

The edge standard is the best choice for directed hypergraphs, as one can draw the hyperedges as two sets connected by lines. In fact this pictorial representation has been used in almost all papers related to the subject.

Directed hypergraphs have a very large number of applications, since hyperedges naturally provide a representation of implication dependencies. Among others [9,
36, 85] were used to solve several problems related to satisfiability in propositional logic, in particular relative to Horn formulas. They also appear in problems relative to network checking [29], chemical reaction networks and more recently algorithmics of convex polyhedra in tropical algebra.

Many algorithmic aspects of directed hypergraphs relating to optimization such as determining shortest paths [75, 76], maximum flows, minimum cardinality cuts or minimum weighted hyperpaths have been studied.

Unfortunately, none of the directed graph algorithms can be extended to directed hypergraphs. The main reason is that the reachability relation of hypergraphs does not have the same structure.

1.5 FUZZY HYPERGRAPHS

The notion of hypergraph has been extended to the fuzzy theory and the concept of fuzzy hypergraph was proposed by Kaufmann [47]. Hyung Lee-Kwang et al.,[52] generalize the concept of fuzzy hypergraph. The proposed fuzzy hypergraphs and concepts can be used to represent and characterize various fuzzy systems. Chen introduced the concept of interval valued fuzzy hypergraphs.

Mordeson and Premchand Nair [66] developed the concept of fuzzy hypergraphs and several analogs of hypergraph theory.
1.6 ORGANIZATION OF THE THESIS

Chapter 1 is a brief overview of the contents of the report, that is, structure and summaries of the manuscripts included in this work. Chapter 2 proposes a standard definition of intuitionistic fuzzy directed graphs and its index matrix representation. Operations on IFDGs are also defined.

On the foundation of the theory of fuzzy hypergraph, the third chapter of the research work extends the traditional research. The definitions of intuitionistic fuzzy hypergraph (IFHG), dual intuitionistic fuzzy hypergraph (DIFHG), \((\alpha, \beta)\) - cut, strength of an edge have been given and they are illustrated with suitable examples. The strength of an edge can be used to decompose the data set in a clustering problem.

Chapter 4 investigates and proposes the operations on IFHGs. Based on the definition of IFHGs given in chapter 2, the operations like complement, join, union, intersection, ringsum, cartesian product, composition are defined and illustrated with suitable examples.

Chapter 5 leads to an extension of intuitionistic fuzzy hypergraph into intuitionistic fuzzy directed hypergraphs. Hypergraphs (Directed hypergraphs) are always represented by the incidence matrix. Based on Atanassov’s index matrix representation of intuitionistic fuzzy relation IFDHGs are defined. This is anotherway of representing IFDHGs. Chapter 5 also includes the operations like
addition, vertexwise multiplication, multiplication and structural subtraction on IFDHGs which are defined in a new form.

Chapter 6 deals with isomorphism on IFDHGs. It is also proved that (i) for any two isomorphic IFDHGs, their order and size are same, (ii) if $G$ and $G'$ are isomorphic IFDHGs then the degree of their vertices are preserved. (iii) isomorphism between IFDHGs preserves equivalence relation.

A new method namely, the score based method for finding shortest hyperpath in a network with intuitionistic fuzzy weights for hyperedges without defining similarity measure and Euclidean distance is proposed and discussed in Chapter 7. The scores of TriIFNs and ranking the paths based on lowest accuracy helps the decision maker to identify the preferable intuitionistic fuzzy shortest hyperpath.

The thesis ends with few suggestions for future work and conclusion.